Lecture 7: Dynamic Games
Backwards Induction and SPNE

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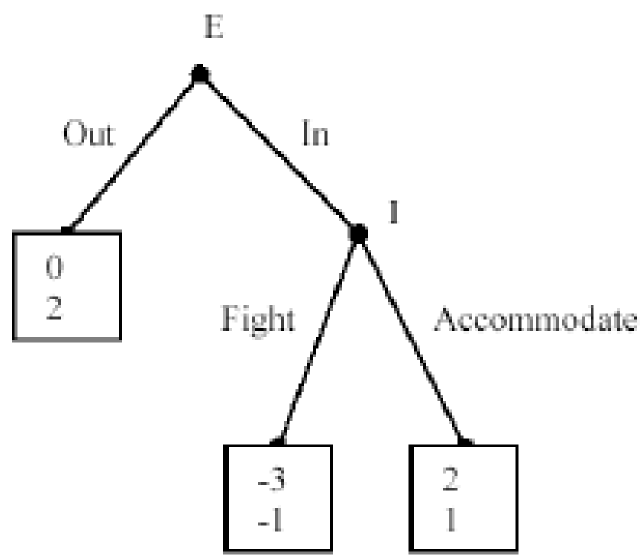
Today's Lecture

- Motivation:
  e.g. firms usually compete for several periods
  in a bargaining process there are offers and counteroffers,...
- Start by representing the normal form and using the Nash Equilibrium concept
- Problem of credibility and the principle of sequential rationality
- Solving by backwards induction in perfect information games
- Subgame Perfect Nash Equilibrium (extension to imperfect information games)

A Simple Entry Game

- Story: A potential entrant E decides whether to enter and if so...
  the incumbent decides whether to fight or accommodate. Payoffs are as follows:



Normal Form and Nash Equilibrium

- Represented in normal form:

E\I	F if In	A if In
Out	0,2	0,2
In	-3,-1	2,1

- Nash equilibria:

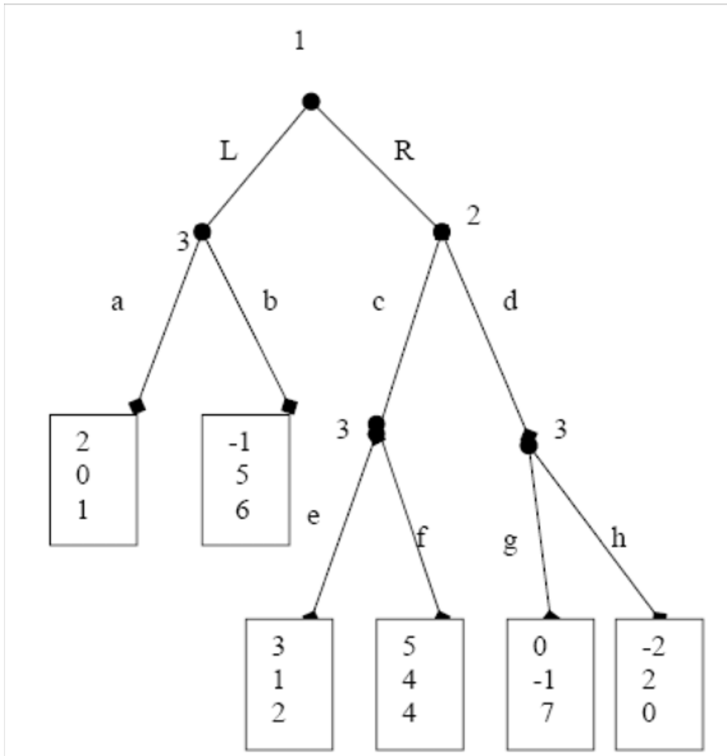
- Is the first reasonable? Problem of credibility. Other examples

- Shouldn't prediction satisfy sequential rationality (i.e. "rational behaviour at each point in time")?

- Yes! Need to refine Nash Equilibrium concept

Backward Induction Method

- Find sequential rational actions in perfect information extensive games:
  a) Find the optimal action at each of the predecessors of the terminal nodes
  b) Associate these nodes with the payoffs of the anticipated terminal node
  c) Start again the process with this reduced game
- Previous example: solve for I's optimal decision in the post-entry game (Accommoda and then, anticipating I's decision, solve for E's optimal decision (In). [(In, A)]
- Example in next slide: one obtains [R,c,(b,f,g)]. This is one of the NE.
- Proposition (Zermelo's theorem): Every finite game of perfect information has a pure strategy NE that can be derived through backwards induction. Moreover, if no player has the same payoffs at any two terminal nodes, then there is a unique NE that can be derived in this way.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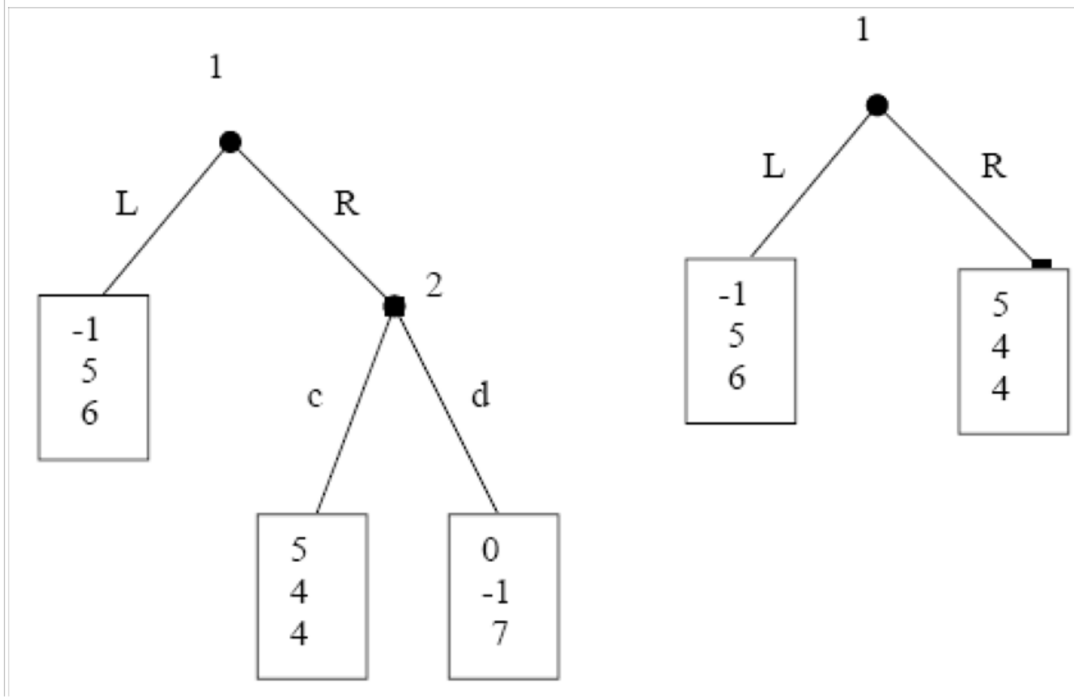
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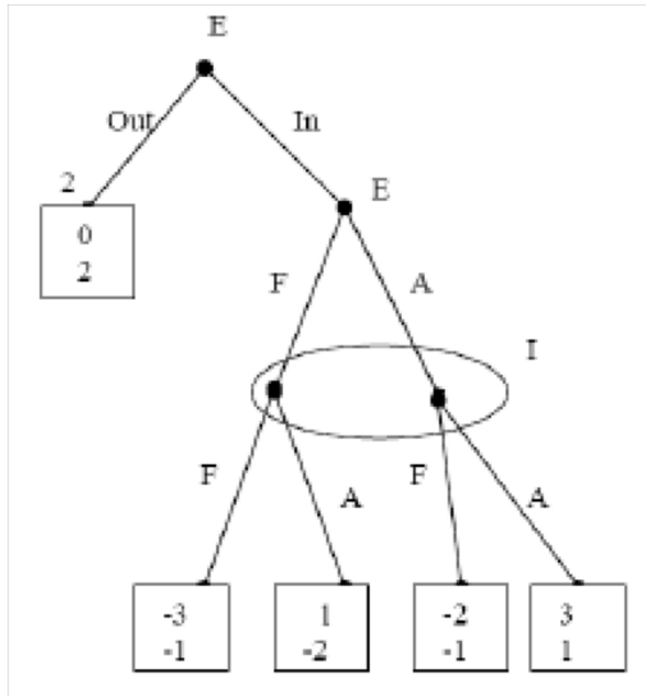
Subgame Perfect Nash Equilibrium: Example

- Extension of backwards induction to imperfect information games



Subgame Perfect Nash Equilibrium

- Definition: A subgame of a game is a subset that satisfies:
  a) Begins at an info set that has only one node and contains all its successors
  b) If a node of an info set is in it, then the other nodes of the info set also are
- Previous example: whole subgame and subgame starting at E decision node
- Definition: A strategy profile $(\sigma_1, ..., \sigma_I)$ is a subgame perfect Nash equilibrium if it induces a NE in every subgame
- By definition every SPNE is a NE (method 1 to find SPNE: select among NE)
- In games of perfect information, SPNE is equal to the set of NE derived by backwards induction
- More generally, SPNE can be found by finding NE in every subgame and substituting backwards (method 2 to find SPNE)

Example (continued)

- Normal form:

E\I	A if In	F if In
Out, A if In	0,2	0,2
Out, F if In	0,2	0,2
In, A if In	3,1	-2,-1
In, F if In	1,-2	-3,-1

- NE:

- Do all of them induce a NE in every subgame? SPNE:

- Other examples in Industrial Organisation: choosing degree of differentiation before competing in prices