Lecture 6: Dynamic Games

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Today's Lecture

- Examples of dynamic games
- Games in extensive form: representation of dynamic and strategic games
- Games in normal form: strategy and representation
- From extensive to normal form representation (and vice versa)

Game in Extensive Form

- Example: "Sequential Matching Pennies"
- 2 players: 1 and 2. Player 1 puts first a penny down, heads up or heads down, and then Player 2, after seeing the move of Player 1, puts her penny down, heads up or heads down. If they match, Player 1 pays £1 to Player 2 and if they do not match , Player 2 pays £1 to Player 1.
- Tree structure representation: decision nodes, branches (actions), payoffs,...
- Strategic interdependence: a player's payoff is not independent of the actions of the others

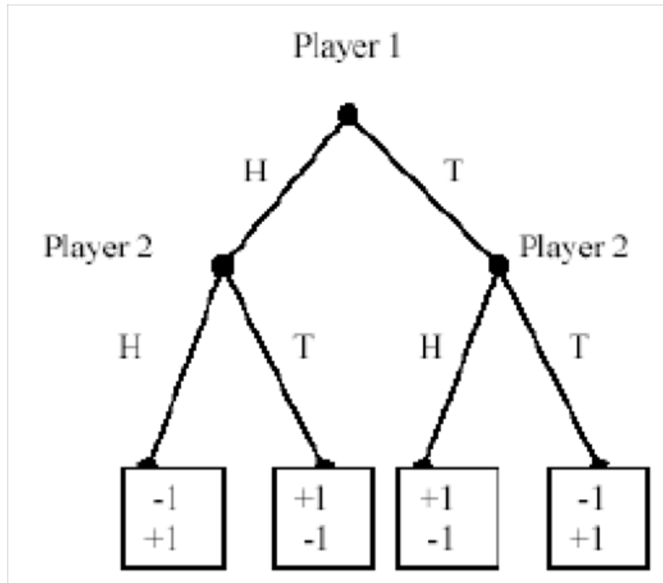


Figure 1:

Simultaneous Moves

- Modified example: suppose that player 1 now hides the penny once put down
- Player 2 does not know what Player 1 played
- Strategically equivalent to "Standard Matching Pennies" (represented equally)
- Representation of an information set

- Game of perfect information: all information sets are singletons
- Game of imperfect information: at least one information set is not a singleton

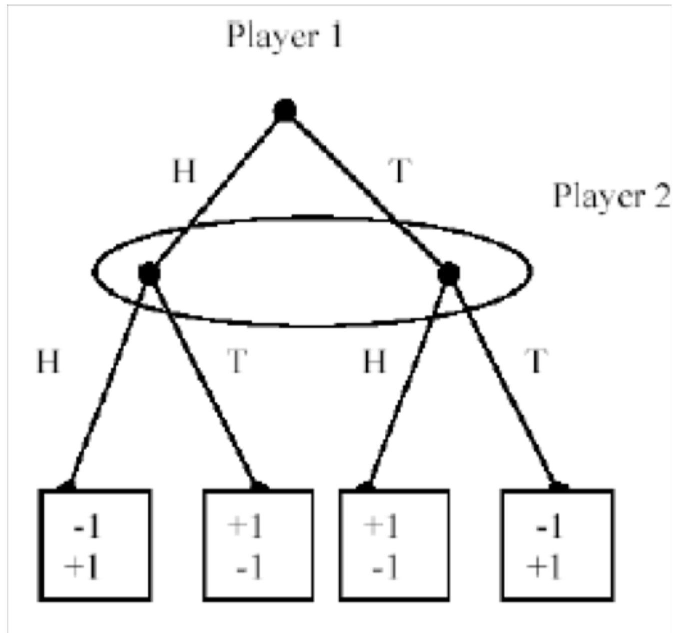
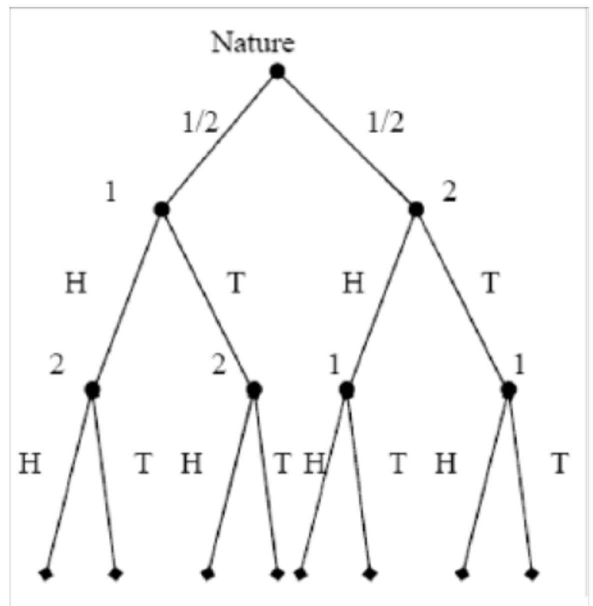


Figure 2:

Randomness

- We can introduce randomness by adding a "Nature" player
- Nature has probabilities to choose each branch



Extensions (Infinite Games)

- Continuous set of actions $[a, b]$.
  Examples: static oligopoly (Bertrand and Cournot games),...

- Infinite set of decision nodes.
  Examples: firms competing indefinite number of periods,...

- Infinite number of players.
  Examples: overlapping generations models,...

Strategy

- Complete contingent plan, specifying how the player will act in every possible situation

- Example 1: "Sequential matching pennies":
  Strategies Player 1: $\{H, T\}$. Strategies Player 2:
  $\{H \text{ if } H, H \text{ if } T\}, \{H \text{ if } H, T \text{ if } T\}, \{T \text{ if } H, H \text{ if } T\}, \{T \text{ if } H, T \text{ if } T\}$
  Notice that, here, an action $\neq$ a strategy

- Example 2: "Matching pennies"
  Strategies Player 1: $\{H, T\}$. Strategies Player 2: $\{H, T\}$
  Notice that, here, an action $=$ a strategy

- Similarly, one can define mixed strategies

Game in Normal Form

- Notice that for each strategy profile $s = (s_1, ..., s_I)$ we have a terminal node (or a distribution over the terminal nodes) and therefore payoffs

- Example: $u_1(\{H\}, \{H \text{ if } H, H \text{ if } T\}) = -1$,$u_2(\{H\}, \{H \text{ if } H, H \text{ if } T\}) = +1$

- A game in normal form can be defined as $\Gamma_N = [I, \{S_i\}, \{u_i\}]$, where...

  I is the set of players

  S_i is the set of strategies for each player $(s_i \in S_i)$ and

  $u_i(s_1, ..., s_I)$ gives the utility level associated with the outcome of $(s_1, ..., s_I)$

- 2-players normal form games can again be summarised in matrices

Game in Normal Form: Examples

- Example: "Sequential matching pennies":

Player1\Player 2	$H,\ H$	H,T	T,H	T,T
H	$-1,+1$	$-1,+1$	$+1,-1$	$+1,-1$
T	$+1,-1$	$-1,+1$	$+1,-1$	$-1,+1$

- Example: "Matching pennies":

1\2	H	T
H	$-1,+1$	$+1,-1$
T	$+1,-1$	$-1,+1$

From Extensive to Normal Form (and Vice versa)

- For any extensive form representation there is an (essentially) unique normal form representation

- The converse is not true. Example: