

Suggested Supervisions Chapter 2

1. (Labour market *continued*) Find the pure and mixed strategy Nash equilibria of the Labour market game in the suggested supervisions for chapter 1 (taking into account the different possible values of w_1 and w_2).

2. (Dividing a pie) Two players have \$10 to divide between themselves. To do so, they use the following procedure: Each player names a number of dollars (a nonnegative integer), at most equal to 10. If the sum of the two numbers is at most 10 then each player receives the amount of money she names (and the remainder is destroyed). If the sum of the two numbers exceeds 10 and the two numbers are different then the player who names the smaller number receives that amount and the other player receives the remaining money. If the sum of the two numbers exceeds 10 and the two numbers are equal each player receives \$5. Determine the best-reply correspondence of each player, plot them in a diagram, and find the Nash equilibria of the game.

3. (Political economy *continued*) Consider again the political economy game in the suggested supervisions for chapter 1.
 - (a) Formulate the situation as a *static* game.
 - (b) Find the set of pure strategy Nash Equilibria when $I = 2$ and the probability distribution is uniform.
 - (c) What would be the equilibrium for a more general probability distribution?
 - (d) Show that there is no Nash Equilibrium when $I = 3$ (*more difficult*).

4. (Auctions) An object is to be assigned to a player in the set $\{1, \dots, I\}$ in exchange for a payment. Player i 's valuation of the object is v_i and $v_1 > v_2 > \dots > v_I > 0$. The mechanism used to assign the object is a (sealed-bid) auction: the players simultaneously submit bids (non-negative numbers), and the object is given to the player with the lowest index among those who submit the highest bid, in exchange for payment.
 - (a) In a first price auction the payment that the winner makes is the price she bids. Formulate a first price auction as a static game and analyze its Nash equilibria. In particular, show that in all equilibria player 1 obtains the object.
 - (b) In a second price auction the payment that the wins makes is the highest bid among those submitted by the players who do not win (so if only one agent submits the highest bid then the price paid is the second highest bid). Show that in a second price auction the bid v_i of any player i is a weakly dominant strategy.

5. (War of attrition *continued*) Consider again the political economy game in the suggested supervisions for chapter 1.

(a) Show that in all (pure strategy) Nash equilibria one of the players concedes immediately.

(b) Do players use weakly dominated strategies in the Nash equilibrium? If not, is it possible to find another example in which players use weakly dominated strategies in the Nash equilibrium? If so, find one!