Lecture 2: Introduction and Elements
Normal Form Representation

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Today's Lecture

- Concept of strategy
- Normal form representation
- Examples
- Extensive and normal form representations
- Randomisation: Mixed and behavioural strategies

Strategy

- Complete contingent plan, specifying how the player will act in every possible situation
- More formally, a strategy for player i is a function

$$s_i : \Xi_i \rightarrow \Lambda \text{ such that } s_i(H) \in C(H) \text{ for all } H \in \Xi_i$$

- Example 1: "Sequential matching pennies":
- Strategies Player 1:
- Strategies Player 2:
- Example 2: "Matching pennies".
- Strategies Player 1: Strategies Player 2:
- Notation: s_{ij} strategy number i player j. $s_{ij} \in S_j$ strategy set of player j
- Strategy profile: $s = (s_1, ..., s_I)$ or $s = (s_i, s_{-i})$, one strategy for each player

Game in Normal Form

- Notice that for each strategy profile s we have a terminal node (or a distribution over the terminal nodes) and therefore payoffs

- Example:

- A game in normal form can be defined as $\Gamma_N = [I, \{S_i\}, \{u_i\}]$, where...

  I is the set of players

  S_i is the set of strategies for each player ($s_i \in S_i$) and

  $u_i(s_1, ..., s_I)$ gives the (von Neumann-Morgensten) utility levels associated with the outcome of $(s_1, ..., s_I)$

- 2-players normal form games can be summarised in matrices

Game in Normal Form: Examples

- Example: "Sequential matching pennies":

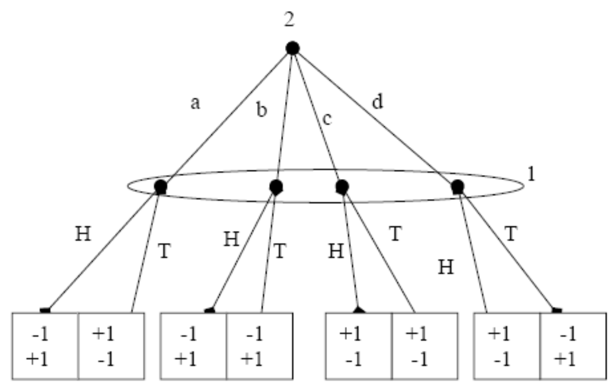
Player1\Player 2	s_{12}	s_{22}	s_{32}	s_{42}
s_{11}	$-1,+1$	$-1,+1$	$+1,-1$	$+1,1$
s_{21}	$+1,1$	$-1,+1$	$+1,-1$	$-1,+1$

- Example: "Matching pennies":

1\2	H	T
H		
T		

From Extensive to Normal Form (and Vice versa)

- For any extensive form representation there is an (essentially) unique normal form representation
- The converse is not true. Example:



Mixed Strategies

- Players may introduce random behaviour (e.g. government auditing taxpayers)
- Notation: $s_i \in S_i$ (deterministic) *pure* strategy and S_i set of pure strategies.
- Definition: A mixed strategy for player i, $\sigma_i : S_i \longrightarrow [0,1]$, assigns to each pure strategy s_i a probability $\sigma_i(s_i)$ that it will be played (where $\sum_{s_i \in S_i} \sigma_i(s_i) = 1$)
- Set of mixed strategies: $\Delta(S_i)$. Representation: simplex.
  Strategy profile: $\sigma = (\sigma_1, ..., \sigma_I)$.
- Given that u_i are of the von Neumann-Morgensten type $u_i(\sigma) \equiv E_\sigma[u_i(s)]$, i.e.,

$$u_i(\sigma) = \sum_{s_{j_1 1},...,s_{j_I I} \in S_1 x ... x S_I} \left(\sigma_1(s_{j_1 1}).\sigma_2(s_{j_2 2})....\sigma_I(s_{j_I I})\right) u_i(s_{j_1 1}, ..., s_{j_I I})$$

- We can redefine again a game in normal form as $\Gamma_N = [I, \{\Delta(S_i)\}, \{u_i\}]$

Behavioural Strategies

- In extensive form games, instead of randomising over set of pure strategies, we could randomise at each information set
- Definition: A behavioural strategy for player i assigns to each information set $H \in \Xi_i$ and action $C(H)$, probability $\lambda_i(a, H) \geq 0$ (where $\sum_{a \in C(H)} \lambda_i(a, H) = 1$ for all $H \in \Xi_i$)
- Proposition (Kuhn 1953): Both types of randomisation are equivalent.

Chapter 1 Summary

- Game Theory: set of tools to analyse behaviour in the presence of strategic interdependence
- Extensive form: Representation of what players can and cannot do and the consequences (tree structure)
- A strategy for a player specifies what to play in each possible circumstance that the player might be called to play
- For each strategy profile (one strategy for each player) there is an outcome and therefore payoffs
- Normal form: Representation using strategies

- Mixed strategies: a player can randomise over the set of deterministic (pure) strategies
- Behavioural strategies: a player can randomise her choice at each information set
- Both types of randomisation are equivalent
- Next chapter: what do people play?