Direcció Financera II

Chapter 2: Investment Decisions

Part (b): Incorporating risk

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("teaching", a la part esquerra)

Cost of capital of a project?

- Often compute the whole firm's cost of capital firm. Why?

- Loads of projects have similar risk as the firm as a whole
- If not, good starting point that can be adjusted for:
  - If project has higher risk relative to the firm as a whole
  - For example, if project has high fixed costs, it will have more risk

- In this part (b), compute the cost of capital of a firm:
  - Expected return of a portfolio of the assets of the firm
  - First, suppose that firm is financed with equity only
  - Second, incorporate possibility that it has debt (WACC)

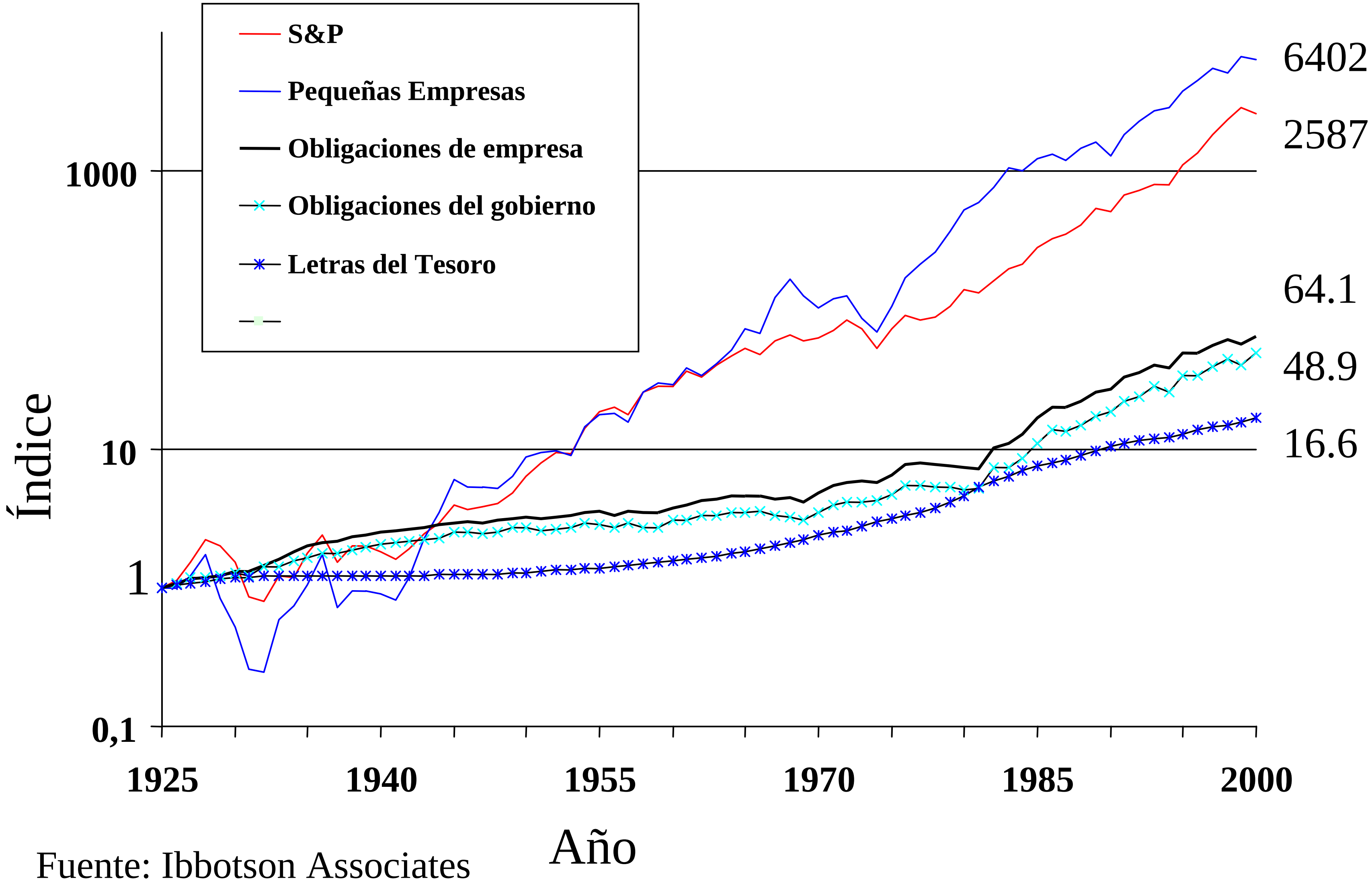
In this part (b) of Chapter 2…

- Estimating the return of equity:
  - Basic tools for "portfolio" theory (risk-return trade-off)
  - Mean variance analysis and portfolio representation
  - The Capital Asset Pricing model (CAPM)
  - Factor models and the Arbitrage Pricing Theory (APT)

- Debt and the Weighted Average Cost of capital (WACC)

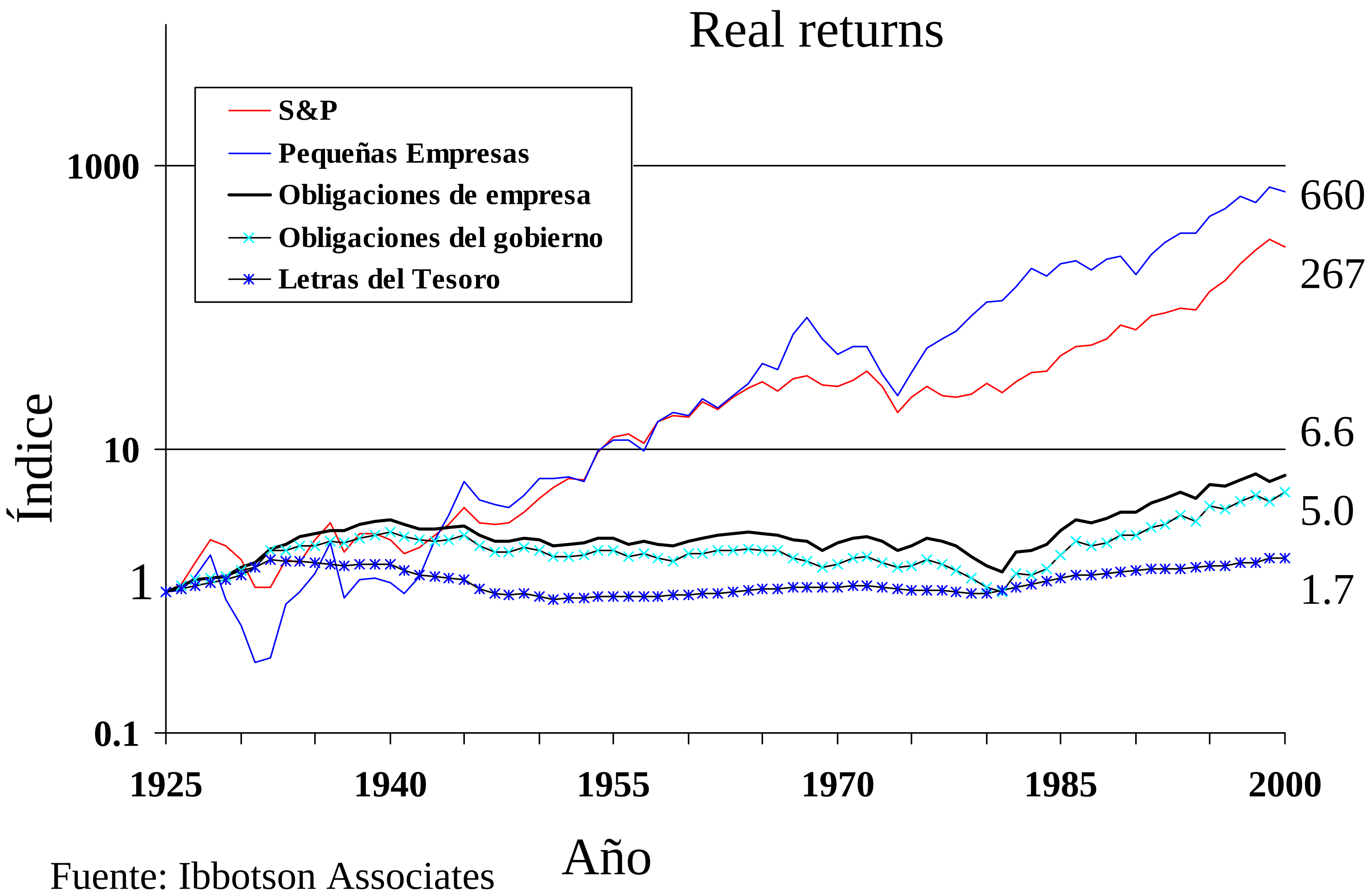
- A final example, separating adjustments for risk and time

Value of 1$ investment in 1926



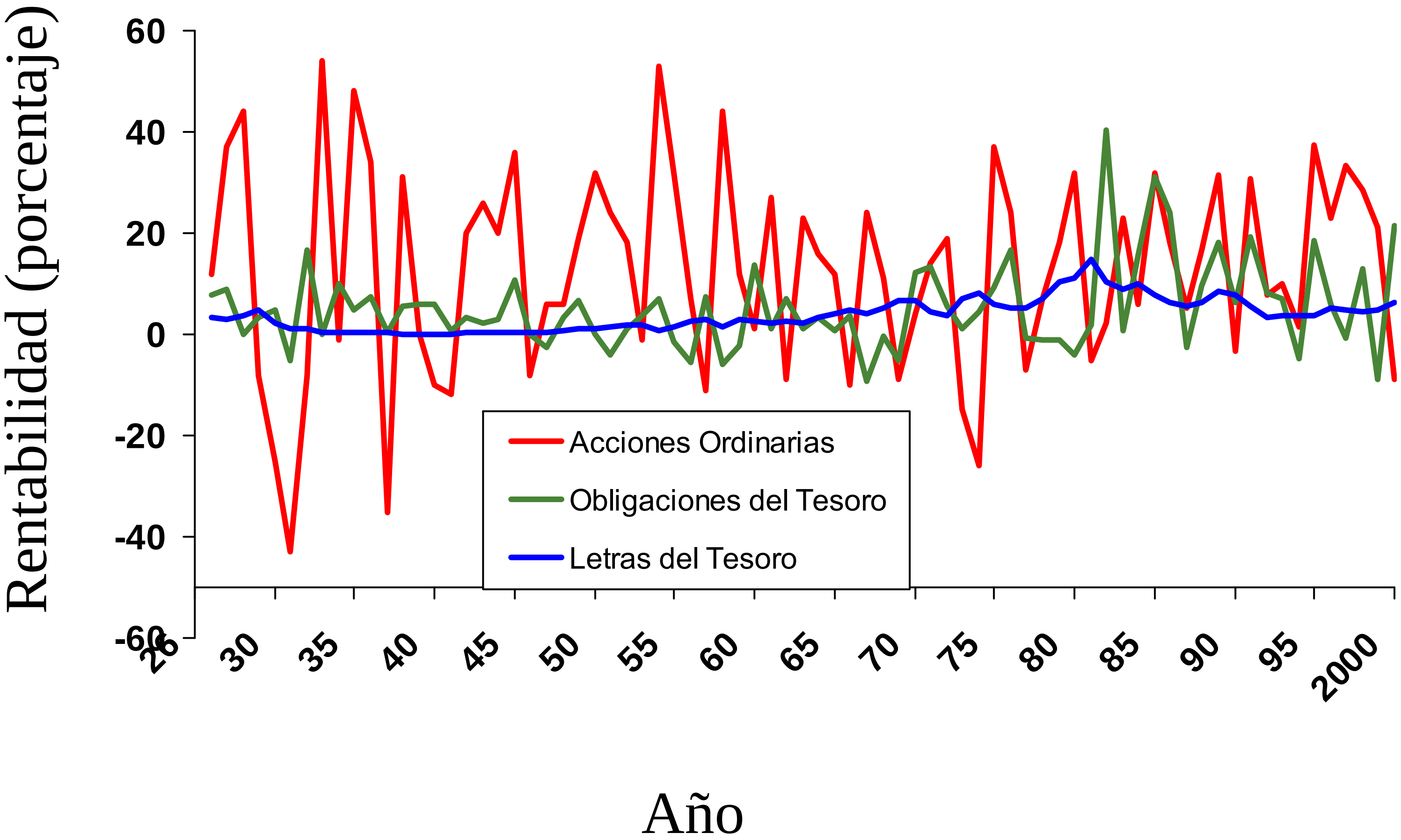
Fuente: Ibbotson Associates

Value of 1$ investment in 1926



Fuente: Ibbotson Associates

Returns 1926-2000



Fuente: Ibbotson Associates

"In investing money, the amount of interest you want should depend on whether you want to eat well or sleep well."

J. Kenfield Morley, *Some Things I Believe*

Basic Tools of Portfolio Analysis

Portfolio Weights

- Portfolio weight for stock j: $x_j = \dfrac{\text{Dollars held in stock j}}{\text{Dollar value of the portfolio}}$
  - Example of a portfolio: £100 in BT and £300 in BP

$$(x_{BT}, x_{BP}) = (1/4, 3/4)$$

- Properties:
  - Weights should add up to 1
  - Weights can be either positive (“long position”) or negative (“short”)
- Short position:
  - A sells shares that does not own to B (formally from C)
  - Position will be closed when A buys from C
  - Example of a portfolio: £400 short in BT and £800 long in BP

$$(x_{BT}, x_{BP}) = (-1, 2)$$

Remember?

- Investment return (historical return) : $r_{i,t} = \dfrac{p_t + d_t - p_{t-1}}{p_{t-1}}$

  $r_1, r_2, r_3, \ldots r_N$ or r_{BT}, r_{BP}

- Expected return (forward looking):

  $\overline{r_1}, \ldots \overline{r_N}$ or $\overline{r_{BT}}, \overline{r_{BP}}$

- Variance and standard deviation of an investment:

  $\text{var}(r_i) = \sigma_i^2 = E[(r_i - \overline{r_i})^2]$ $\qquad \sigma_i = \sqrt{\text{var}(r_i)}$

  - Standard deviation has same units as returns

- Covariance of two investments 1 and 2:

  $\text{cov}(r_i, r_j) = \sigma_{i,j} = E[(r_i - \overline{r_i})(r_j - \overline{r_j})]$

  - Interpretation: measure of relatedness. Move together?
  - Depends on units but…

Remember?

- A useful measure of the co-movement of two returns is the correlation coefficient ρ.

- $\rho_{i,j} = \frac{\mathrm{cov}(r_i, r_j)}{\sigma_i \sigma_j} = \frac{\sigma_{i,j}}{\sigma_i \sigma_j}$ and $\rho_{i,j}$ ε [-1,1]

  - When $\rho_{i,j}$ = 1 (or -1), the assets' returns are perfectly positively (or negatively) correlated, i.e. always move together (or in opposite directions)

  - When $\rho_{i,j}$ = 0, the assets' returns are uncorrelated

Obtaining the expected return, standard deviation, and covariances from historical data

Suppose that you have T observations on any two stocks, $r_{i,1} \ldots r_{i,T}$ and $r_{j,1} \ldots r_{j,T}$.

An estimate for the expected return is $$E(r_i) = \overline{r_i} = \frac{1}{T}\sum_{t=1}^{T} r_{i,t}$$

An estimate for the variance is $$\sigma_i^2 = \frac{1}{T-1}\sum_{t=1}^{T}(r_{i,t} - \overline{r_i})^2$$

The standard deviation is the square root of the variance.

An estimate for the covariance is $$\sigma_{i,j} = \frac{1}{T-1}\sum_{t=1}^{T}(r_{i,t} - \overline{r_i})(r_{j,t} - \overline{r_j})$$

Expected Return

- Portfolio return:
  - Portfolio-weighted average of returns of assets in the portfolio
  - Example: if BT's return has been 10% and BP's 5% and portfolio (0.25,0.75) then…
  - Portfolio return: 0.25*0.10+0.75*0.05=0.0625 or 6.25%

- Expected portfolio return:
  - Portfolio-weighted average of expected returns
  - If the portfolio is $P=(x_1,\ldots, x_N)$ then

$$E(r_P) = \sum_{t=1}^{N} x_i \overline{r}_i$$

- How can you maximise expected portfolio return? (e.g. in example above)

Variances and Covariances of a Portfolio

- For any two-stock portfolio…

$$\sigma_p^2 = \text{var}(x_1 r_1 + x_2 r_2) = x_1^2\sigma_1^2 + x_2^2\sigma_2^2 + 2x_1 x_2 \sigma_{1,2}$$

- Hence… larger covariance leads to higher portfolio variance

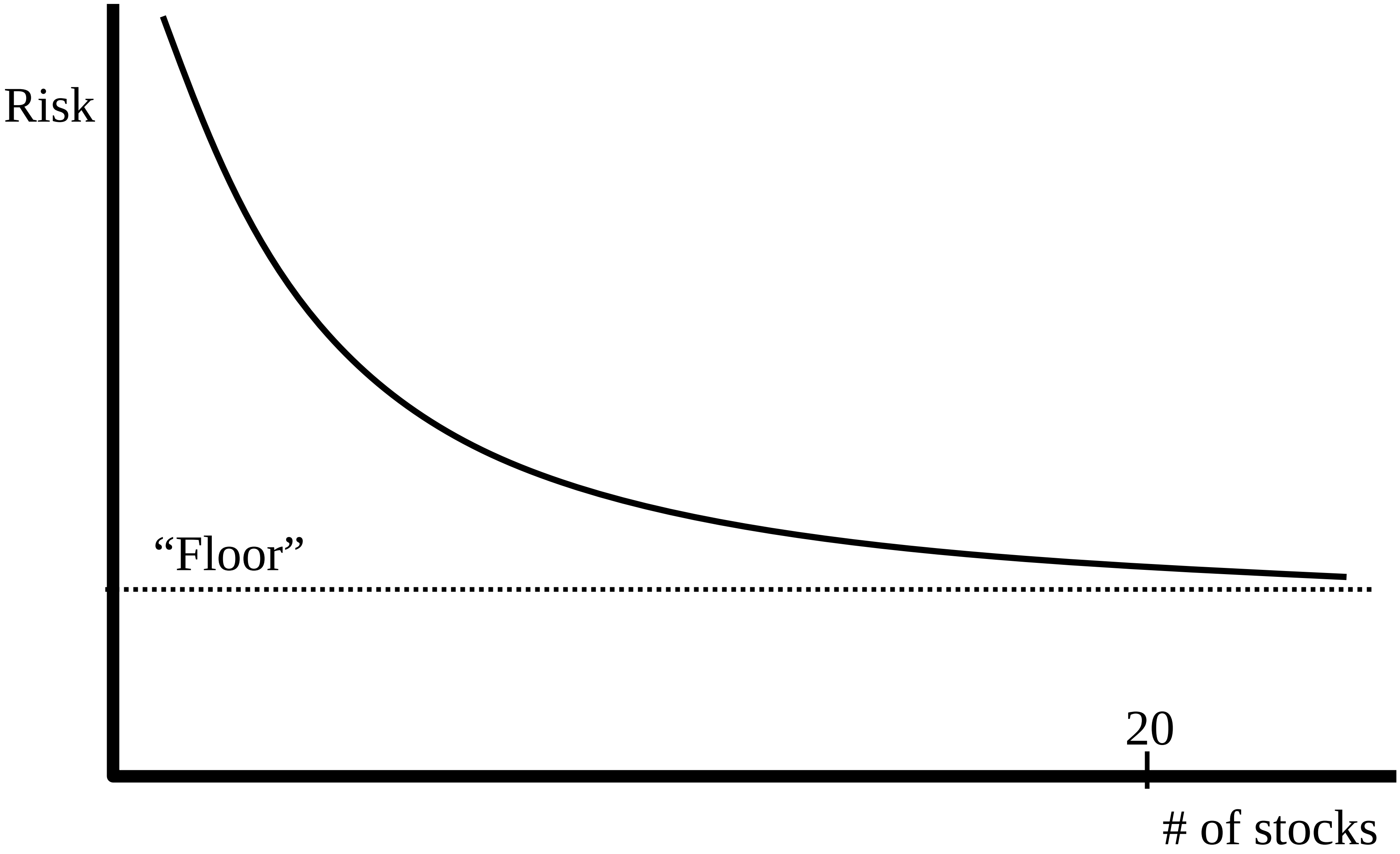
$$= x_1^2\sigma_1^2 + x_2^2\sigma_2^2 + 2x_1 x_2 \rho \sigma_1 \sigma_2$$

$$\leq x_1^2\sigma_1^2 + x_2^2\sigma_2^2 + 2x_1 x_2 \sigma_1 \sigma_2 = (x_1 \sigma_1 + x_2 \sigma_2)^2$$

- With strict inequality if ρ<1
- Thus..

$$\sigma_p \leq x_1\sigma_1 + x_2\sigma_2$$

How Large Diversification Benefits are?



Mean Variance Analysis

Portfolio problem

- How best to combine many assets in order to…
  - maximize expected return for a given variance (i.e. risk)
  - minimize variance (i.e. risk) for a given expected return.

- In other words, how to construct the set of “mean-variance efficient” portfolios?

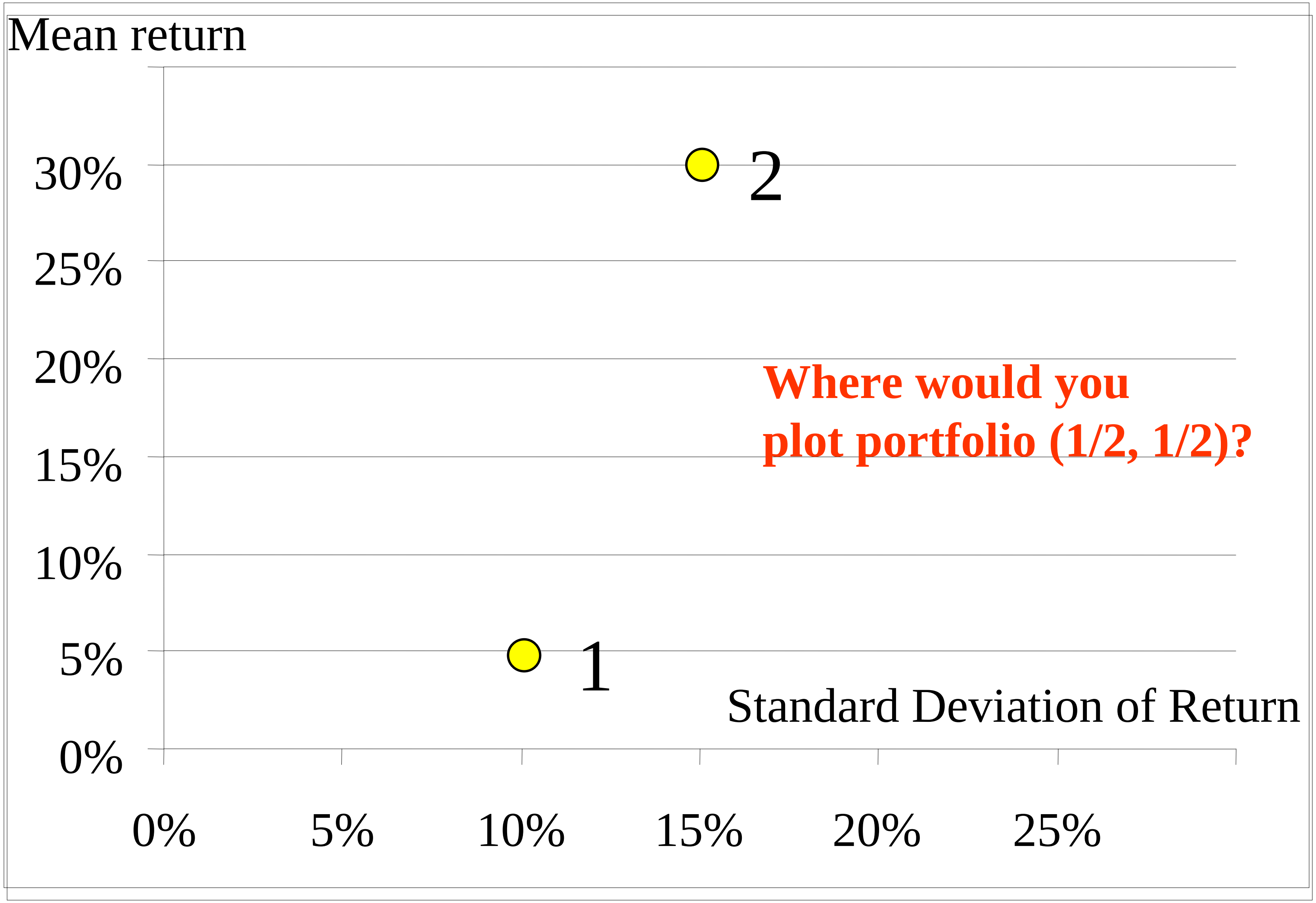
- We assume frictionless markets:
  - all investments are tradable in any quantity (no restrictions on short-positions)
  - no transaction costs, regulations or tax consequences

- Two cases: a risk-free asset is not and is available

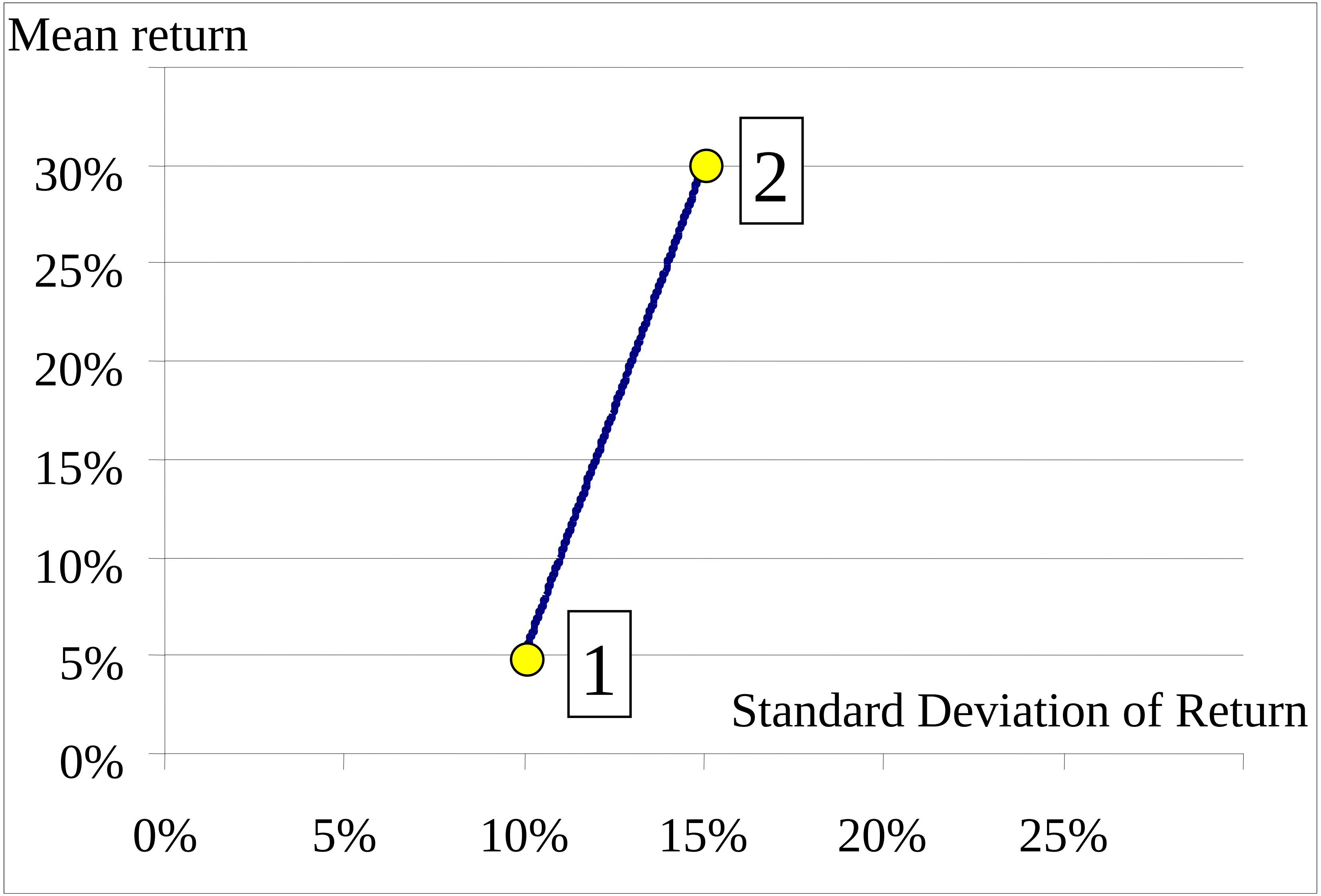
In this section

- Risky environment:
  - Representing risky portfolios
  - Minimum variance portfolio and efficient frontier
  - Properties
- Introducing a risk-free asset:
  - Representing portfolios including a risk-free asset
  - New efficient frontier
  - Properties
- The risk and return equation

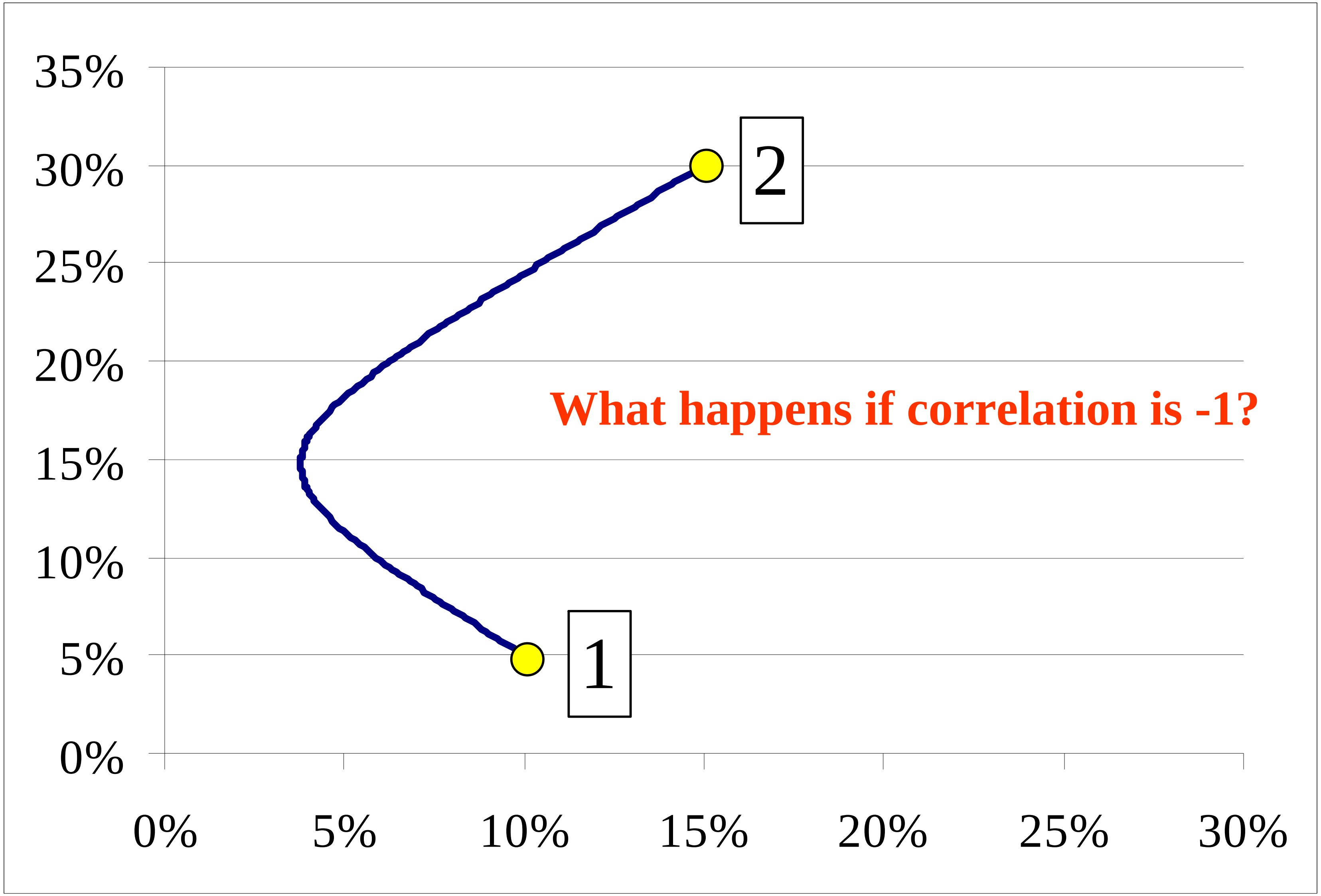
Representation: Mean-Variance Diagrams



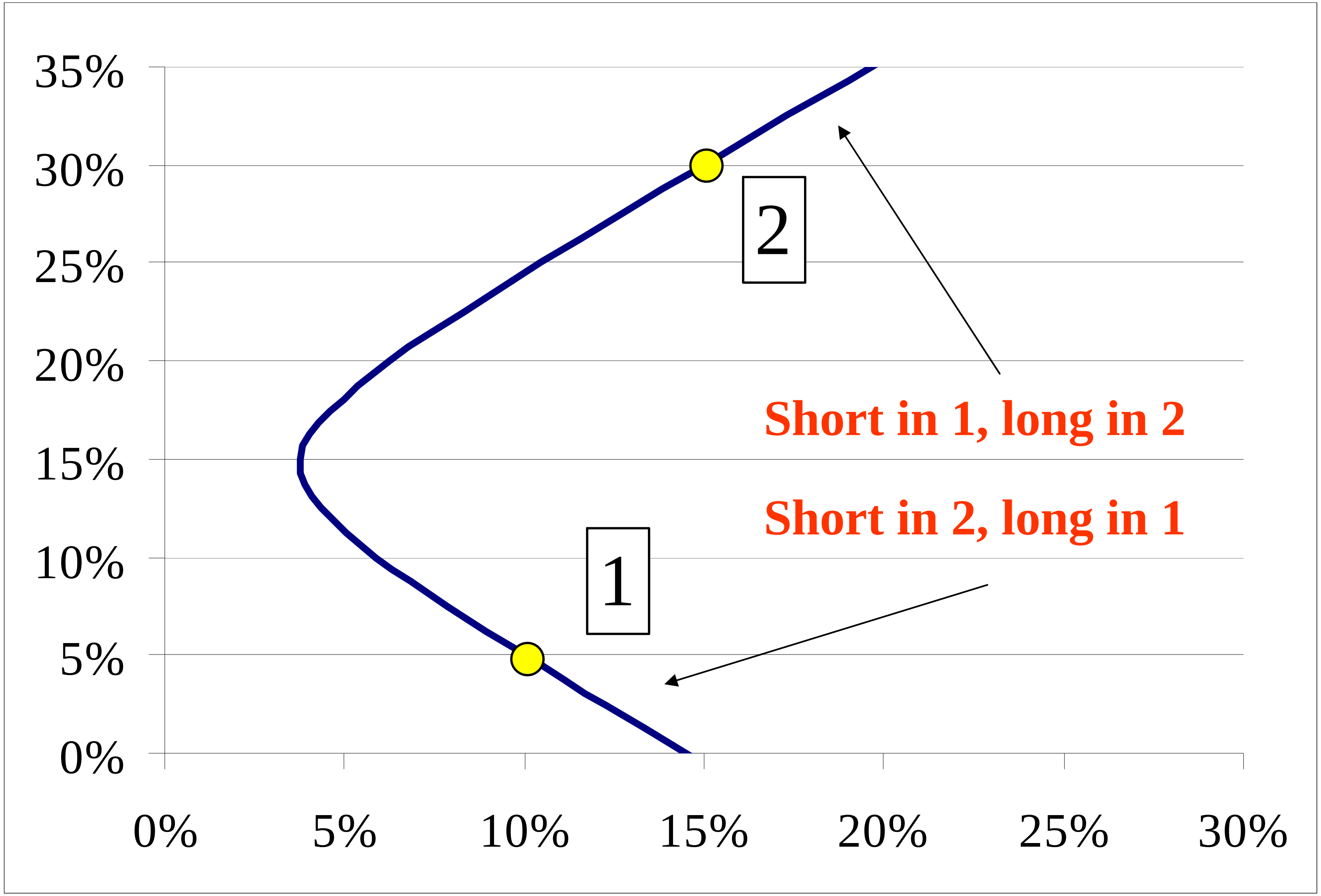
Two-Asset Portfolio (Perfect Correlation)



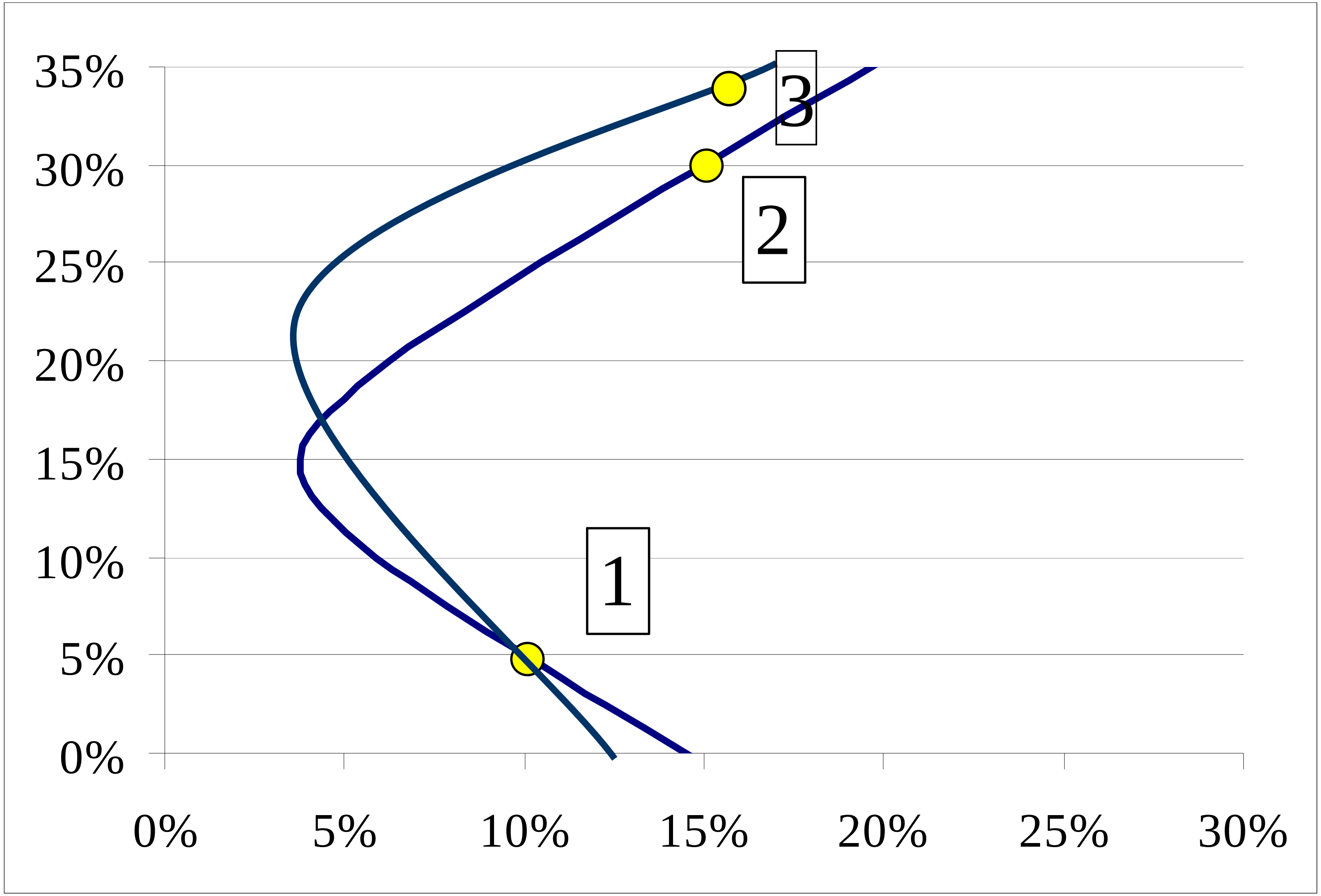
Two-Asset Portfolio (Imperfect Correlation)



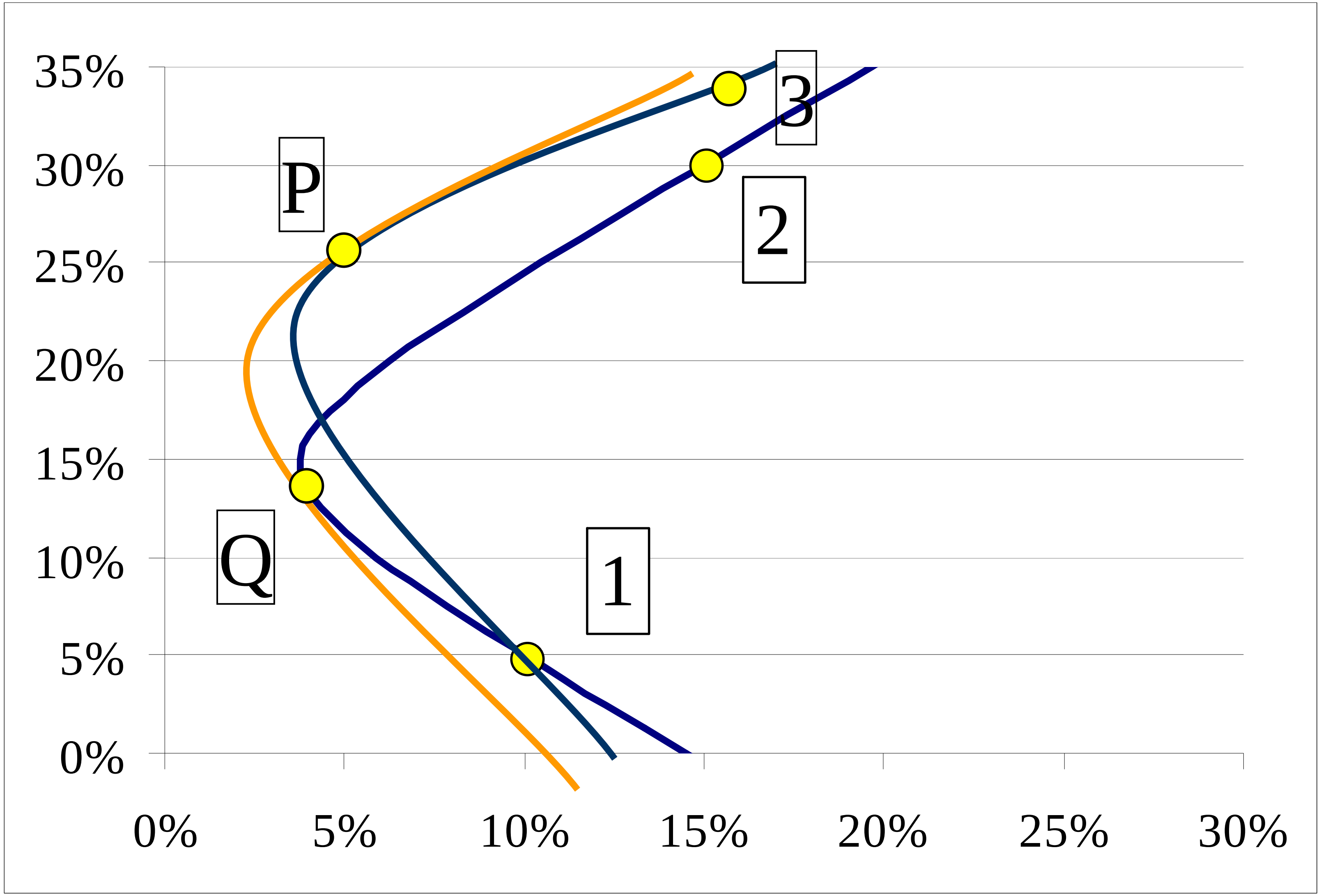
Two-Asset Portfolio (with Short-sales)



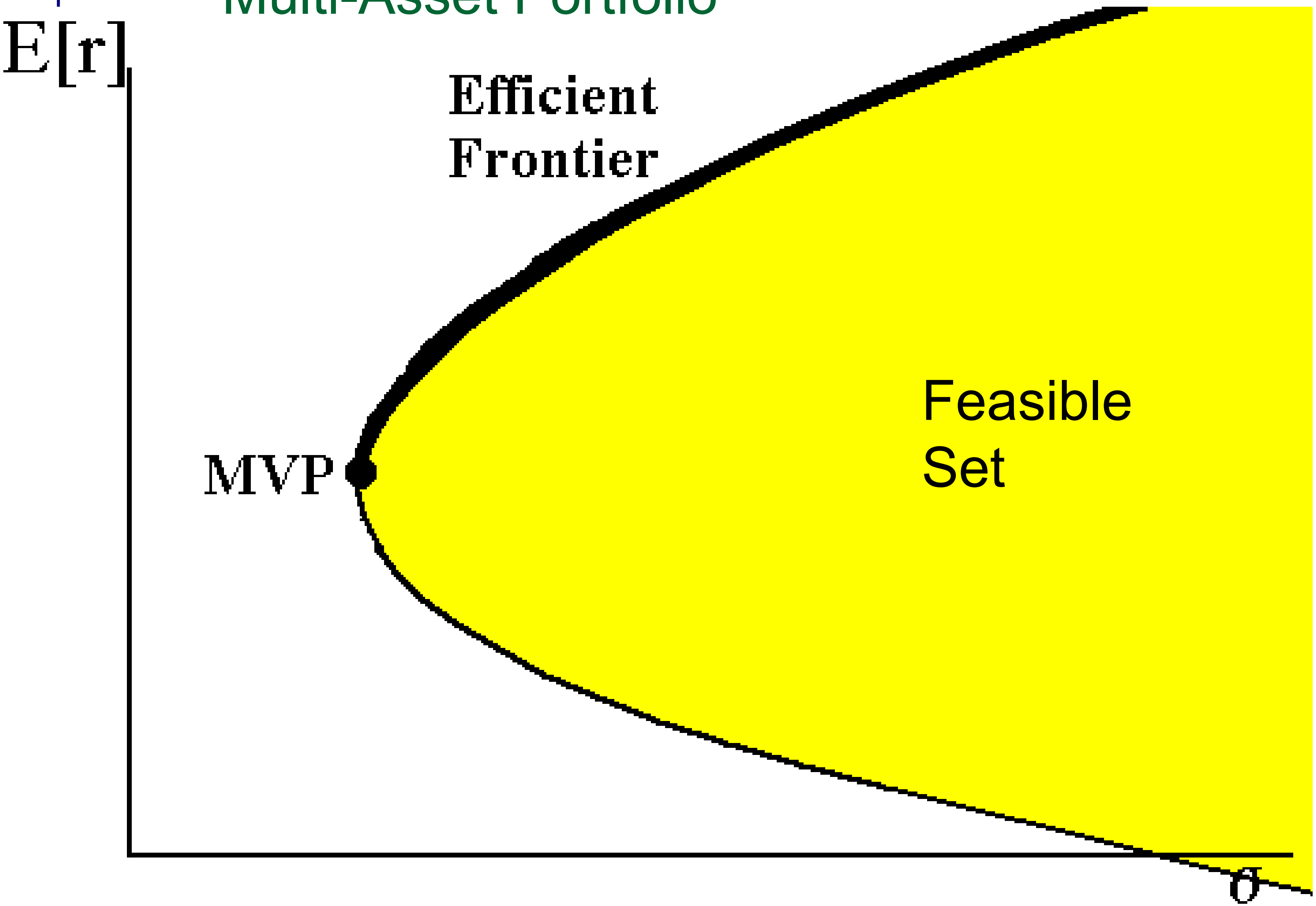
New Asset



Portfolio of Portfolios is another Portfolio!



Multi-Asset Portfolio



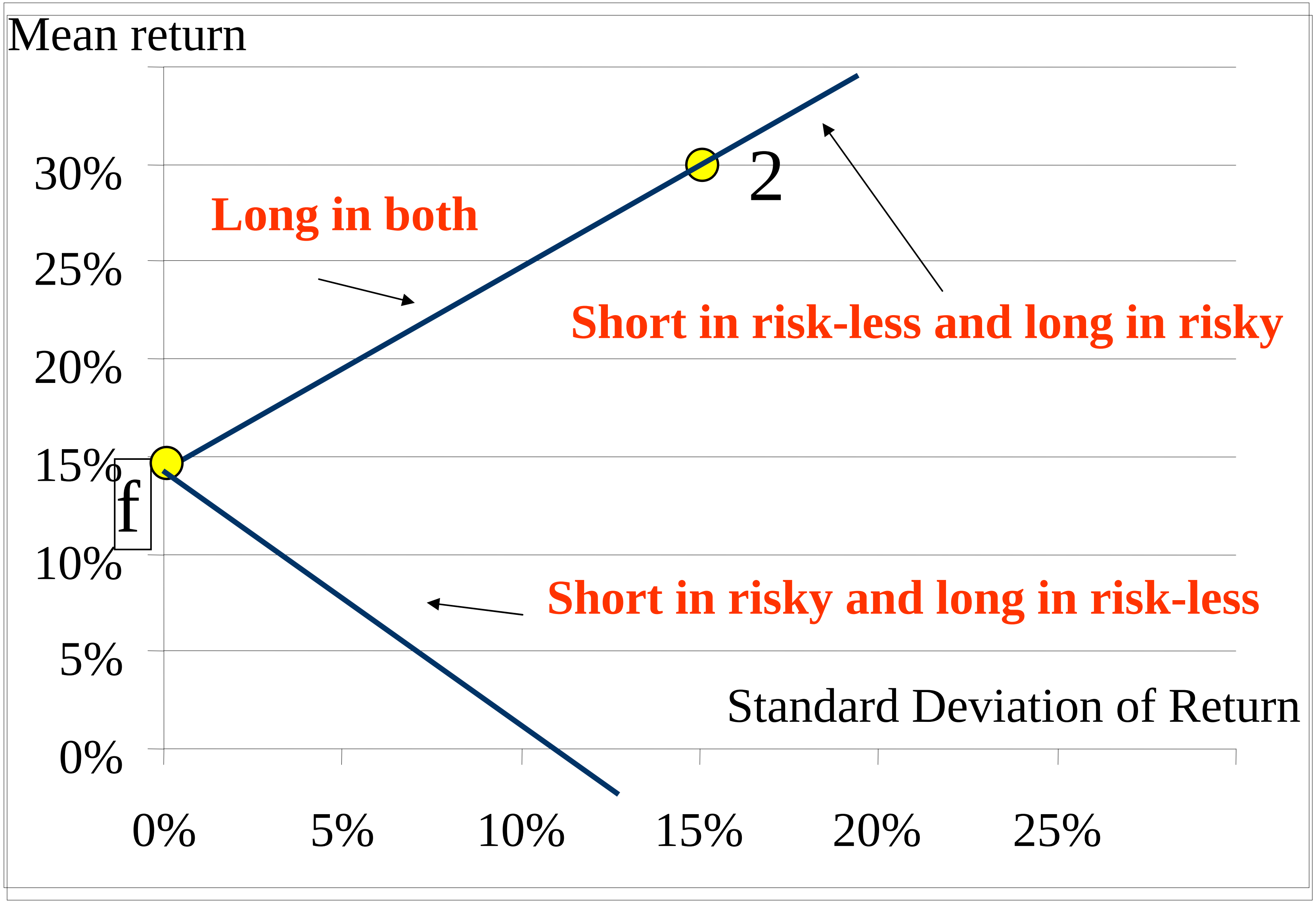
Properties

- Portfolio frontier:
  - "The locus of points in mean-standard deviation space of all portfolios minimising standard deviation for a given expected return"
  - The portfolio frontier is a hyperbola
  - **All** risky assets and **all** portfolios lie inside the portfolio frontier ("feasible set")
  - A portfolio on the frontier is called a frontier portfolio
  - Frontier above the Minimum Variance Portfolio is called efficient

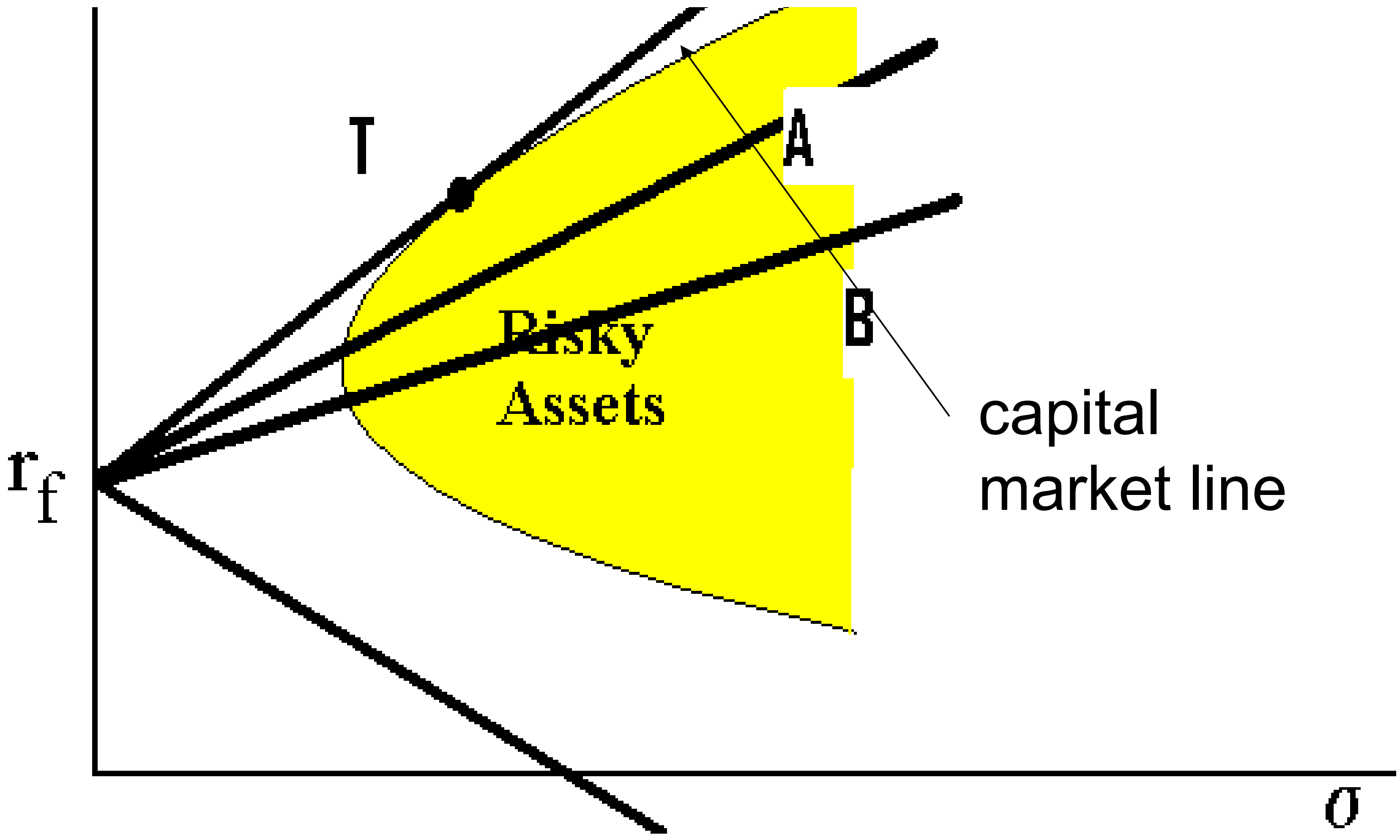
- All investors want to hold efficient portfolios

- **All** possible efficient portfolios can also be created by taking **any** 2 efficient portfolios and combining them…
- and vice versa, any combination of the two is going to be either in the boundary (***two-fund separation property)***

Portfolios of a Risk-less Asset and a Risky Asset

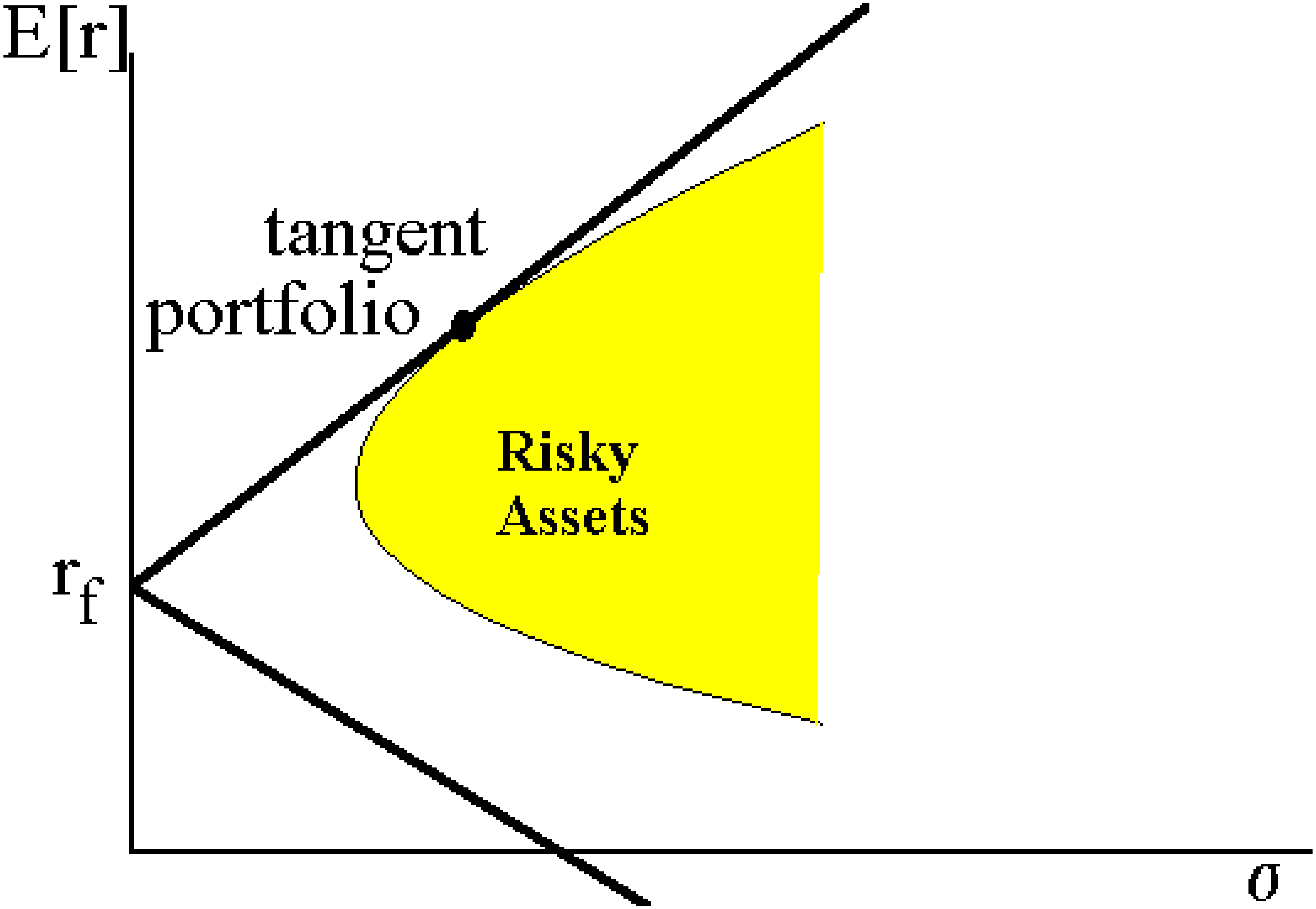


Adding a Risk-less Asset to the Risky Set?



Properties (with a Risk-free Asset)

- The frontier is no longer the hyperbola, but it consists of 2 half lines emanating from r_f, symmetric about r_f
- "Capital market line": Half line with positive slope tangent to hyperbola of risky assets
- Frontier portfolios above r_f are efficient
- The two portfolios out of which all efficient portfolios can be created are now the risk free asset and the **tangent portfolio**
- All investors should hold the same proportion of risky assets:
  - Example: suppose that tangent is (BT,BP)=(2/3,1/3)
    What are the set of efficient portfolios?


E[r]
tangent
portfolio
Risky
Assets
r
f
σ

Nice Mathematical Result: Risk & Return

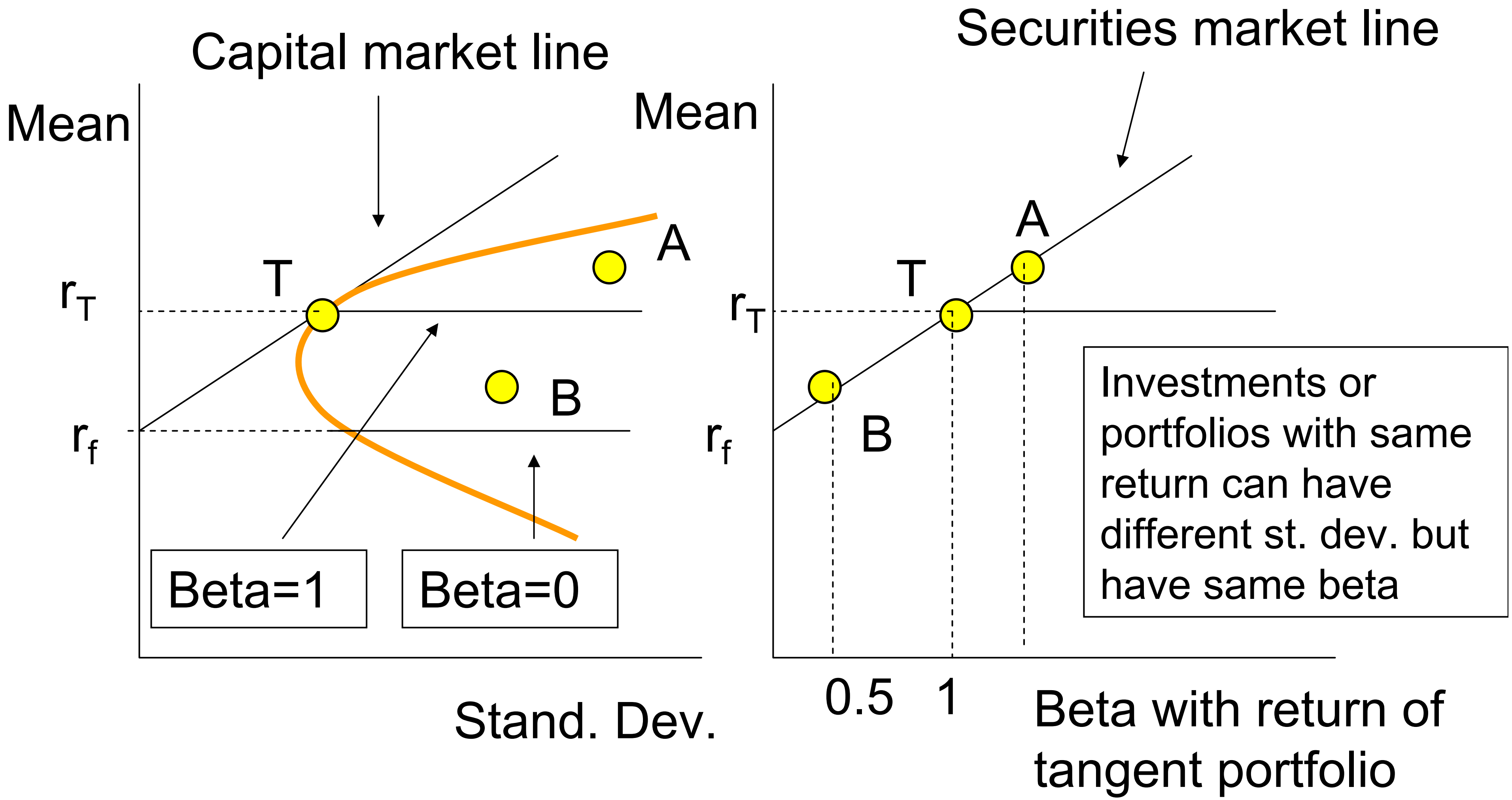
- For any investment i we have,

$$E[r_i] - r_f = \beta_{iT} (E[r_T] - r_f) \text{ where } \beta_{iT} = \frac{\sigma_{i,T}}{\sigma_T^2}$$

$$E[r_i] = r_f + \beta_{iT} (E[r_T] - r_f) \text{ where } \beta_{iT} = \frac{\sigma_{i,T}}{\sigma_T^2}$$

- *Tangency portfolio key to relate expected return on any investment with a measure of its risk: the covariance.*
- *Covariance is the relevant measure of risk*
- *Allows us to use return risk estimates to estimate return*

Capital and Security Market Lines



Finding the Tangency Portfolio

- Derive as shown in the appendix:
  - Relatively easy for investment across countries or asset types
  - Low number of investments and good parameter estimates
- But may be computationally demanding individual investment selection:
  - Loads of individual investments!
  - Need to estimate all means and covariances
- CAPM tell us which should be the tangency portfolio

The CAPM

CAPM and some Applications

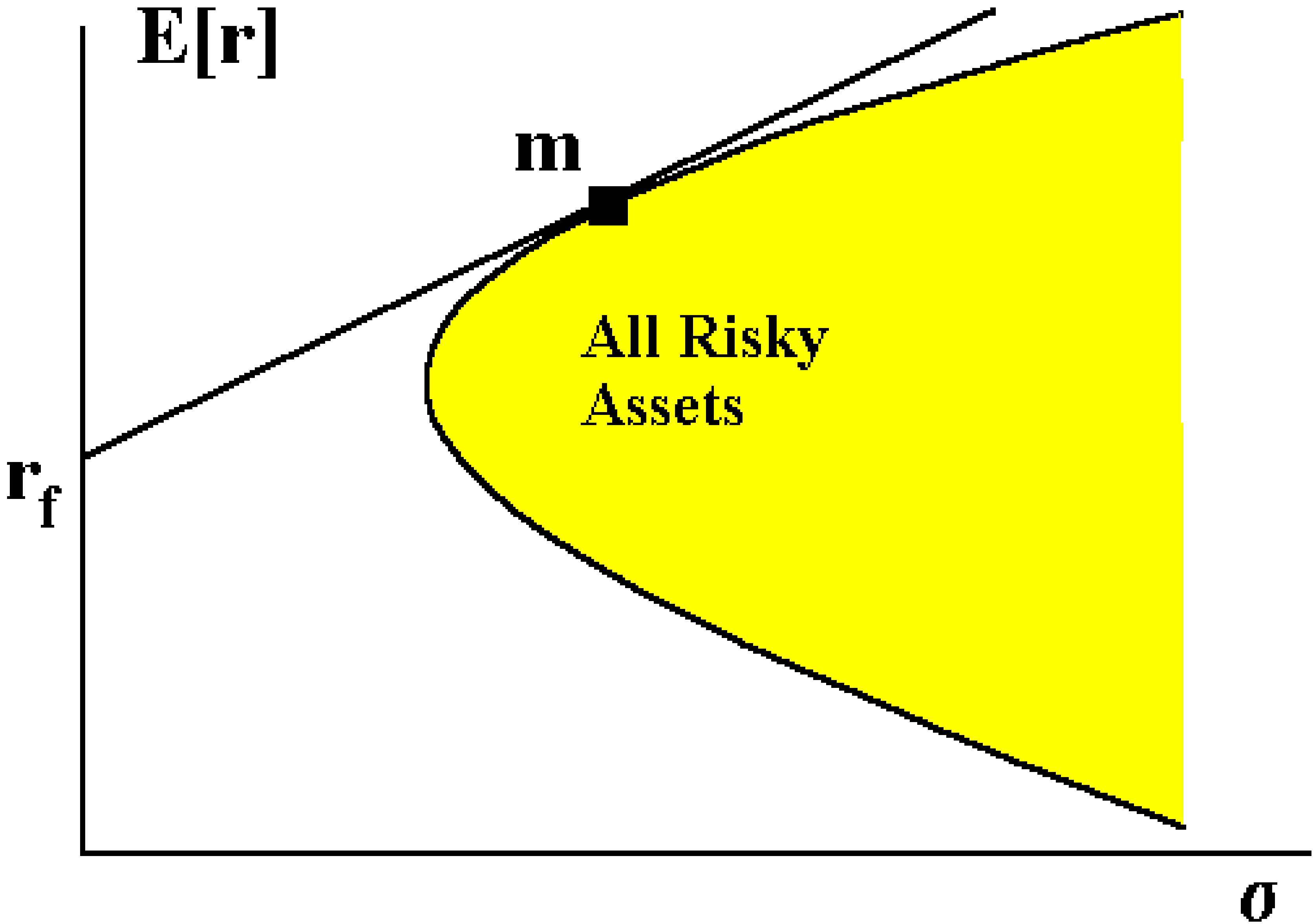
- Main questions that will be answered:
  - What is the equilibrium relation between risk and return?
  - What is risk (i.e. which uncertainty “matters”)?
  - What is the price of a unit of risk?

Assumptions and Conclusion

1. Markets are frictionless
2. There is a risk-free asset that returns r_f
3. All investors want to hold efficient frontier portfolios;
4. Supply equals demand in financial markets (we are in *equilibrium*)
5. Investors have homogenous beliefs about means and st deviations

- These assumptions are sufficient to apply some of the previous results

- And, we also get…
  - Two fund separation between risk-free asset & **market portfolio**
  - The market portfolio (m) is the portfolio of all risky assets, where the weight of each asset is the market value (market capitalisation) divided by the total market value


E[r]
m
All Risky
Assets
r
f
σ

Example

Consider a three-stock economy: HP, IBM, CPQ a risk-free asset (US Treasuries) and only two investors, A and B.

	HP	IBM	CPQ
Price per share	$33	$95	$20.25
Shares outstanding	2 bill	1.758 bill	1.7 bill

- Market portfolio: (UST, HP,IBM,CPQ)=(0, 0.25, 0.62, 0.13)
  Risk free asset: (UST, HP,IBM,CPQ)=(1, 0, 0, 0)

- Both A and B hold a combination of the risk-free and the market portfolio, i.e. the same relative positions in risky assets, e.g.:

The Big News

- Since investors all hold the market portfolio, the only thing to worry about for any asset is what the addition of that asset would do to the market portfolio
- The additional risk of any asset, when added to the market portfolio, is entirely captured by the covariance of that asset with the market.
- Mathematically, since we know that the market portfolio is mean-variance efficient, we get

$$E[r_q] = r_f + b_q (E[r_m] - r_f) \quad \text{where } b_q = b_{qm} = \frac{\sigma_{mq}}{\sigma_m^2}$$

- This is the "beta" of a security or portfolio or any other risky asset

More Terminology

- That is why the positively sloped efficient frontier is called the “capital market line” (or CML). It should contain all portfolios that will be held by investors.
- The measure of risk *for an efficient frontier portfolio* is its standard deviation, and the CML gives the trade-off between risk and (expected) return.
- The price of a unit of risk for an efficient frontier portfolio is simply the slope of the CML. It is the market risk premium per unit of market standard deviation (the market **Sharpe Ratio**):

Estimating the CAPM

- **What risk free rate?**
  - **Theory**: The government bond rate in the same currency with the same maturity as the investment under consideration.
  - **Practice**: Some people choose the short rate (because the long rate fluctuates when inflation expectations change). Some people choose the long rate to match maturity with the asset in question. From this long rate, a liquidity premium (historically ~ 1%, currently closer to 0) should be subtracted.

- **What is the Market Portfolio?**
  - **Theory**: The basket of all assets that the investors can invest in.
  - **Practice**: Generally, choose an index of equities, because (i) we only have reliable return information on equities and (ii) the firms themselves own enough 'other assets' (real estate, oil, gold, etc.) to argue that equity index really reflects behaviour of the entire **market**.

Which Country's Market Portfolio?

- **Theory**: The one representing the investment opportunities for the *representative* **investor in the company**

- **Practice**: Often the home country's equity index of a company

- **Best Practice**: You have to look at the world through the eyes of your firm's investors
  - If your investors have access (i.e. routinely use) the world market portfolio (proxy: MSCI), this is the correct portfolio.
  - If your investors only invest in a particular portfolio - say their home country - then this is the correct portfolio.
  - I.e. even if you have a firm with operations in **Germany**, whose investors are mainly **Germans**, you would still use the World Market Portfolio if the investors have holdings around the world.

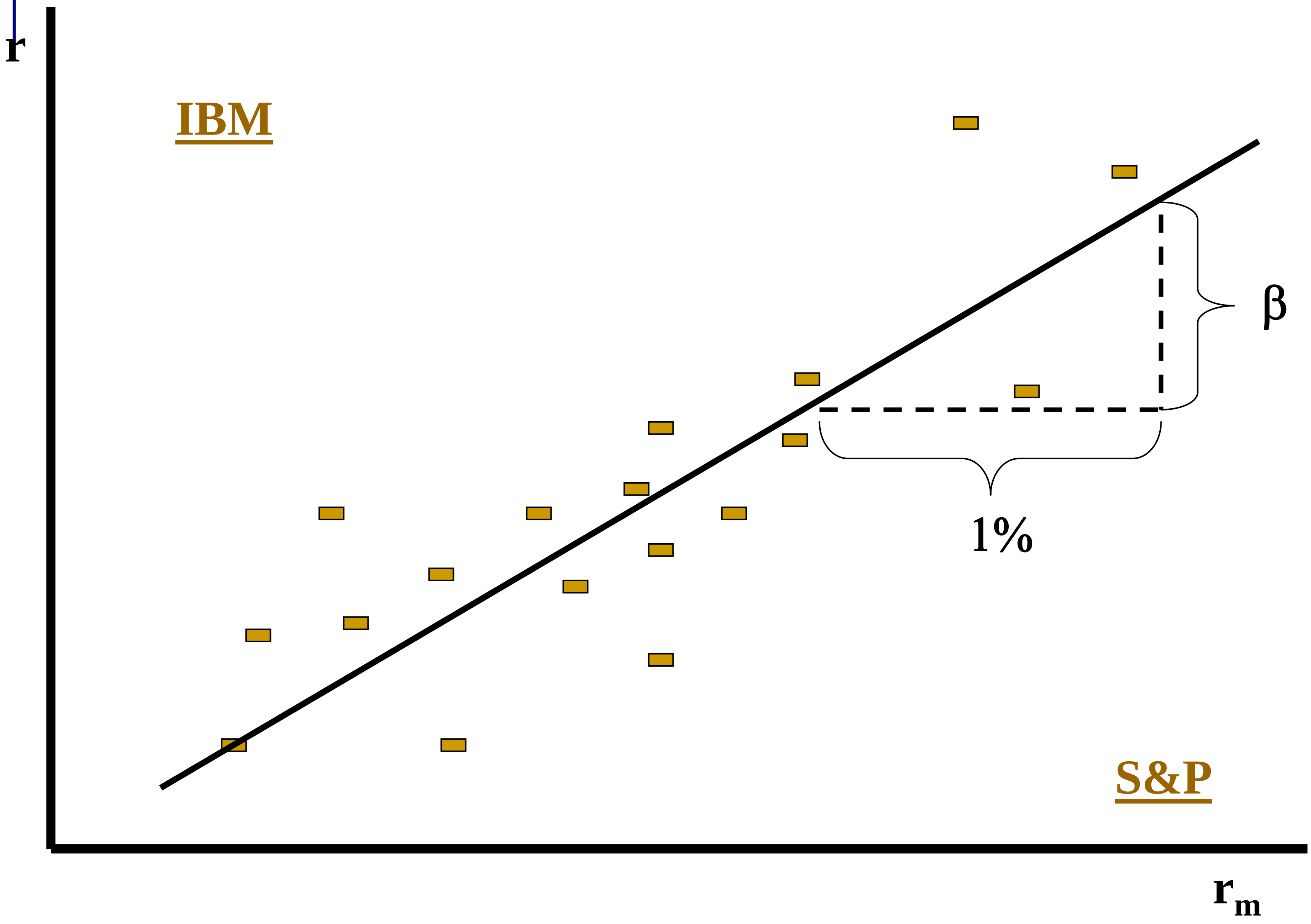
Where do we get a beta?

- Define…
  - r_{jt} : realised return (including dividends) of stock j in period t
  - r_{mt} : realized return (including all dividends) of market index (e.g. S&P500 or FTSE100) in t;
  - r_{ft} : risk-free rate over period t;

- Run one of the following regressions:

$$r_{jt} - r_{ft} = a + b \times (r_{mt} - r_{ft}) + e_{jt} \quad \text{or} \quad r_{jt} = a + b \times r_{mt} + e_{jt}$$

- Under some conditions, ordinary least squares estimate of b is an unbiased estimate of the beta of stock j


r
IBM
β
1%
S&P
r_m

Example 1

Dell Computer

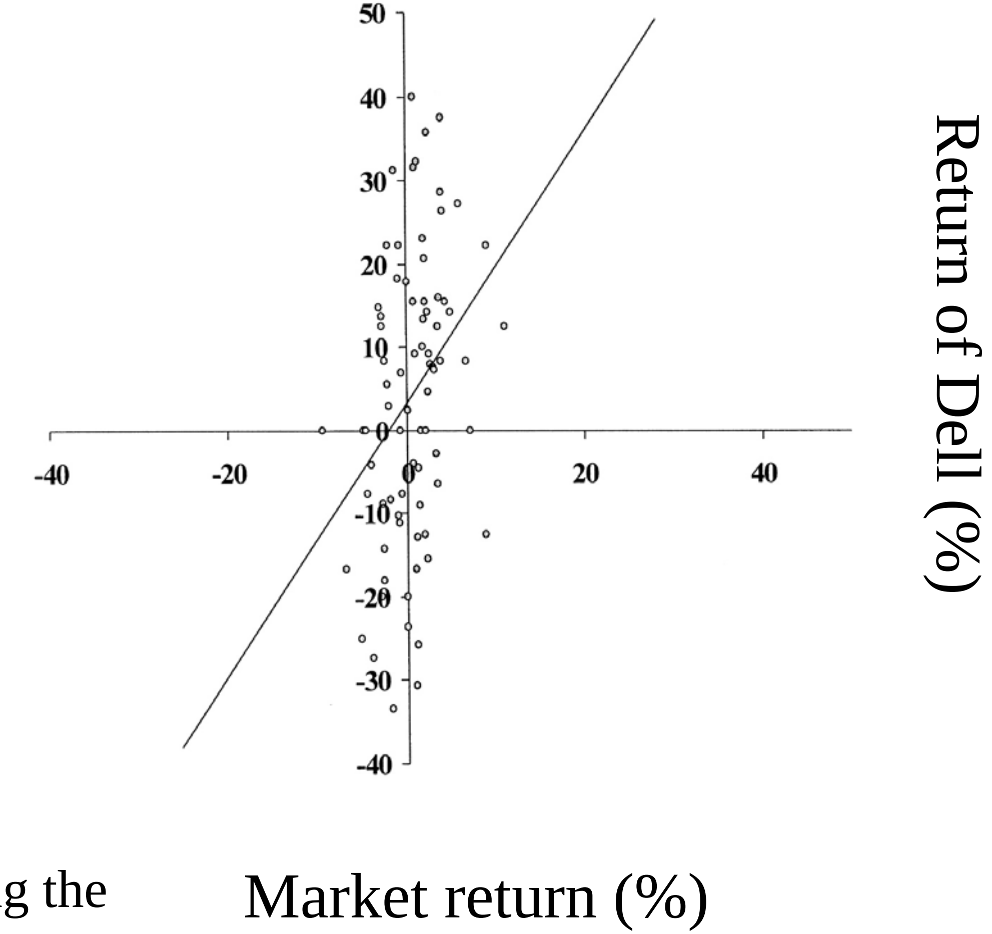
Price data

Aug 88- Jan 95

$R^2 = .11$

β= 1.62

Std dev= 0.52



Slope determined when plotting the best-line fit

Example 1b

Dell Computer

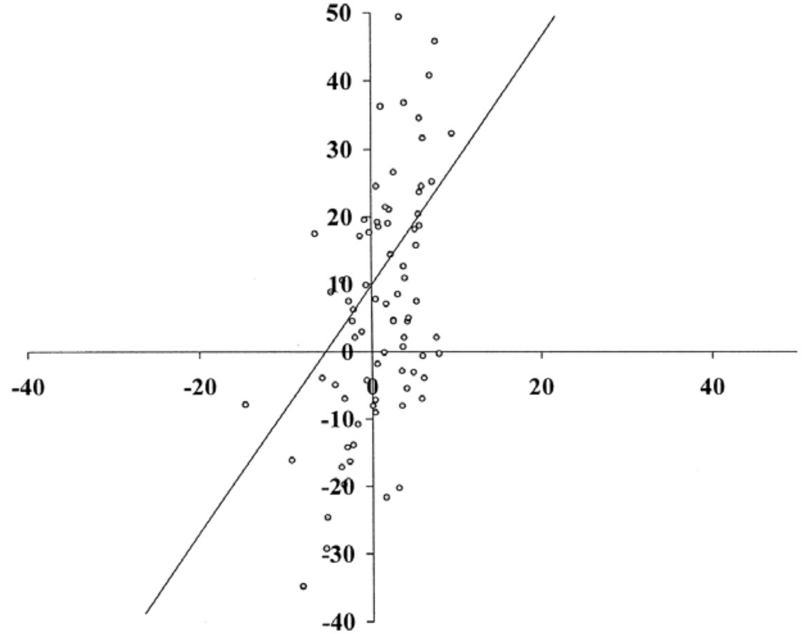
Price data

Feb 95 – Jul 01

$R^2 = .27$

$\beta = 2.02$

Std dev= 0.38



Slope determined when plotting the best-line fit

Example 2

General Motors

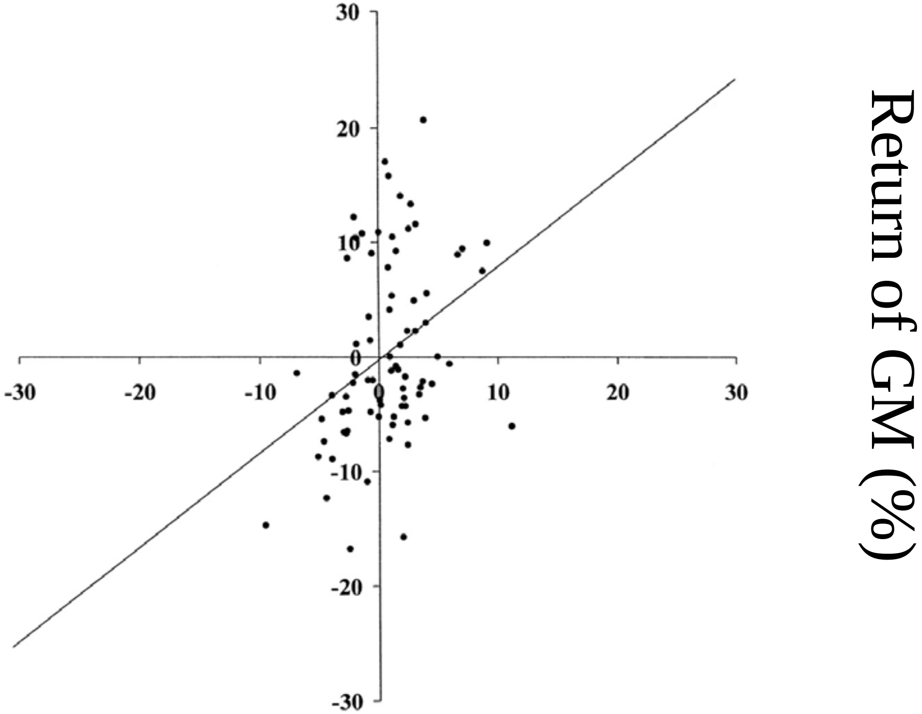
Price Data

Aug 88- Jan 95

$R^2 = .13$

$\beta = 0.80$

Std dev= 0.24



Slope determined when plotting the best-line fit

Market return (%)

Example 2b

General Motors

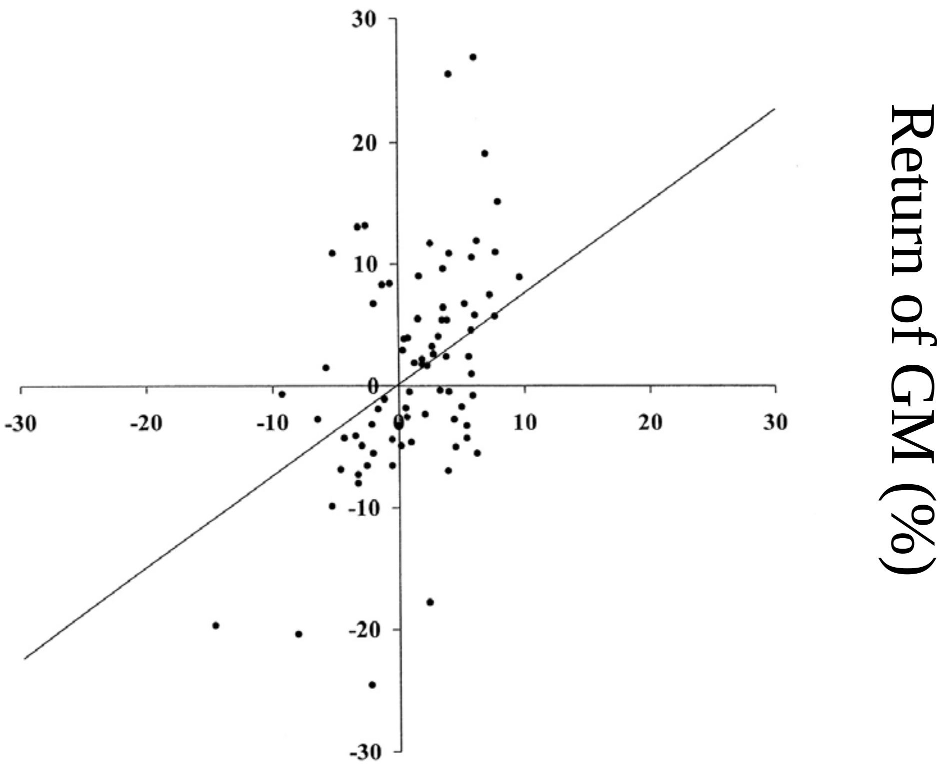
Price data

Feb 95 – Jul 01

$R^2 = .25$

β= 1.00

Std dev= 0.20



Slope determined when plotting the best-line fit

Market return (%)

Market Risk Premium (Equity Premium)

- Once we have r_f and beta, we need $(r_m - r_f)$,

- What is $(r_m - r_f)$?
- Historically ~ 8% (US)
- Academics: ~ 6% (US)
- Practitioners: ~ 4% (US) - recently

Finally…

- With an estimate of b_j and an estimate of the market risk premium, r_m - r_f, we can now form an estimate of the required rate of return on stock or project using the CAPM.

- Example: AT&T (a few years back)
  - Suppose AT&T's beta is estimated (using the S&P500 as a market proxy) at about 0.81
  - Using 5% as current risk-free rate and 5% as market premium
  - W would estimate AT&T's required rate of return as $0.05 + 0.81\times 0.05 = 0.0905$ or 9.05%.

Factor Models and APT

Factor models

- The return of a risky investment is determined by:
  - Common factors (e.g. interest rates, inflation, productivity…)
  - A firm-specific component (new R&D results, fire in a plant,…)
- Return variances of large portfolios are determined by common factors, firm-specific ones can often be ignored
- Common factors do not affect all investments equally: each has its sensitivities to the factors ("factor betas")
  - E.g stock of car company more sensitive to changes in interest rate than stock of a soft drink firm
  - Car companies are highly affected by interest rate (factor) risk
- Factor models can be used to estimate the expected rate of return of an investment, as an alternative to the CAPM:
  - Arbitrage pricing theory: relation of factor risk to expected return

A One-Factor Model: the Market Model

- Run the following regression:

$$r_{Dell} = \alpha_{Dell} + \beta_{Dell} r_{S\&P500} + \varepsilon_{Dell}$$

- If $r_{S\&P}$ and ε_{Dell} are uncorrelated:

$$\sigma^2_{Dell} = \beta^2_{Dell}\sigma^2_{S\&P500} + \sigma^2_{\varepsilon_{Dell}}$$

- Risk can be divided in two:
  - Systematic, market risk: part explained by market movements
  - Unsystematic risk: part not explained by market movements

Unsystematic and Diversifiable Risk

- Unsystematic risk may be related to other factor risks:
  - Car company highly affected by interest rate risk
  - Part of this effect shows up in the residual of previous equation
  - As a result, not all unsystematic risk is diversifiable
- However, if for all the investments i we had

$$r_i = \alpha_i + \beta_i r_{\mathrm{m}} + \varepsilon_i$$

such that all ε_i were uncorrelated then ε_i would be firm-specific and therefore the related risk would be diversifiable

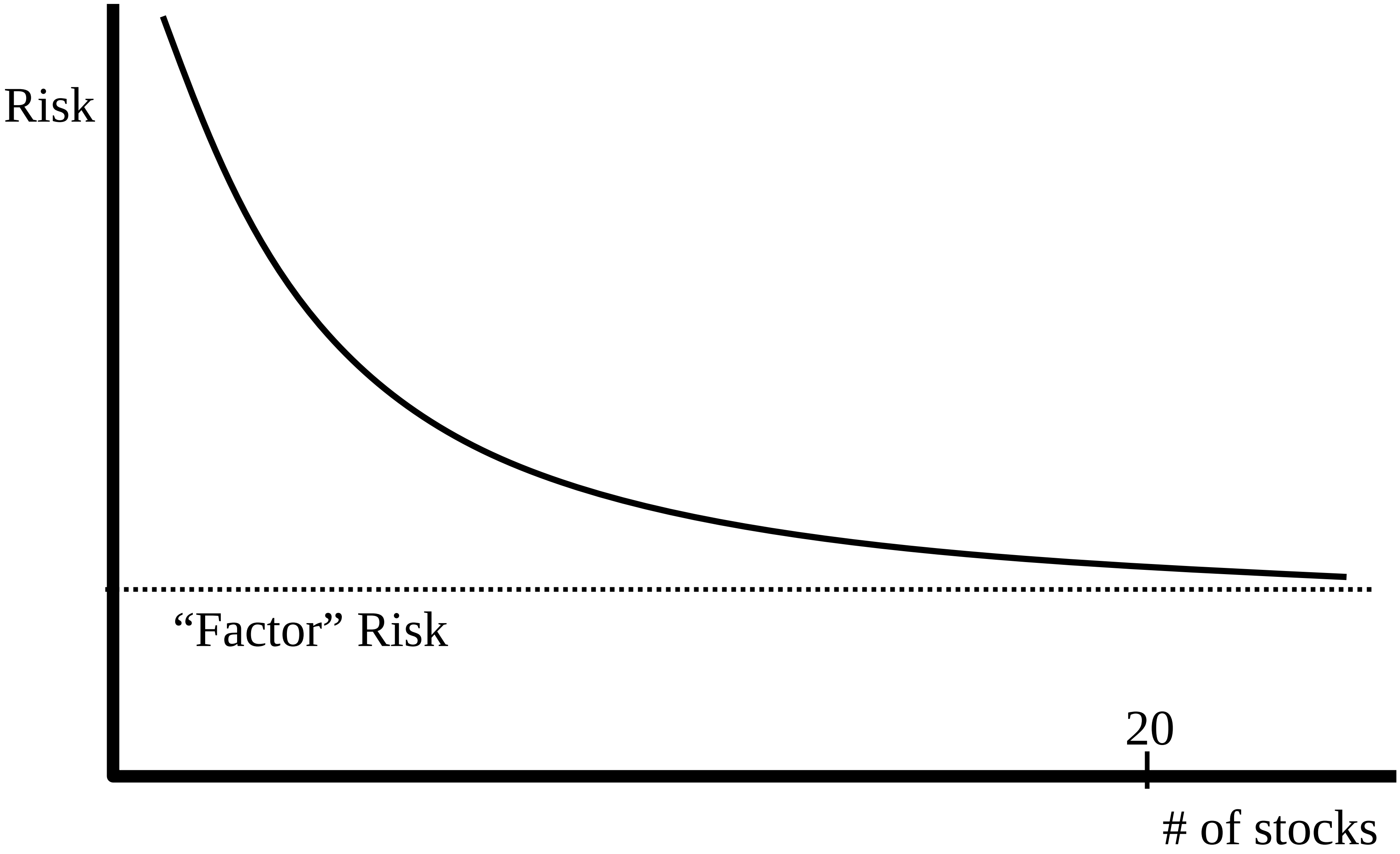
More Factors

- The last assumption however is unrealistic
- Need more factors. More generally, one could specify:

$$r_i = \alpha_i + \beta_{i,1} r_{\text{Factor 1}} + \ldots + \beta_{i,K} r_{\text{Factor K}} + \varepsilon_i$$

  - Returns are assumed to be generated by relatively small number of factors
  - Betas are the sensitivities to each factor
  - ε_i are uncorrelated firm-specific components
- Factors:
  - Other examples: industrial production, oil prices,..
  - Usually rescaled to have mean of zero
- Risk from…
  - Common factors cannot be eliminated by diversification
  - Unique factors can be eliminated and should be ignored

How Large are Diversification Benefits?



Arbitrage Pricing Theory

- Under a set of assumptions:

$$\bar{r}_i - r_f = \beta_{i,1}(\bar{r}_{\text{Factor 1}} - r_f) + \ldots + \beta_{i,K}(\bar{r}_{\text{Factor K}} - r_f)$$

- A diversified portfolio with 0 sensitivity to each macro factor…
  - Is essentially risk-free and should offer no market premium
  - If the return is higher or lower than the risk-free rate then profits can be made by arbitrage

- A diversified portfolio with sensitivity to the factors…
  - Should offer a risk premium proportional to its sensitivity to the factor
  - Otherwise, profits from arbitrage can be made!

Example: Arbitrage Pricing Theory

Estimated risk premiums for taking on risk factors (1978-1990)

Factor	Estimated Risk Premium ($r_{factor} - r_f$)
Yield spread	5.10%
Interest rate	-.61
Exchange rate	-.59
Real GNP	.49
Inflation	-.83
Market	6.36

The weighted average cost of capital

Capital Structure

- So far we have assumed firm (or project) equity financed
- No debt interest payments and therefore only required return of shareholders matter
- CAPM or APT then gives opportunity cost of capital

- How do things change if firm has debt?

Weighted Average Cost of Capital

- Return that "an average" investor would have earned if she invested in any other firm with a comparable level of risk (opportunity cost of capital)
- Cost of capital is the cost of debt (e.g. bonds) and equity (e.g. common stocks) weighted by their market values

$$r = d\frac{D}{D+E} + k\frac{E}{D+E}$$

where d is the cost of debt, k the cost of equity and D the value of debt outstanding and E the value of equity outstanding

The proportion of debt over total value (E+D) is called gearing

Cost of debt and cost of equity

- Cost of equity already analysed

- Cost of debt…
  - Repayments due to interest on debt issued
  - Possible to estimate from current debt outstanding
  - Lower than cost of equity (since debt payments are senior)
  - Depends on risk-free rate and the risk premium of debt (related to the company rating assigned by credit rating agencies)

- More on the WACC later!!

A final example

Project A is expected to generate CF=$100milllion per year for three years. Given a risk-free rate of 6%, a market risk premium of 8% and a beta of 0.75, what is the PV of the project?

$$r = r_f + \beta(r_m - r_f)$$
$$= 6 + .75(8)$$
$$= 12\%$$

Project A

Year	Cash Flow	PV @12%
1	100	89.3
2	100	79.7
3	100	71.2
	PV Total	240.2

Separating adjustments for risk and time?

Suppose alternative project generates lower but risk-free cash flows. Which reduction would you accept?

$$\frac{\text{CEQ}_1(\text{certainty - equivalent cash flow}_1)}{1.06} = 89.3$$

Project A

Year	Cash Flow	PV @12%
1	100	89.3
2	100	79.7
3	100	71.2
	PV Total	240.2

Project B

Year	Cash Flow	PV @ 6%
1	94.6	89.3
2	89.6	79.7
3	84.8	71.2
	PV Total	240.2

In general…

$$PV = \frac{C_t}{(1+r)^t} = \frac{CEQ_t}{(1+r_f)^t}$$

Certainty equivalences…

Year	Cash Flow	CEQ	Risk deduction
1	100	94.6	5.4
2	100	89.6	10.4
3	100	84.8	15.2

Larger risk deduction for later periods

Not necessary to discount at higher rates distant periods to generate a larger risk deduction

Risk premium

Risk premium can be defined as the ri such that…

$$\frac{C_i}{1 + r_i} = CEQ_i$$

In our case,

$$r_i = \frac{100}{94.6} - 1 = 0.57 \text{ or } 5.7\%$$

Appendix

Finding the MVP of Risky Assets

- Method to obtain MVP:
  - Find "weights" (w_1,.., w_N) of a "portfolio" w such that:
    - $Cov(r_w,r_i) = 1$ for all i=1,N
    - (weights may not add up to 1)
    - N equations with N unknowns
  - Rescale them to sum 1

- Intuition:
  - We look for a portfolio whose return has an equal covariance with every individual stock return
  - If they were not, we could reduce the variance by increasing the weight on the low covariance stock and reducing that of the high covariance stock

Example

A company wants to invest in the following countries at the minimum risk.

	India	Russia	China
India	.002	.001	0
Russia	.001	.002	.001
China	0	.001	.002

What are the “weights” that make the covariance equal to 1 for each country?

What is the MVP?

Finding the tangency portfolio

- Method to obtain T:
  - Find "weights" $(w_1,.., w_N)$ of a "portfolio" w such that:
    - $Cov(r_w,r_i) = E(r_i)- r_f$ for all i=1,N
    - weights may not add up to 1
    - N equations with N unknowns
  - Rescale them to sum 1

- Intuition:
  - We look for portfolios whose covariance with the return of each stock is equal to the risk premium of the stock
  - If they were not, we could increase the mean while lowering the variance

Example (continued)

Suppose that India, Russia and China have expected returns of 15%, 17% and 17% resp. and the risk-less asset is 6%.

- What are the “weights”?
- What is the tangent portfolio?

Finding Efficient Frontier of Risky Assets

- We only need to find two assets in the frontier (two-fund separation)
  - Hypothesise a risk-less asset with lower return than MVP
  - Compute the hypothetical tangency portfolio using the previous return as a risk-free asset
  - Take weighted averages of the hypothetical tangency portfolio and the MVP

- What is the efficient frontier for the previous example?