Lecture 3: Reputation

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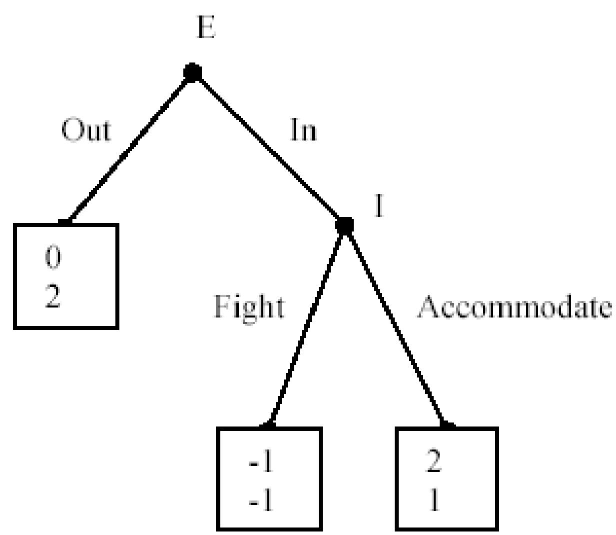
Today's Lecture

- In dynamic games of incomplete information, actions can reveal information about players' types
- Knowing this, players have incentives to tailor actions to manipulate inference
- Others anticipate this manipulation
- They attempt to make inference subject to the knowledge that they are being manipulated
- Examples: reputation, signalling, cheap talk

Example

- Another version of the "entry game":



Reputation

- Only plausible outcome (SPNE) is (In, Accommodate)
- In practice, incumbent may fight in order to establish a reputation for toughness
- Definition:
  "An individual has reputation if she is expected to behave in a certain way in the current environment because she has behaved similarly in similar environments"
- Model:
  (a) repeated game
  (b) facing different opponents at each period

Reputation in the Entry Game

- Suppose that the incumbent plays the game...
  repeatedly (infinitely), discounting the future at rate δ
  against a sequence of different opponents appearing only once
- In one SPNE, entrant enters and incumbent accommodates in each period
- Can a tough reputation be established in equilibrium? Consider the following...
- Strategy opponents:
  if either all opponents stayed out in the past or if the incumbent has never accommodated entry in the past, then play Out
  otherwise, play In
- Strategy incumbent (if the current opponent enters):
  if either all opponents stayed out in the past or if the incumbent has never accommodated entry in the past, then play $Fight$
  otherwise, play $Acco\operatorname{mod}ate$

Reputation in the Entry Game

- Is the previous strategy profile a SPNE?
- Opponent: (a) when either all opponents have stayed out in the past or the incumbent has never accommodated entry:
  Incumbent will play $Fight$, therefore Out is the best choice
- Opponent: (b) when some opponent entered and incumbent accommodated:
  Incumbent will play $Acco\bmod ate$, therefore In is the best choice

- Incumbent: (a) when either all opponents have stayed out in the past or the incumbent has never accommodated entry:
  Assuming entry occurs, payoff from equilibrium strategy $(Fight)$: $\{-1, 2, 2, ...\}$ since opponents stay Out. Utility of $-1 + \frac{2\delta}{1-\delta}$
  Assuming entry occurs, payoff from deviation $(Acco\operatorname{mod}ate)$: $\{1, 1, 1, ...\}$, since opponents play In (this is the best deviation). Utility of $\frac{1}{1-\delta}$
  $Fight$ is optimal whenever

$$-1 + \frac{2\delta}{1-\delta} \geq \frac{1}{1-\delta} \qquad \text{or} \quad \delta \geq \frac{2}{3}$$

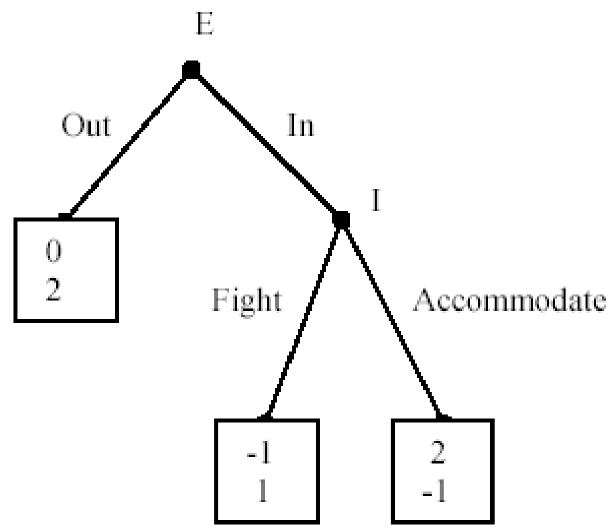
- Incumbent: (b) when some opponent entered and incumbent accommodated: Regardless of what the incumbent does, all future opponents will play In, therefore the best choice is to $Acco\operatorname{mod}ate$

Conclusions

- Incumbent benefits from reputation for toughness (she will fight if someone enters to sustain reputation)
- However, in equilibrium it *never* fights and does nothing to create or maintain the reputation
- The previous pair is only one of the SPNE of the game. In other SPNE the incumbent does not benefit from reputation
- Impossible to sustain if the game is finitely repeated

Incomplete Information

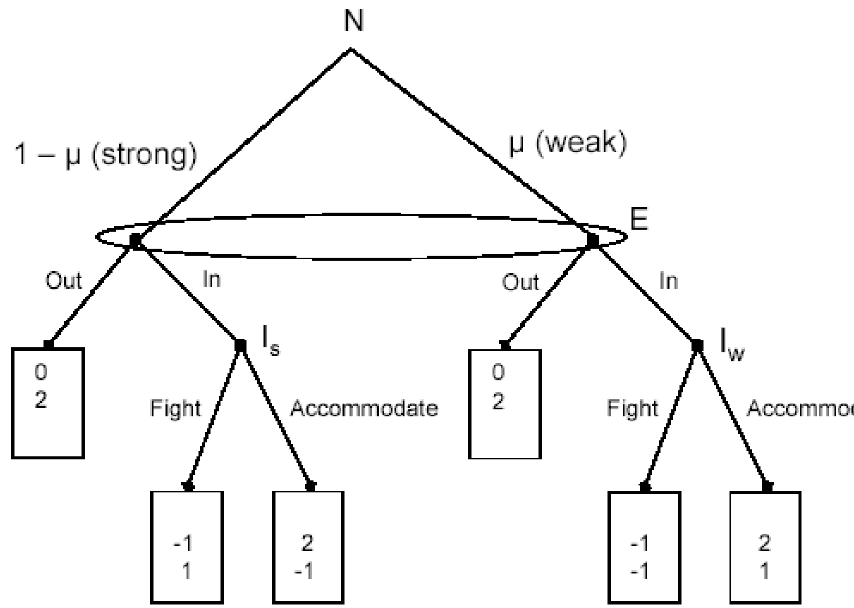
- Two types of incumbents: weak (W) or strong (S)
  If the incumbent is weak the game is as before. If she is strong...



- SPNE here: $(Out, Fight)$

Extensive-Form Game

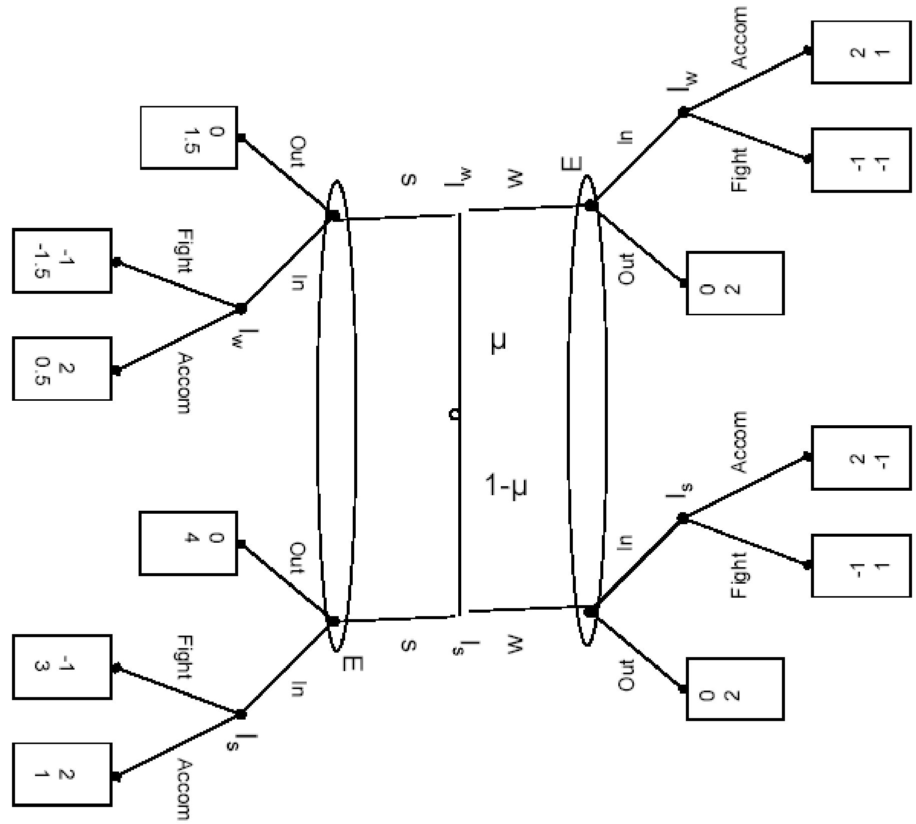
- Assume entrant gives probability μ to weak ($1-\mu$ to strong)



- SPNE: Entrant enters iff $\mu \geq \frac{1}{3}$, $Fight$ if strong, $Acco\,\text{mod}\,ate$ if weak

Reputation in Incomplete Information

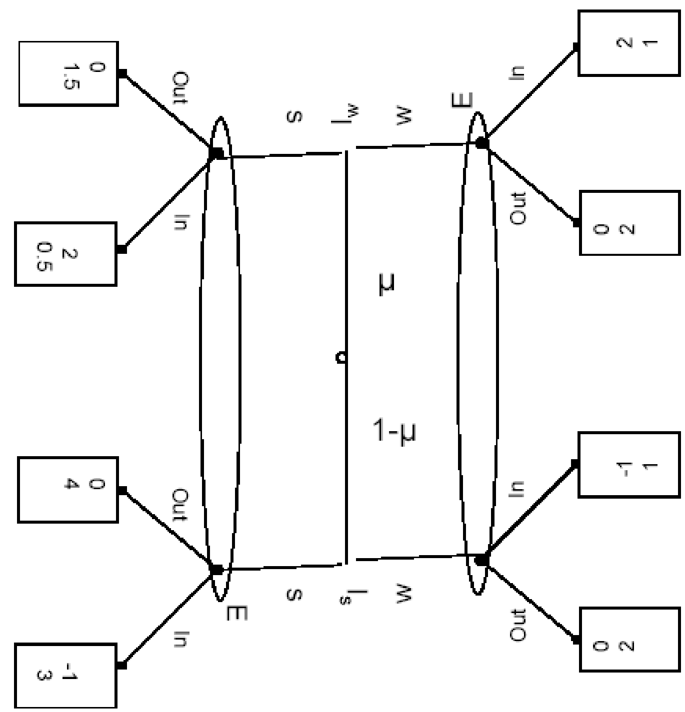
- Incumbent has been in the industry for a while
- Entrant looks at past behaviour to make inferences about incumbent's type
- Incumbent know this and may behave to mislead entrants
- Simple model: before the entry game, incumbent can raid another market (s, acting strong) or not (w, acting weak)
- Payoffs:
  in first period: for I_s 2 from s and 0 from w; for I_w $-1/2$ from s and 0 from w
  in second period, as before
  assume no discounting



E
In
Out
Fight
Accom
I_w
I_s
w
s
o
μ
1-μ
0 1.5
-1 -1.5
2 0.5
0 4
-1 3
2 1
2 1
-1 -1
0 2
2 -1
-1 1
0 2

Sequential Equilibria

- Since sequential equilibrium is subgame perfect...



Sequential Equilibria for $\mu < 1/3$

- Claim: there is a SE in which...
  both I_w and I_s play s
  E plays In if I has played w and Out if I has played s
  upon w, E believes she faces I_w with prob 1 (beliefs off-equilibrium-path)

- Proof:
  s is optimal for I_w: she obtains 1.5 from s and 1 from w
  s is optimal for I_s: she obtains 4 from s and 1 from w
  Out if s is optimal for E: she obtains $3\mu - 1$ from In and 0 from Out ($\mu < 1/3$)
  In if w is optimal for E: she obtains 2 from In and 0 from Out
  beliefs are consistent: if I_w plays w with prob ε and I_s plays w with prob ε^2

$$\Pr ob(I = I_w \mid w) = \frac{\mu\varepsilon}{\mu\varepsilon + (1-\mu)\varepsilon^2} \text{ which converges to 1 as } \varepsilon \to 0$$

Sequential Equilibria for $\mu < 1/3$ (2)

- Claim: there is no other SE. Proof:
- I_s plays s with certainty: lowest payoff if it plays s is 3 and highest for w is 2
- I_w cannot play w with certainty: s would indicate I_s with prob. 1 and E would play Out. w would indicate I_w with prob. 1 and E would play In. But then deviating to s, I_w would increase her payoff from 1 to 1.5
- I_w cannot mix between w and s: I_w would mix if she were indifferent. Upon observing w, E would infer that I was I_w. E would choose In, and E would receive a payoff of 1. Upon s, E would infer that I was I_w with prob. $\lambda < \mu < 1/3$ and would choose Out, which means that E would receive 1.5. Contradiction!!
- Conclusion: I_w imitates I_s to deter entry
  I_w succeeds because E fears that incumbent might be strong when observing s
  I_w acts as if she was strong to disguise her true type

Sequential Equilibria for $\mu > 1/3$

- Claim: there is no pure strategy SE. Proof:
  again I_s plays s with certainty and I_w cannot play w with certainty
  I_w cannot play s with certainty: Upon s, E would infer that I is I_w with prob. $\mu > 1/3$ and E would play In. Thus, I_w, by playing s gets 0.5 and by playing w, she gets no less than 1. She would have incentives to deviate

- Constructing the unique mixed strategy SE:
  again I_s plays s with certainty
  if I_w mixes between s and w, E will infer that I is I_w upon w and play In
  to make I_w indifferent between s and w, E should play In with prob 1/2
  suppose that I_w plays s with prob ϕ, and $\phi = (1-\mu)/2\mu$
  (given that $1/3 \leq \mu \leq 1$ then $1 \geq \phi \geq 0$)
  the posterior belief upon s is $\Pr(I = I_w \mid m) = \frac{\phi\mu}{1-\mu+\phi\mu} = \frac{1}{3}$

Sequential Equilibria for $\mu > 1/3$ (2)

- Conclusion: I_w deters entry in some cases by randomly imitating I_s
  deterrence is not complete
  I_s is hurt by this imitation
  probability of imitation declines with μ but effect on I_s does not change with μ

- Exercise: what happens for $\mu = 1/3$?

- See Fudenberg and Tirole for other reputation games