Abstract

Private equity partnerships are commonly regarded as examples of long-term commitment contracts. Limited partners commit the entire capital of the fund up-front and may face expensive default penalties for not honoring a capital call. Commitment, however, may be perceived to be stronger than it actually is, as explicit early termination provisions are also common. We develop a model to show how the degree of commitment can be used to screen general partners according to their managerial ability. Our model generates new predictions on the use of default penalties, on the investment strategy, and on the fee structures of general partners.

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1 Introduction

Private equity partnerships are commonly regarded as examples of long-term commitment contracts. They are structured as closed-end funds with a limited tradeability of shares, and a fixed life of about 10 years. The investors, known as limited partners, make a full capital commitment at the fund’s inception. As the fund managers, known as general partners, need cash to invest, they call in some of the committed capital. If a limited partner fails to honor a capital call, she may be required to pay a default penalty. Default penalties can be harsh, typically implying the loss of some of the profits, and sometimes resulting in the cancellation of the defaulter’s account (Lerner, Hardymon, and Leamon (2005), Litvak (2004)).

While the degree of investor commitment in private equity is probably higher than in other classes of investments, it may be perceived to be stronger than it actually is. The default penalties of some funds are very mild and bear very little consequences for the defaulter. Partnership agreements may even include explicit early termination provisions, such as the no-fault divorce clauses, which allow limited partners to default on a capital call with no need to provide a “just cause.” According to Schell (1999), no-fault divorce clauses are often invoked “simply because investors change their views about the private equity fund’s investment strategy or because investment strategies or economic terms offered by other private equity funds seem in retrospect more appealing.”

The inclusion of harsh default penalties in some funds and of no-fault divorce clauses in others reveals a significant degree of heterogeneity in investor commitment. Sometimes it seems optimal to have a high degree of commitment, while in other cases it seems better to give limited partners the freedom to withdraw from the fund at no cost. The question that naturally arises is in which situations is it best to commit, and does the degree of commitment bear implications for the investment strategy of the fund and on the fee structure of the general partners? To tackle these questions, this paper develops a model that illustrates how

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2 Defaults on capital calls became an “issue of significance” during 2001 and 2002, when hundreds of limited partners sought to reduce their remaining capital commitments. See Brett Byers, Secondary Sales of Private Equity Interests, Venture Capital Fund of America (Feb 18, 2002).
commitment, investment sizes, and fees, form part of an optimal contract between limited and general partners.

The logic of our argument is that, by limiting ex-post renegotiation, commitment reduces the cost of screening general partners. Managerial ability is initially difficult to assess, but becomes known over the various rounds of investments. Limited partners can use the information revealed in previous rounds to their own advantage, generating a *ratchet effect* (Laffont and Tirole (1987), Laffont and Tirole (1988)). In particular, limited partners can threaten to default on future capital calls unless the general partners accept worse contractual terms. Anticipating renegotiation in later rounds, general partners conceal their managerial ability in earlier rounds, thus hardening the initial screening process. To ensure effective screening, limited partners might need to inefficiently distort the investment size. This chain of events breaks down if limited partners can commit to not renegotiating in later rounds. Default penalties are meant to achieve precisely this objective, as they raise the cost of defaulting on a capital call, and reduce the limited partners’ ability to threaten and impose undesirable renegotiations on the general partners.

To understand the pros and cons of commitment, we develop a two-period repeated adverse selection model of a private equity partnership. Limited and general partners contract on the amount of capital that should be invested in each round, the compensation of the general partners, and the possible default penalties incurred by limited partners if they do not honor a capital call. The terms of a partnership agreement are chosen to induce general partners to self-select according to their ability. Better able general partners end up running partnerships with larger capital investments and higher fees, but are also expected to deliver greater returns. As in the real-world, general partners are compensated partly with fees that are proportional to the capital under management (*management fees*) and partly with fees that are non-proportional (such as *monitoring* and *transaction fees*). We show that the proportional fees should represent a smaller percentage of capital under management for larger funds. This is consistent with the finding of Gompers and Lerner (1999a) that management fees decrease
with fund size. To induce better general partners to run larger funds with lower proportional fees, limited partners offer them larger non-proportional fees.\(^3\)

We show that the size of the first capital call (and the associated investments) is distorted away from first-best. As in a one-period adverse selection model, a certain amount of distortion is needed to ensure self-selection. With two periods, the selection problem is compounded and separation between general partners can be achieved only by promising very high non-proportional fees to better general partners. Although they face lower returns per unit of capital under management, less able general partners will be tempted to set up a larger fund than they should, as the high non-proportional fees given to the managers of large funds more than offset the higher costs of managing a large fund. This will especially be the case if the differences in the managerial ability among general partners, and therefore the costs for less able managers of running a fund designed for more able managers, are low. In these circumstances, separation across general partners can once again be ensured by distorting the size of the fund in the first period. Larger funds are required to over-invest in the first period, while smaller funds under-invest.

The above mechanism breaks down if the limited partners commit their capital over several investment rounds, because less able general partners find it too costly to run a large fund in the long run. Thus, commitment improves efficiency by restoring the right level of investments for each type of fund. The main drawback of commitment is that limited partners lose the option to divert their capital into alternative investment opportunities when these arise. Therefore, default penalties as an instrument of commitment should be employed when the possible forgone investment opportunities are low, and the degree of asymmetric information is low. Indeed, if the differences in managerial ability are high, the distortion in investment is not necessary, and the default penalties carry no value.

Our results are in line with existing empirical evidence, and suggest new predictions for

\(^3\)Consistent with this, Toll and Vayner (2012) state that: “Along with management fees, many private equity funds can generate quite a bit of income by charging transaction fees, consulting fees and related fees to their portfolio companies. Should investors give their blessing to letting the general partner keep all or a large share of these fees, the general partner might well be willing to concede to a lower management fee.”
empirical research. For instance, our model implies that partnerships that invest in domestic markets, which are relatively less prone to problems of asymmetric information, should have higher default penalties and fewer early termination clauses than international funds. Consistent with this prediction, Toll and Vayner (2012) show that a high percentage of North American venture funds (73%) and North American buyout funds (72%) include the “forfeiture of a portion of the capital balance,” the most severe default penalty surveyed. This provision is in contrast much less common among international venture funds (47%) or international buyout funds (47%), and, no-fault divorce clauses are much more common among international funds (51%) than among North American funds (39%).

The literature on private equity has dedicated a greater deal of attention to the relationship between venture capitalists and entrepreneurs (Casamatta (2003), Cornelli and Yosha (2003), Gompers (1995), Hellmann (1998), Kaplan and Strömberg (2003), Kaplan and Strömberg (2004), and Schmidt (2003)). However, there has been little research on the design of partnership agreements.\(^4\) Axelson, Strömberg, and Weisbach (2009) (ASW) show how committing capital for multiple investments reduces the general partner’s incentives to make bad investments. Relative to financing each deal separately, compensating a general partner on aggregate returns reduces his incentives to invest in bad deals, since bad deals contaminate his stake in the good deals. ASW are primarily concerned with the provision of incentives to general partners to make the right investments. We, on the other hand, focus on how contracts are designed to screen general partners with a heterogeneous ability. The returns of investments are uncorrelated over time in ASW’s model (deals proposed to the general partner) but are correlated in ours (ability of the general partner). The sequential nature of the investment rounds is crucial in our model whereas in ASW’s model the investments can be simultaneous. We regard our findings as complementary to those of ASW.

Earlier papers have stressed the role of performance-based fees, such as the carried interest, to reduce the mismanagement of the fund by the general partner (Lerner and Schoar (2004)).

\(^4\)See Gompers and Lerner (1999b) and Sahlman (1990) for an overview of the structure and main characteristics of private equity partnerships.
In our setting, we deliberately abstract from moral hazard issues and focus on how adverse selection problems drive the choice of management and deal fees. Deal fees have recently attracted attention in the media because they have increased substantially in the aftermath of the crisis, while management and performance fees have been reduced. To our knowledge, our paper provides a first rationale for the use of deal fees.

Our model offers new predictions on the split of capital commitments into capital calls, and shows how these investment dynamics over the life of the fund are related to the size of the fund and to the inclusion of default penalties. Both Gompers and Lerner (1999a) and Lerner and Schoar (2004) examine how the partnership agreement evolves from one fund to a successor fund. Contrary to theirs, our analysis focuses on the relationship between compensation and investments during the life of a fund and not across successor funds. By breaking the life of the fund into individual investment rounds, we are able to provide an explanation for the illiquidity of the securities held by limited partners, which is alternative to that of Lerner and Schoar (2004).

In terms of the theoretical setup, we benefit primarily from Laffont and Tirole (1987, 1990) and from Laffont and Martimort (2002). Our model offers a generalization of the model proposed by Laffont and Tirole (1987, 1990), by allowing the reservation utility of the principal to vary stochastically over time, and allowing for the inclusion of termination fees. These new elements introduce a novel trade-off between long- and short-term contracts which was not present in the original model of Laffont and Tirole. Without stochastic reservation utilities, long-term contracts always dominate short-term contracts. We show that with stochastic reservation utilities, the ranking might be reversed.

The rest of the paper is organized as follows: in Section 2 we introduce the model. In Section 3 we describe the optimal contracts of a benchmark one period setting. In Section 4

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5 The Economist, "Private equity: Fee high so dumb. Some buy-out firms’ fees have gone up", November 12th 2011.

we examine the full-fledged two-period model, and describe the optimal contracts with and without default penalties. In Section 5 we carry out a comparison between the two contracting strategies both analytically and numerically. In Section 6, we provide a series of empirical predictions generated by the model. Section 7 concludes.

2 The model

2.1 Set up

Consider a two-period set up in which there is a limited partner (the principal) that has capital to invest and a general partner (the agent) that has investment opportunities. In each period \( i \), with \( i = 1, 2 \), a round of investments of total amount \( k_i \) generates a net payoff \( \hat{R}(k_i, \theta) \equiv R(k_i) - \theta k_i \), where \( R(\cdot) \) is publicly observable, and satisfies \( R'(0) > \theta, R'(\cdot) > 0, R''(\cdot) < 0 \), but \( \theta \) is only observable to the general partner.

The parameter \( \theta \) reflects the general partner’s overall level of efficiency: general partners with a lower \( \theta \) have the ability to generate better investment opportunities at the same cost and/or the same investment opportunities at a lower cost. We assume that there are two types of general partners, efficient and inefficient, so that \( \theta \in \{ \underline{\theta}, \bar{\theta} \} \) where \( \underline{\theta} < \bar{\theta} = \theta + \Delta \theta \). The general partner is efficient (type \( \underline{\theta} \)) with a common knowledge probability \( \nu_1 \) and inefficient (type \( \bar{\theta} \)) with probability \( 1 - \nu_1 \).

Furthermore, we assume that the general partner’s outside option is zero in both periods, while that of the limited partner is zero in the first period, and \( I \geq 0 \) in the second period. At the outset, \( I \) is distributed with a commonly known density \( f(I) \) and a cumulative distribution function \( F(I) \). The realization of \( I \) is privately known only to the limited partner. Both the limited partner and the general partner are risk neutral and protected by limited liability.

This setup captures four important features of a private equity partnership (also referred

\footnote{We assume that the probability of dealing with an inefficient manager is not too small. For some threshold level of \( \nu_1 \) the limited partner would choose not to contract with the inefficient manager at all.}
to as the *fund*). First, there is asymmetric information regarding the efficiency of the general partner because fund managers are generally better informed than investors about the fund’s investment opportunities, and about the ability and costs of the management team. In the model, the main source of information costs comes from the unobservability of $\theta$, which implies that the net payoff $\hat{R}(k, \theta)$ is not fully observable by the general partner.

Second, partnerships typically make several rounds of investments over their life. Therefore, at least two periods are needed in the model to capture the dynamics of the investment process. The two investment rounds may partially overlap, in the sense that the second round may occur before the first has been fully exited. However, it is important that they do not occur completely simultaneously. The interpretation of the model is unaffected by whether a round comprises an investment only in one company or in several.

Third, the model includes a stochastic outside option for the limited partner, which means that the opportunity cost of investing in the fund changes over time. $I$ may be interpreted as the positive net present value of an alternative (and mutually exclusive) investment opportunity that becomes available to the limited partner during the life of the fund.\(^8\) For the purpose of our model, the realization of $I$ must be known before the second investment begins. However, it is not important that $I$ is observed before the realization of the first payoff.

Fourth, the outside opportunity $I$ should not be contractible. If a contract could be written contingent on the realization of $I$ – for example, because $I$ was publicly observable and verifiable –, including a contingent default penalty would always be beneficial. In our application, however, we regard the non-contractibility of $I$ as a realistic feature of the model, which fits in well with the privately-observable nature of investment opportunities and liquidity needs that limited partners may face during the life of the fund.

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\(^8\)We model the outside investment opportunity similarly to the “deepening investment” of Holmström and Tirole (1998).
2.2 Investments, compensations, and default penalties

We assume that partnership agreements are designed by the limited partner and come as a take-it-or-leave-it offer for the general partner, whose role is to simply pick one contract from the menu. Contractual agreements should specify the size of the investment in each round, $k_i$, and a fee $t_i$ for the general partner in exchange of a repayment $r_i$ to the limited partner for each of the two rounds. At the end of investment $i$, the general partner obtains $t_i + R(k_i) - \theta k_i - r_i$ whereas the limited partner obtains $r_i - t_i$. Without loss of generality (transferring payments from $t_i$ to $r_i$ if necessary), for any $k_i$ we set $r_i = R(k_i)$, and therefore the general partner obtains a net payoff of $t_i - \theta k_i$ whereas the limited partner obtains $R(k_i) - t_i$. Also, without loss of generality, we assume that payments to the general partner occur round by round but the general partner cannot pick a contract that may entail a loss over the two rounds.

Due to the uncertain nature of $I$, a limited partner may want to default after one round. This suggests that a contract should also include a default penalty, $P$. The default penalty can be strictly positive or zero. The case of $P = 0$ can also be interpreted as that of a no-fault divorce provision. Unilateral default should be distinguished from consensual renegotiation that may occur in the second round with the agreement of both the general partner and the limited partner.

The timing of contracting is summarized in Figure 1. At time $t_0$, the limited partner offers a contract (or a menu of contracts) including capital commitments ($k_1, k_2$) and payments ($t_1, t_2$) for each of the two rounds, and a default penalty, $P$. Upon acceptance, round one begins and the general partner invests $k_1$. At time $t_1$, the limited partner observes $I$.

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In many finance applications, bargaining power is allocated to the agent rather than to the principal. However, we believe that the opposite assumption is more appropriate in our context. During the last decade, the contracting position of limited partners has become increasingly stronger due to a widespread use of gatekeepers, to the wider role played by institutional investors, and to the introduction of standardized sets of principles, such as those proposed by the Institutional Limited Partners Association (ILPA). Although our results do not crucially depend on the allocation of bargaining power, it is important that contracts are designed by the principal rather than by the agent. See Albert J. Hudec “Negotiating Private Equity Fund Terms. The Shifting Balance of Power”, Business Law Today, Volume 19, Number 5 May/June 2010; D. Peninon “The GP-LP Relationship: At the Heart of Private Equity.” AltAssets, 22 January, 2003; and ILPA Private Equity Principles (January 2011) downloadable from the ILPA website.
At time $t_2$, the limited partner can: (i) abide by the original contract stipulated in $t_0$; (ii) consensually renegotiate the original contract with the general partner, by (ii.a) offering the general partner an alternative one-period contract $(k'_2, t'_2)$, or (ii.b) offering a sufficiently high default compensation to the general partner; (iii) unilaterally terminating the original contract by paying the default penalty $P$ to the general partner, allowing the limited partner to (iii.a) offer a new one-period contract (or menu of contracts) $(k''_2, t''_2)$ to the general partner or (iii.b) obtaining the outside option $I$.

At time $t_3$, the first round ends and the general partner obtains a net payoff of $t_1 - \theta k_1$ whereas the limited partner obtains a profit $R(k_1) - t_1$. At time $t_4$, the second round ends. Second-round payments are similar to first-round ones if the partnership “continues”, either because the original contract is maintained (option i), or because the general partner accepts the renegotiation offer (option ii.a), or because the general partner accepts the new contract (option iii.a). Instead, if default occurs, either because the general partner accepts the new offer (option ii.b), or because the limited partner pays the default penalty (option iii.b), the general partner obtains either the new offer or $P$; whereas the limited partner obtains $I$ in both cases.

3 Benchmark one-period contracts

This section uses a reduced one-period problem to introduce (i) the adverse selection problem between general partners and limited partners, (ii) the type of optimal contract that can address this problem, and (iii) some notation to be employed in the rest of the paper.

Assume in this section that there is a single round of investments. The limited partner might be willing to invest only if the general partner is efficient. In this case, the limited partner optimally offers a single contract with a first-best size of investment. If instead the
limited partner is willing to set up a partnership with both types of general partners, then she offers a menu of fully separating contracts. The following proposition shows the optimal menu of one-period contracts, which include investment sizes and compensations (but not yet default penalties).

**Proposition 1 (Benchmark One-Period Contracts)** If the limited partner wants to employ only efficient general partners, she offers the following contract:

\[(t, k) = (\theta k^F B, k^F B)\],

whereas if the limited partner is willing to employ both types of general partners, she offers the following menu of contracts:

\[(t, k) = \left(\theta k^F B + \Delta \theta k^S B(\nu_1), k^F B\right), \quad (\bar{t}, \bar{k}) = \left(\bar{\theta} k^S B(\nu_1), \bar{k}^S B(\nu_1)\right),\]

where the investment sizes for an efficient and an inefficient general partner, \(k^F B\) and \(k^F \bar{B}\), are defined as the unique \(k\) that satisfies \(R'(k) = \theta\) and \(R'(k) = \bar{\theta}\), respectively. And, the second-best (SB) investment size for the inefficient general partner for a given belief of the limited partner, \(k^S B(\nu)\), is defined as the unique \(k\) that satisfies, for any \(\nu\),

\[R'(k) = \bar{\theta} + \frac{\nu}{1 - \nu} \Delta \theta.\]

The surplus from implementing first and second-best investment sizes are

\[R^F B \equiv R(k^F B) - \theta k^F B, \quad R^F \bar{B} \equiv R(\bar{k}^F B) - \bar{\theta} k^F B \quad \text{and} \quad R^S B(\nu) \equiv R(k^S B(\nu)) - \bar{\theta} k^S B(\nu).\]

Ideally, the limited partner would like to assign a first-best level of investment to both types of general partner, \(k = k^F B\) and \(k = \bar{k}^F B\), and compensate each of them with the level of fees that allows them to break even, \(t = \theta k^F B\) and \(\bar{t} = \bar{\theta} k^F B\). If both of these contracts
are offered, however, the efficient general partner would have an incentive to claim to be an inefficient type so as to obtain a net payoff equal to $\Delta \theta \bar{k}$, because she would be able to run the fund at a lower cost or generate better investment opportunities. In order to induce the efficient general partner to pick the contract with the largest capital investment, $\bar{k}$, the limited partner needs to offer her rents, non-proportional to the amount invested, and equal to the payoff she would obtain by claiming to be inefficient, $\Delta \theta \bar{k}$.

The optimal contract addresses the trade-off between efficiency and rents. To reduce the rent paid to the efficient general partner, the optimal menu of contracts requires a downward distortion of the optimal investment for the inefficient general partner ($\bar{k} = \bar{k}^{SB} (\nu_1) < \bar{k}^{FB}$).

In the benchmark one-period contracts, the inefficient general partner does not have an incentive to claim to be efficient, as the gains in terms of non-proportional fees or rents ($\Delta \theta \bar{k}^{SB} (\nu_1)$) are outweighed by the costs of running the fund designed for the efficient general partner ($\Delta \theta \bar{k}^{FB}$). That will change in the presence of multiple investment rounds, to which we turn to in the next section.

In sum, in the case of benchmark one-period contracts, efficient general partners run larger funds, $k^{FB} > \bar{k}^{SB} (\nu_1)$, and generate more surplus, $R^{FB} > R^{SB} (\nu)$, than inefficient general partners. The limited partner also expects larger profits from contracting with efficient general partners and therefore from running larger funds, $R (k^{FB}) - \ell = R^{FB} - \Delta \theta \bar{k}^{SB} (\nu_1) > R^{SB} (\nu) = R (\bar{k}^{SB} (\nu_1)) - \ell$.\(^{10}\) Efficient general partners also receive larger net payoffs (or “rents”), $\ell - \theta \bar{k}^{FB} = \Delta \theta \bar{k} > 0 = \ell - \theta \bar{k}^{SB} (\nu_1)$. The contract designed for the efficient general partner requires that part of the fees should be proportional to the size of investment, $\theta \bar{k}^{FB}$, and part of them should be non-proportional ($\Delta \theta \bar{k}$). The proportional fees cover the exact cost of managing the fund.\(^{11}\) Non-proportional fees provide general partners with incentives to truthfully reveal their type and to accept lower proportional fees.

\(^{10}\)This can be shown by contradiction. Suppose that $R (k^{FB}) - \ell < R^{SB} (\nu)$. Then, given that $R^{SB} (\nu) < R^{FB}$, the expected profits of the LP are lower than $R^{FB}$. But then the LP would prefer to offer a pooling contract rather than a separating contract, which contradicts the assumption that $\nu$ is small enough so that separation is optimal (see footnote 7).

\(^{11}\)It is standard practice for management fees to pay for the salaries of the partners and supporting employees, to pay for office space, and to pay for other expenses related to operating the partnership.
4 Designing optimal partnerships

We now turn to the full-fledged two-period model. To highlight the costs and benefits of including default penalties, we proceed by first assuming in Section 4.1 that default penalties cannot be included in the partnership agreement. We will relax this assumption in Section 4.2 and allow for the inclusion of default penalties. We provide a comparison of the benefits and costs of each contracting strategy in Section 5.

4.1 Partnerships without default penalties

As there are no default penalties, the limited partner can (and will) withdraw from the partnership at no cost at the end of the first round, or offer a new one-period contract (or menu of contracts) at the beginning of the second round, independently of what was agreed upon at the beginning of the first round. The partnership is effectively structured as a series of (two) one-period contracts. We can therefore proceed by backward induction: first, we derive the optimum second-period contract. Second, we identify the thresholds for \( I \) above which the limited partner finds it optimal to default. Third, we derive the optimal contract for the first round.

Denote the limited partner’s updated belief that the general partner is efficient at \( t_2 \) as \( \nu_2 \). The problem is that of the one-period contract, analyzed in the previous section, based on the revised expectations \( \nu_2 \).

**Proposition 2 (Optimum Contracts for the Second Investment Round)** The optimal contracting strategy is as described in Proposition 1 replacing \( \nu_1 \) with the revised expectations \( \nu_2 \).

Anticipating the second-round contracts, we now compute the thresholds for \( I \) that define the optimum default strategy at \( t_2 \). We compare the limited partner’s profits of (i) continuing with both types (offering the menu of contracts (2)), (ii) continuing with the efficient
type (offering contract (1)) or (iii) terminating (offering no contract). The next proposition summarizes the limited partner’s optimal decision for each realization of $I$.

**Proposition 3 (Default Thresholds for Contracts without Default Penalties)** The limited partner will (i) continue with both types if $I \leq I(\nu_2)$, (ii) continue only with the efficient general partner if $I(\nu_2) < I < \overline{I}$, and (iii) default with either type if $I \geq \overline{I}$, where

$$I(\nu_2) \equiv \overline{R}^{SB} - \frac{\nu_2}{1 - \nu_2} \Delta \overline{R}^{SB} \quad \text{and} \quad \overline{I} \equiv \overline{R}^{FB}.$$

As outside investment opportunities improve, limited partners first default with inefficient general partners, and then they terminate all partnerships, including those with efficient general partners. Default is unavoidable if the realization of $I$ is larger than the profits that the limited partner makes at first-best with an efficient general partner, which are the highest possible profits the partnership can produce. If the limited partner knows with certainty that she is employing an inefficient general partner ($\nu_2 = 0$), continuation occurs if and only if $I \leq I(0) = \overline{R}^{FB}$. Whereas, if the limited partner knows the general partner is efficient ($\nu_2 = 1$), continuation occurs if and only if $I \leq \overline{I} = \overline{R}^{FB}$.

At $t_0$ the limited partner can offer (i) a menu of fully separating contracts, which induces the general partners to truthfully reveal their types in the first round, allowing the limited partner to fully update her beliefs to $\nu_2 = 1$ or $\nu_2 = 0$, (ii) a partially separating menu of contracts, which depends on the induced probability of the efficient and the inefficient general partner telling the truth about their types (Laffont and Tirole, 1987), or (iii) a single pooling contract, which does not allow any updating of beliefs, $\nu_2 = \nu_1$. To simplify the exposition, we describe the fully separating contracts in the text, and relegate the complete set of contracts to the proofs. As we will show in Section 5, the fully separating contracts are optimal in most circumstances.

The following proposition summarizes the optimum first-round separating contracts.
Proposition 4 *(Optimum Contracts for the First Investment Round)* Let

\[
\Delta \theta \bar{k}_{FB} > \Delta \theta \bar{k}_{SB}^{*}(\nu_{1}) + F\left(\bar{R}_{FB}\right) \Delta \theta \bar{k}_{FB}.
\] (4)

If condition (4) holds the optimal menu of fully separating contracts offered in the first round is

\[
(t_{1}, \bar{k}_{1}) = \left(\theta \bar{k}_{FB} + \Delta \theta \bar{k}_{SB}^{*}(\nu_{1}) + F\left(\bar{R}_{FB}\right) \Delta \theta \bar{k}_{FB}, \bar{k}_{FB}\right), \quad (\bar{t}_{1}, \bar{k}_{1}) = \left(\bar{\theta} \bar{k}_{SB}^{*}(\nu_{1}), \bar{\theta} \bar{k}_{SB}^{*}(\nu_{1})\right).
\]

If it is not satisfied, there exists a unique \( \bar{k}_{1}^{*} \) and \( \bar{k}_{1}^{*} \), with \( \bar{k}_{1}^{*} < \bar{k}_{FB} < \bar{k}_{FB} < \bar{k}_{1}^{*} \), such that the optimal menu is

\[
(t_{1}, \bar{k}_{1}) = \left(\theta \bar{k}_{1}^{*} + \Delta \theta \bar{k}_{1}^{*} + F\left(\bar{R}_{FB}\right) \Delta \theta \bar{k}_{FB}, \bar{k}_{1}^{*}\right), \quad (\bar{t}_{1}, \bar{k}_{1}) = \left(\bar{\theta} \bar{k}_{1}^{*}, \bar{k}_{1}^{*}\right).
\]

As in the second part of Proposition 1, the limited partner would like to choose a first-best investment size for the efficient general partner, and a downward distortion of investment for the inefficient general partner. However, to induce the efficient general partner to accept the investment strategy designed for her, the limited partner needs to offer non-proportional fees related to both investment rounds. The minimum level of non-proportional fees that should be offered to the efficient general partner is equal to the net gains she would get by concealing her type and taking the contract designed for the inefficient general partner \( (\Delta \theta \bar{k}_{SB}^{*}(\nu_{1}) + F\left(\bar{R}_{FB}\right) \Delta \theta \bar{k}_{FB}). \)

Unlike Proposition 1, the high non-proportional fees might induce the inefficient general partner to take the contract designed for the efficient general partner. The inefficient partner would take the efficient’s contract if the non-proportional fees offered to the efficient partner offset the cost of running the fund of the efficient partner \( (\Delta \theta \bar{k}_{FB}), \) i.e., if condition (4) is not satisfied. If the limited partner was offered the contract of the first part of the proposition, there would be *countervailing incentives*, in the sense that the inefficient general partner would
conceal her type and pretend to be efficient for one period and reject the second one-period offer at the beginning of the second period.\footnote{A related phenomenon is that of \textit{grandstanding} described in Gompers (1996), according to which lesser known general partners take companies public earlier than more established counterparts in order to establish a reputation and raise capital for successor funds.} Separation of types requires further distortion in investment size, more than it occurs in one-period contracts. As shown in the second part of the proposition, the efficient type is distorted upward above first-best, while the inefficient type is distorted downward below second-best.

The contracts of this section suggest two possible sequences of events that are described in Figure 2.

Figure 2 approximately here

4.2 Partnerships with default penalties

In this section, we allow for contracts with default penalties, and show that these contracts can be used to eliminate countervailing incentives. We show that the inclusion of default penalties removes any advantage of using the information obtained in the first round against the general partner. In other words, it eliminates the \textit{ratchet effect} that arises from renegotiation in the second round. In the terminology of Laffont and Tirole (1989), the contracts that we examine in this section are referred to as \textit{long-term renegotiation-proof contracts}. We show that with these contracts investment distortion is smaller, but default is more costly. Ultimately, the choice between contracts with and without default penalties is dictated by a trade-off between investment distortions and default costs.

We proceed again in three steps: first, we derive the contractual terms for the second round, which now requires computing the renegotiation-proof contracts with the lowest possible rents for the efficient general partner and the associated default penalties. Second, we identify the default thresholds for $I$. Third, we derive the optimal contracts that cover both first and second investment rounds.

To prevent a ratchet effect, an optimum contract must satisfy two properties. First, the
second-period part of the contract should be renegotiation-proof, which means that no Pareto improving changes can be made to the contract at time $t_2$.\footnote{To illustrate a possible renegotiation that is mutually accepted by the general partner and the limited partner, consider a contract in which the general partner invests at the optimum level of the one-period framework, i.e., $k^{FB}$ and $k^{SB}(\nu_1)$ depending on the type. Then, at time $t_3$ the limited partner can raise the investment level for an inefficient general partner to $k^{FB}$. This change in the contract would be Pareto improving as it increases returns for the limited partner while leaving the utility of the general partner unchanged.} Second, the contract should include positive default penalties to prevent strategic defaults of the contract by the limited partner. By strategic defaults we refer to those that are done for the sole purpose of worsening the contractual terms for the general partner.

**Proposition 5 (Optimum Contracts for the Second Round of Investments and Default Penalties)** The optimal second-period contracts are (1) and (2) as described in Proposition 1 replacing $\nu_1$ with the revised expectations $\nu_2$. Default penalties are set to zero ($P = 0$) if contract (1) is used, while $P = \Delta \theta k^{SB}(\nu_2)$ if contract (2) is used.

The investment size and fees are exactly the same as in the case of contracts without default penalties, described in Proposition 2. This means that the benchmark contracts, described in Proposition 1, are renegotiation-proof. The two parties never agree on a new one-period contract at the beginning of the second investment round that substitutes the second-period part of the original two-period contract. To avoid strategic default by the limited partner, the contract needs to include a default penalty. The limited partner might reduce the net profits paid to the efficient general partner by paying the default penalty and offering her a new one-period contract. To prevent such renegotiation, the default penalties are set equal to the net profits of the efficient general partner, i.e., $P = \Delta \theta k^{SB}(\nu_2)$ if contract (2) is used.

However, if the outside option is sufficiently attractive, the limited partner prefers to default and pay the default penalty. If the second part of the contract is the first described in Proposition 5, the limited partner can, at $t_2$, either (i.a) abide by the original contract; or (i.b) unilaterally terminate by paying the default penalty $P = 0$. If the second-part of the contract is the second contract of Proposition 5, at $t_2$ the limited partner can either (ii.a) abide by the
original menu of contracts; (ii.b) continue with the efficient type only,\textsuperscript{14} or (ii.c) unilaterally terminate by paying $P = \Delta \theta k^{SB}(\nu_2)$. The next proposition summarizes the optimal decision of the limited partner for each realization of $I$ for both contracts.

**Proposition 6 (Default Thresholds for Contracts with Default Penalties)** For contract (2) the limited partner will (i) continue with both types of general partner if $I \leq \underline{\bar{I}}(\nu_2)$, (ii) continue only with the efficient general partner if $\underline{\bar{I}}(\nu_2) < I < \bar{I}(\nu_2)$, and (iii) default with either type if $I \geq \bar{I}(\nu_2)$, where

$$\underline{\bar{I}}(\nu_2) \equiv R^{SB}(\nu_2) \quad \text{and} \quad \bar{I}(\nu_2) \equiv R^{FB} + \frac{1 - \nu_2}{\nu_2} \Delta \theta k^{SB}(\nu_2).$$

For contract (1) the thresholds are the same as above with $\nu_2 = 1$.

Default here occurs less often than in Proposition 3, as both default thresholds are now higher. In addition, default is costly if $I \geq \bar{I}(\nu_2)$ because the limited partner pays a strictly positive default penalty, $P = \Delta \theta k^{SB}(\nu_2)$. However, in the special case of full separation (i.e. either $\nu_2 = 1$ or $\nu_2 = 0$) the default thresholds are the same as in Proposition 3, and the default penalties are never paid.

As in the previous section, we now offer a partial characterization of the first round contracts. At $t_0$ the limited partner can offer (i) a menu of fully separating contracts, which induces general partners to truthfully reveal their types in the first round. Upon acceptance of one of these contracts the limited partner fully updates her beliefs to $\nu_2 = 1$ or $\nu_2 = 0$. Alternatively, the limited partner can offer (ii) a partially separating menu of contracts, in which both types of general partners can lie with some probability. In the text, we restrict the analysis to fully separating contracts.

**Proposition 7 (Optimum Contracts for the First Round of Investments)** The opti-

\textsuperscript{14}This is achieved by making a termination offer at 0 (or slightly above 0), which only the inefficient general partner will accept.
A comparison with Proposition 4 highlights the differences between contracts with and without default penalties. The contracts of Proposition 7 are identical to the first set of contracts of Proposition 4 with the following two differences. On the one hand, there are no countervailing incentives in the presence of commitment. The inefficient general partner does not have an incentive to take the efficient general partner’s contract as she would face the high cost of running a large fund for two periods. Therefore, it is not necessary to inefficiently distort investment size, as it happens in the second part of Proposition 4. On the other hand, the contracts with default penalties have higher non-proportional fees. This happens because in the case of contracts without default penalties, an efficient general partner claiming to be inefficient obtains a positive net payoff from the second round ($\Delta \theta k^F B$) only if continuation occurs, which happens with probability $F(R^{FB})$. Instead, in the case of default penalties an efficient general partner always obtains positive profits from the second round: in the form of net payoffs in case of continuation, and in the form of a default penalty when default occurs.

The contracts of this section suggest two possible sequences of events that are described in Figure 3.

5 Comparison across different contracting strategies

This section compares the contracting strategies with and without default penalties, both for the case of fully and partially separating contracts. To illustrate the main trade-off, the
following proposition provides sufficient conditions for the optimality of contracts in the case of full separation.

**Proposition 8 (Optimality of the Use of Default Penalties)** The comparison across contracts with and without default penalties is reduced to a trade-off between fees and investments. Contracts without default penalties carry lower fees because \( \Delta \theta^F \) is only paid with probability \( F(\overline{R}^F) \) but might require an investment distortion if condition (4) is not satisfied. In contracts with default penalties, \( \Delta \theta^F \) is paid with certainty but there is no distortion of the size of the investment round. Therefore, fully separating contracts with default penalties yield equal or larger profits than those without default penalties if \( F(\overline{R}^F) = 1 \). Fully separating contracts without default penalties yield strictly larger profits than those with default penalties if condition (4) is satisfied.

The above proposition highlights the importance of condition (4). Default penalties can only add value if condition (4) is not satisfied. The following proposition states when the condition is less likely to hold.

**Proposition 9 (Comparative Statics on Key Condition)** Condition (4) is less likely to be satisfied, and therefore default penalties can add value if, all else equal, the degree of asymmetric information \( (\Delta \theta) \) is smaller, the probability of observing an efficient type \( (\nu_1) \) is lower, and the outside opportunities are smaller \( (F(\cdot) \) is larger).

As mentioned earlier, condition (4) is not satisfied if for the inefficient partner the costs of running a larger fund are outweighed by the non-proportional fees offered to the efficient general partner. This happens if the difference in investment sizes between efficient and inefficient general partners in one-period contracts is sufficiently small. As shown in Proposition 1, the difference in investment sizes is smaller if the degree of asymmetric information is small and the probability of observing an efficient type is small. i.e., when the degree of asymmetric information is high.
In general, the two contracting strategies cannot be compared analytically. As in Laffont and Tirole (1987), the computation of the optimal contract requires a numerical procedure. Both for contracts with and without default penalties, the optimal contract might be a partially separating contract, i.e., a contract that depends on the induced probability that an efficient general partner chooses the contract designed for himself, $x$, and the induced probability that the inefficient general partner chooses the contract designed for himself, $y$. The equivalent of condition (4) for the general case of partially separating contracts is as follows (see the proof of Proposition 4, condition (12)):

$$
\Delta \theta^F \geq \Delta \theta^{SB}(\nu_1 x) + F(I(\varphi_2(x))) \Delta \theta^{SB}(\varphi_2(x)).
$$

In the case of contracts without default penalties, the optimal contract might also be a pooling contract, i.e., a contract in which the efficient and inefficient general partners are required to invest the same amount, and receive the same compensation.

We now carry out numerical simulations to compare contracts with and without default penalties for a general $x$ and $y$. We assume the following specification for the revenue function, $R = bk^a$, with $0 < a < 1$ and $b > 0$. We also assume that investment opportunities follow a uniform distribution function between 0 and $Z(\geq 0)$ so that $f(I) = 1/Z$ and $F(I) = I/Z$ for $0 \leq I \leq Z$.

Figure 4 approximately here

We report our results in Figure 4. In all panels the red line plots the monetary value of the contracts with default penalties. The green line identifies (partially and fully) separating contracts without default penalties that satisfy condition (5). The yellow line represents separating contracts without default penalties that do not satisfy condition (5). The black line represents pooling contracts without default penalties.
Panels A and B provide comparatives statics for different values of the outside option \( Z \) for the parameter specification: \( a = 0.4, b = 1.2, \) and \( \theta = 1 \). In Panels A and B we set \( \bar{\theta} = 1.2 \) and \( \bar{\theta} = 1.5 \) respectively to identify the cases of low and high asymmetric information. Consistent with Proposition 9, contracts without default penalties do not satisfy condition (5) because the degree of asymmetric information is low. In this panel, contracts with default penalties dominate in most cases but for high values of the outside option. Panel B shows that for a high level of asymmetric information, condition (5) is more easily satisfied, and contracts without default penalties dominate. In both panels, an increase in the value of the outside option generates as a first order effect that contracts without default penalties dominate, because they are more flexible. The two panels also highlight the key role of condition (5): when the condition is not satisfied, contracts without default penalties require a distortion in investment that might make them dominated. This effect can clearly be seen in Panel C, where we fix the outside options by setting \( Z = 0.4 \), and let \( \bar{\theta} \) vary. As \( \bar{\theta} \) increases, the degree of asymmetric information also increases (higher \( \Delta \theta \)). For higher values of \( \Delta \theta \), contracts without default penalties dominate, because condition (5) becomes slack. In Panel D, we fix the outside option and the degree of asymmetric information, and let the percentage of efficient general partners, \( \nu_1 \), vary along the horizontal axis. As \( \nu_1 \), condition (5) becomes less stringent, and contracts without default penalties dominate.

We have generated the same graphs for the particular case of fully separating contracts, i.e., restricting \( x = 1 \) and \( y = 0 \). They are visually identical to the optimal partially separating contracts, indicating that the optimal contract is almost always (if not always) the fully separating one.

6 Empirical predictions

We now lay out the empirical predictions generated by the model in the case of full separation contracts. Following Proposition 1, efficient general partners run funds with larger investment
sizes. Therefore, in what follows we refer to large funds as those run by efficient general partners, and to small funds as those run by inefficient general partners.

6.1 Default Penalties

One of the main objectives of the paper is to understand when it is optimal to commit. Prediction 1, which follows from Propositions 8 and 9, and from the numerical analysis, states when default penalties as an instrument of commitment should be employed.

**Prediction 1** Default penalties are more likely to be employed when (i) there are worse outside investment opportunities; (ii) the degree of asymmetric information is lower, and (iii) the proportion of efficient general partners is smaller.

Assuming that funds which invest in domestic markets are relatively less prone to problems of asymmetric information that funds which invest internationally, Toll and Vayner (2012) provide survey evidence supporting our prediction. Most North American venture funds (73%) and North American buyout funds (72%) include the “forfeiture of a portion of the capital balance,” the most severe default penalty surveyed. In contrast this provision is much less common among international venture funds (47%) or international buyout funds (47%). Also consistent with the prediction, no-fault divorce clauses are much more common among international funds (51%) than among North American funds (39%).

An alternative test of the hypothesis could be based on the sequence number of the fund, which can also be a proxy of information asymmetry because it reflects the amount of prior information that is available for each fund (Lerner and Schoar (2004)). We are not aware of any empirical evidence on the effect of the outside opportunities. A potential test could be based on how the use of default penalties vary over the business cycle. Boom times are periods of low investment opportunities and therefore, according to our prediction, the use of default penalties should be high. Following Gompers and Lerner (2000), periods of high capital commitments can also be related to a lower average quality of managerial talent, as more inexperienced general partners enter into the industry. According to our prediction,
default penalties should be observed more often in periods of high capital supply.

Our second prediction relates to the optimal degree of commitment across funds. Proposition (7) and Figure 3 show that default penalties should be relatively larger for contracts designed for inefficient general partners, i.e., for small funds.

**Prediction 2** Default penalties are relatively more important in smaller funds.

### 6.2 Investment strategy

The model shows that limited partners might distort investment strategies to screen general partners. Default penalties, as an instrument of commitment, can be used as a substitute of inefficient distortion. In particular, Proposition 7 shows that, in the presence of default penalties, the total capital invested by large and small funds is \( k = k^{FB} + k^{FB} \), and \( \bar{k} = \bar{k}^{SB}(\nu_1) + \bar{k}^{FB} \), respectively. In the absence of default penalties, Proposition 4 shows that the total capital invested by large and small funds is distorted upwards and downwards, \( \bar{k} = k^{i} + k^{FB} \) and \( \bar{k} = \bar{k}^{i} + k^{FB} \), respectively, if condition (4) is not satisfied. This result allows us to generate a prediction for the relative differences of capital under management.

**Prediction 3** Capital under management has less dispersion across funds in the presence of default penalties.

The same result allows us to draw predictions on how to split capital commitments into capital calls. In the absence of default penalties, the size of large funds might need to be distorted upwards, and the one of small funds downwards, to ensure separation.

**Prediction 4** Capital commitment of large funds should be equally split across capital calls in the presence of default penalties. In the absence of default penalties, earlier capital calls might be larger than later ones. In small funds, earlier investment rounds should be smaller both in the presence and absence of default penalties.
Ljungqvist and Richardson (2003) show that there is substantial cross-sectional variation in draw down rates. According to our prediction, larger funds should draw down the same proportion of capital committed faster than smaller funds. Ljungqvist and Richardson find a negative, albeit not significant, effect of the size of the fund on the duration between a fund being raised and it having drawn down at least 80 or 90 percent of its committed capital. Unfortunately, they do not split the funds on their inclusion of default penalties.

6.3 Fee structure

The model calls for fees that are proportional to the capital under management, as well as for non-proportional fees. The real-world counterpart of the proportional fees would be the management fees, which typically represent 1.5 to 2.5 of the capital invested. Propositions 7 and 4 show that, both in the presence and in the absence of default penalties, proportional fees are \( \theta k \) and \( \bar{\theta} k \) for the efficient and inefficient general partners, respectively.

**Prediction 5** Management fees represent a higher percentage of investment in small funds.

Prediction 5 is supported by the findings of Gompers and Lerner (1999a). They compute the size of a fund as the ratio of the capital invested in the fund to the total amount raised by all other funds, and identify three size groups: partnerships i) with a ratio of 0 – 0.2 percent, ii) with a ratio of 0.2 – 0.7, iii) with a ratio greater than 0.7. They find that the present value of management fees for each of these classes is respectively, 19.9, 18.2, 15.1 percent of capital under management. Their evidence provides support for the model’s prediction that large funds receive lower management fees per unit of capital.

The model shows that limited partners might also need to pay positive non-proportional fees, such as monitoring or transaction fees, to efficient general partners, i.e., to larger funds. In the presence of default penalties, the non-proportional fees for the efficient and inefficient general partner are \( \Delta \bar{k}^{SB}(\nu_1) + \Delta \bar{k}^{FB} \) respectively, whereas, in the absence of default
penalties they are \( \Delta \theta^k + F(\nu_1) \Delta \theta^F \), as long as condition (4) is satisfied.

**Prediction 6** Monitoring and transaction fees are larger in (i) large funds, (ii) in the presence of default penalties, and (iii) in the early years of the fund.

Metrick and Yasuda (2010) report that the general partners of buyout funds charge transaction fees which vary between 1 and 2 percent of the transaction value. They also provide practitioners’ estimates of annual monitoring fees which vary between 1 and 5 percent of EBITDA, with smaller companies at the high end of this range and larger companies at the low end. Unfortunately, there is still very little empirical evidence on the frequency and size of these fees, and on how they relate to the observable characteristics of the funds.

### 7 Conclusions

This paper provides a two-period model which describes the interaction between limited partners and general partners as a dynamic principal-agent adverse selection problem. Limited partners set the contractual terms and conditions to screen general partners of heterogeneous and unobservable ability. Contracts may include default penalties that limited partners have to pay if they do not honor a capital call. The degree of commitment depends on if, and to which extent, the agreement includes default penalties.

We show that limited partners distort investment size to better screen general partners. The extent of distortion is smaller if default penalties are included in the agreement. Fees paid to general partners, however, are larger. Therefore, there is a trade-off between lowering fees and investing more efficiently. The mechanism that drives this result is that in the absence of default penalties, efficient general partners need to be provided with a high set of fees to account for the possible default of the limited partners. The high fees promised to efficient general partners distort the incentives of inefficient general partners, making adverse selection more severe. A high degree of distortion in the size of the investment rounds is then necessary to ensure separation.
The model generates several empirical implications regarding the use of default penalties, and how the investment strategy and the fees paid to general partners should be set in relation to fund size and upon the use of default penalties. The model predicts that default penalties should be more common when (i) there are worse outside investment opportunities, (ii) the degree of asymmetric information is lower, and (iii) the proportion of efficient general partners is smaller. It predicts the use of fees that are proportional to investment, which resemble the commonly employed management fees. Also, it predicts the use of other fees that are non-proportional to investment, and that may find an empirical counterpart in the deal fees. In our model, proportional fees represent a higher percentage of investment in small funds, while non-proportional fees are higher in large funds, consistent with the empirical evidence in Gompers and Lerner (1999a).

The predictions of the model offer possible avenues for future empirical research in the field of private equity contracts. One of these lines of research relates to how contractual choices vary over the business cycle. The parameters of our model can be employed to characterize different stages of the business cycle, as the average quality and heterogeneity of the population of general partners changes over time (Gompers and Lerner (2000)).
References


Proof of Proposition 1

The first statement is immediate. As for the second statement, for the case in which the limited partner chooses to employ both types of general partner, the limited partner offers a menu of contracts, \((t, k), (\bar{t}, \bar{k})\), one for each type, that solves

\[
\max_{\{t, \bar{t}, k, \bar{k}\}} \nu_1 [R(k) - t] + (1 - \nu_1) [R(\bar{k}) - \bar{t}]
\]

subject to

\[
\bar{t} - \bar{k} \geq 0 \ (TR); \ t - \theta k \geq \bar{t} - \bar{k} + \Delta \theta \bar{k} \ (IC);
\]

where \((TR)\) stands for the individual rationality constraint of the inefficient general partner, and \((IC)\) for the incentive compatibility constraint of an efficient general partner. The individual rationality constraint of the efficient general partner, \(t - \theta k \geq 0\), and the incentive compatibility constraint of the inefficient general partner, \(\bar{t} - \bar{k} \geq t - \theta k - \Delta \theta \bar{k}\), are satisfied when \((TR)\) and \((IC)\) hold. Cost minimization requires that both \((TR)\) and \((IC)\) bind at the optimum. Insert them into the objective function of the limited partner,

\[
\max_{\{k, \bar{k}\}} \nu_1 [R(k) - \theta k - \Delta \theta \bar{k}] + (1 - \nu_1) [R(\bar{k}) - \bar{k}].
\]

The first order conditions yield \(k = k^{FB}\) and \(\bar{k} = \bar{k}^{SB}(\nu_1)\), from which the statement of the proposition follows.

Proof of Proposition 3

If the limited partner decides to continue with both types (case (i)), substituting the optimal contract (2) into the objective function, her expected profits are \(\nu_2[R_{FB}^{FB} - \Delta \theta \bar{k}^{SB}(\nu_2)] + (1 - \nu_2) \bar{R}_{FB}^{SB}(\nu_2)\). If the limited partner decides to continue with the efficient general partner only (case (ii)), substituting the optimal contract (1) in the objective function, her profits are
\( \nu_2 R^{FB} + (1 - \nu_2) I \). If the limited partner decides to default (case (iii)) her profits are equal to \( I \).

A comparison across these three cases yields the statements of the proposition.

**Proof of Proposition 4**

The optimal menu of partially separating contracts in round one depends on \( x \), the probability that an efficient general partner tells the truth about his type, and \( y \), the probability that an inefficient general partner lies about his type. For given \( x \) and \( y \), the limited partner’s (second-period) updated beliefs that the general partner is efficient, conditional on the general partner respectively choosing either the efficient contract or the inefficient contract in the first round, are

\[
\nu_2 = \frac{\nu_1 x}{\nu_1 x + (1 - \nu_1)y} \quad \text{and} \quad \nu_2 = \frac{\nu_1 (1 - x)}{\nu_1 (1 - x) + (1 - \nu_1)(1 - y)}.
\]

Substituting \( \nu_2 \) and \( \nu_2 \) into Proposition 2 and Proposition 3, we obtain the optimal second-period contracts and default thresholds. If \( x = 1 \) and \( y = 0 \), both types of general partner are truthfully revealing their type in the first period, in which case \( \nu_2 = 1 \) and \( \nu_2 = 0 \); whereas if \( x = y \), the limited partner is not able to update her beliefs, in which case \( \nu_2 = \nu_2 = 0 \).

The incentive compatibility constraints of the efficient and inefficient general partners take into account the effects that telling the truth or lying have on the expected second-period fees. An efficient general partner tells the truth as long as

\[
t_1 - \theta k_1 + F \left( I (\nu_2) \right) \Delta \theta k^{SB} (\nu_2) \geq \tilde{t}_1 - \overline{\theta k}_1 + F \left( I (\nu_2) \right) \Delta \theta k^{SB} (\nu_2).
\]  (9)

The intertemporal incentive constraint of an inefficient general partner, accounting for the fact that in the second round the general partner always receives a zero rent, depends solely on first-period payoffs, and is given by

\[
\tilde{t}_1 - \overline{\theta k}_1 \geq \tilde{t}_1 - \overline{\theta k}_1 - \Delta \theta k_1.
\]  (10)
The participation constraint of an inefficient general partner is always binding to minimize costs, which means that \( \bar{t}_1 = \bar{\theta} k_1 \).

The incentive constraints of the efficient and inefficient general partners may or may not be binding. We analyze each case in turn.

**Case 1: Constraint (9) binding and (10) not binding**  An inefficient general partner always declares his true type, while the efficient one is indifferent between declaring the truth or lying. Therefore, when a general partner declares to be efficient, the general partner is automatically identified with certainty as efficient (full separation), \( \nu_2 = 1 \). Under these conditions, an efficient general partner’s second-period rent is \( \Delta \theta \bar{k}^{SB} (1) = 0 \). Using this result and the participation constraint of the inefficient general partner, the objective function of the limited partner at \( t_0 \) is equivalent to

\[
\max_{k_1, \bar{k}_1} \nu_1 x \left[ R(k_1) - \theta k_1 - \Delta \theta \bar{k}_1 - F(I(\bar{\nu}_2)) \Delta \theta \bar{k}^{SB} (\bar{\nu}_2) \right] + (1 - \nu_1 x) \left[ R(\bar{k}_1) - \bar{\theta} k_1 \right] \tag{11}
\]

As only the first-period payoffs depend on \( k_1, \bar{k}_1 \), we have ignored second-period payoffs in this maximization problem. From the first order conditions of this maximization we obtain that the optimum investment sizes are \( k_1 = \bar{k}^{FB} \) and \( \bar{k}_1 = \bar{k}^{SB} (\nu_1 x) \). As the solution depends on \( x \), a numerical maximization with respect to \( x \) yields the optimum productions and transfers.

This case applies as long as (10) is not binding. At the optimum, this constraint requires

\[
\bar{k}^{FB} \geq \bar{k}^{SB} (\nu_1 x) + F(I(\bar{\nu}_2(x))) \bar{k}^{SB} (\bar{\nu}_2(x)) . \tag{12}
\]

When \( x = 1 \), we have \( k_1 = \bar{k}^{FB} \) and \( \bar{k}_1 = \bar{k}^{SB} (\nu_1) \) and \( t_1 = \theta k_1 + \Delta \theta \bar{k}^{SB} (\nu_1) + F(\bar{R}^{FB}) \Delta \theta \bar{k}^{FB} \) and the condition becomes \( k^{FB} \geq \bar{k}^{SB} (\nu_1) + F(\bar{R}^{FB}) \bar{k}^{FB} \).
Case 2: Constraint (9) and (10) are both binding. Both types are indifferent and may lie. In this case the updated beliefs are

\[ \nu_2(x, y) = \frac{x\nu_1}{x\nu_1 + y(1 - \nu_1)} \quad \text{and} \quad \nu_2(x, y) = \frac{(1 - x)\nu_1}{(1 - x)\nu_1 + (1 - y)(1 - \nu_1)} \]

As (10) is binding and \( \bar{t}_1 = \overline{\theta k_1} \), we have that \( t_1 = \theta k_1 + \Delta \theta k_1 \). From (9) we then have

\[ \bar{k}_1 = k_1 + (F(I(\nu_2))\overline{\theta}^{SB}(\nu_2) - F(I(\nu_2))\overline{\theta}^{SB}(\nu_2)) \]  

(13)

We then obtain a maximization in one variable

\[ \max_{\bar{k}_1} (x\nu_1 + y(1 - \nu_1)) [R(k_1(\bar{k}_1)) - \overline{\theta}k_1(\bar{k}_1)] + (1 - x\nu_1 - y(1 - \nu_1)) [R(\bar{k}_1) - \overline{\theta}k_1] \]

In the maximization we have ignored the second round, because it does not depend on \( \overline{\theta}k_1 \). Observe that the derivative of \( k_1(\bar{k}_1) \) equals one. The first order condition of the maximization then is

\[ (x\nu_1 + y(1 - \nu_1)) [R'(k_1(\bar{k}_1)) - \overline{\theta}] + ((1 - x)\nu_1 + (1 - y)(1 - \nu_1)) [R'(\bar{k}_1) - \overline{\theta}] = 0 \]

A solution can be found via numerical optimization over \( x \) and \( y \).

When \( x = 1 \) and \( y = 0 \) we have that the first order condition reduces to

\[ \nu_1 R'(k_1(\bar{k}_1)) + (1 - \nu_1) R'(\bar{k}_1) = \overline{\theta} \]

(14)

where \( k_1 = \bar{k}_1 + F(R^{FB})k^{FB} \). Substituting this equation into the profits of the first round, and adding and subtracting \( \theta k_1 \), we have that

\[ \nu_1 \left[ R(k_1(\bar{k}_1)) - \theta k_1 - \Delta \theta k_1 - F\left( R^{FB} \right) \Delta \theta k^{FB} \right] + (1 - \nu_1) \left[ R(\bar{k}_1) - \overline{\theta}k_1 \right] . \]
Given that the maximum of $R(k) - \theta(k)$ with respect to $k$ is equal to $R^{FB}$, and the maximum of $-\nu_1 \Delta \theta k + (1 - \nu_1) [R(k) - \bar{\theta} k]$ with respect to $k$ is equal to $R^{SB} (\nu_1)$, we have that the previous equation is lower than

$$\nu_1 \left[ R^{FB} - \Delta \theta \bar{k}^{SB} (\nu_1) - F \left( R^{FB} \right) \Delta \theta \bar{k}^{FB} \right] + (1 - \nu_1) R^{SB} (\nu_1),$$

which are the profits of the first round in case 1.

Therefore, there exists a unique $k_1^*$ and $\bar{k}_1^*$, as claimed in the text. We now show by contradiction that $k_1^* > k^{FB}$ and $\bar{k}_1^* < \bar{k}^{SB} < k^{FB}$. Suppose that $k_1^* \leq \bar{k}^{FB}$ then $R'(k_1^*) \geq \bar{\theta}$ and replacing at (14), we have

$$R' \left( \bar{k}_1^* \right) \leq \bar{\theta} + \frac{\nu_1}{1 - \nu_1} \Delta \theta$$

and therefore $\bar{k}_1^* \geq \bar{k}^{SB}$ and $k_1^* - \bar{k}_1^* \leq k^{FB} < k^{SB} < F(R^{FB}) \bar{k}^{FB}$, which contradicts that $k_1^* = \bar{k}_1^* + F(R^{FB}) \bar{k}^{FB}$. Using the same argument, given that $k_1^* > \bar{k}^{FB}$ we have that $\bar{k}_1^* < \bar{k}^{SB}$.

**Case 3: Constraint (9) not binding and (10) binding** An efficient general partner always declares his true type, while the inefficient one is indifferent between declaring the truth or lying. Therefore, when a general partner declares to be inefficient, the general partner is automatically identified with certainty as inefficient (full separation), i.e. $\nu_2 = 0$. On the contrary, if in the first round a general partner declares his type as efficient, the updated belief about the distribution of efficient types at $t_2$ is

$$\nu_2 = \frac{\nu_1}{\nu_1 + \gamma (1 - \nu_1)}$$

As (10) is binding and $\bar{t}_1 = \bar{\theta} \bar{k}_1$, we have that $\bar{t}_1 = \theta \bar{k}_1 + \Delta \theta \bar{k}_1$. Using the fact that $\nu_2 = 0$ and $I(0) = R^{FB}$, the condition imposed by (9) then requires
\[ k_1 \geq \bar{k}_1 + F\left(\bar{R}^{FB}\right) \bar{k}^{FB} - F\left(I\left(\nu_2\right)\right) \bar{k}^{SB}\left(\nu_2\right). \]  

Notice that \( F(\bar{R}^{FB}) > F\left(I\left(\nu_2\right)\right) \) and \( \bar{k}^{FB} > \bar{k}^{SB}\left(\nu_2\right) \), the term in round brackets of (16) is positive. Then, we conclude that (10) requires at least \( k_1 > \bar{k}_1 \). Noticing that second period profits do not depend on \( k_1 \) and \( \bar{k}_1 \), the maximization then is equivalent to

\[
\max_{\{\bar{k}_1, \bar{k}_1\}} \left(\nu_1 + (1 - \nu_1) y\right) \left[R\left(\bar{k}_1\right) - \bar{\theta}_1\right] + (1 - \nu_1) (1 - y) \left(R\left(\bar{k}_1\right) - \bar{\theta}_1\right)
\]

From the first order conditions of this maximization we obtain \( \bar{k}_1 = \bar{k}_1 = \bar{k}^{FB} \) which violates (16). We conclude that condition (10) is always binding. Therefore this case never applies.

**Proof of Proposition 5**

Suppose that the limited partner is willing to employ both types of general partners. To examine the renegotiation proof contracts, we first need to analyze the possible renegotiations that could occur at \( t_2 \). Any new contract offered at \( t_2 \) must provide an efficient general partner with at least the same rents as in the original contract offered at \( t_0 \) (renegotiation condition \((RC)\)). We define these rents as \( M_2 \). at \( t_2 \), the limited partner can offer a menu of new contracts that solves

\[
\max_{\{\ell_2, k_2, t_2, \bar{k}_2\}} \nu_2 \left[R\left(\bar{k}_2\right) - \ell_2\right] + (1 - \nu_2) \left[R\left(\bar{k}_2\right) - \ell_2\right]
\]

subject to

\[
\bar{t}_2 - \bar{\theta}_2 \geq 0 \ (TR); \ t_2 - \theta k_2 \geq \bar{t}_2 - \bar{\theta} \bar{k}_2 \ (IC); \ t_2 - \theta k_2 \geq M_2 \ (RC).
\]

Cost minimization by the limited partner requires \( \bar{t}_2 = \bar{\theta} \bar{k}_2 \), therefore the first condition binds. Defining \( \gamma_1 \) and \( \gamma_2 \) as the Lagrange multipliers respectively associated with \((IC)\) and
the first-order conditions of the Lagrangian w.r.t. $t_2, k_2, \bar{k}_2$ are given by

$$-\nu_2 + \gamma_1 + \gamma_2 = 0,$$

(19)

$$\nu_2 R' (k_2) - \gamma_1 \theta - \gamma_2 \theta = 0,$$

(20)

and

$$(1 - \nu_2) \left[ R' (\bar{k}_2) - \bar{\theta} \right] - \gamma_1 \Delta \theta = 0.$$

(21)

Substituting (19) into (20), we have that $R' (k_2) = \theta$ and therefore an efficient general partner invests at the efficient level, $k_2 = k^{FB}$. We now distinguish three cases, depending on whether $\gamma_1$ and $\gamma_2$ are positive or zero. From (19), it cannot be that $\gamma_1 = \gamma_2 = 0$ unless $\nu_2 = 0$.

**Case 1:** $\gamma_1 > 0$ (**(IC) is binding**), $\gamma_2 = 0$ (**(RC) is slack**) Substituting $\gamma_2 = 0$ into (19) we have that $\nu_2 = \gamma_1$. Substituting into (21), we have that $R' (\bar{k}_2) = \bar{\theta} + \frac{\nu_2}{1 - \nu_2} \Delta \theta$ and therefore $\bar{k}_2 = \bar{k}^{SB} (\nu_2)$. From the participation constraint of the inefficient general partner binding we have that $\bar{t}_2 = \bar{\theta} k^{SB} (\nu_2)$. Finally, from (IC), we have that $\bar{t}_2 = \bar{\theta} k^{FB} + \Delta \theta \bar{k}^{SB} (\nu_2)$. Notice that this case is possible as long as (RC), which can potentially be slack, is satisfied. That is, as long as, $\Delta \theta \bar{k}^{SB} (\nu_2) > M_2$. But if this condition is slack then a renegotiation contract can be agreed upon. In order to have a renegotiation-proof contract, we need that $M_2 = \Delta \theta \bar{k}^{SB} (\nu_2)$.

**Case 2:** $\gamma_1 > 0$ (**(IC) is binding**), $\gamma_2 > 0$ (**(RC) is binding**) Substituting $\bar{t}_2 = \bar{\theta} k_2$ and $t_2 = \bar{\theta} k^{FB} + M_2$ ((RC) is binding) into (IC), we have that $M_2 = \Delta \theta \bar{k}_2$ from which we obtain $\bar{t}_2 = \bar{\theta} M_2 / \Delta \theta$. We now look for the conditions under which this case applies. On the one hand, (21) can be written as $\gamma_1 = (1 - \nu_2) \left[ R' (\bar{k}_2) - \bar{\theta} \right] / \Delta \theta$. In order to have $\gamma_1 > 0$ we need that $R' (\bar{k}_2) > \bar{\theta}$, i.e., that an inefficient general partner invests less than at first best. On the other hand, substituting $\gamma_1$ into (20) we have that $\gamma_2 = (\nu_2 \Delta \theta - (1 - \nu_2) \left[ R' (\bar{k}_2) - \bar{\theta} \right]) / \Delta \theta$. This is positive as long as $R' (\bar{k}_2) < \bar{\theta} + \frac{\nu_2}{1 - \nu_2} \Delta \theta$. Hence, this case holds as long as $\bar{\theta} < R' (\bar{k}_2) < \bar{\theta} + \frac{\nu_2}{1 - \nu_2} \Delta \theta$ or in other words, as long as $R' (k^{FB}) < R' (\bar{k}_2) < R' (k^{SB} (\nu_2))$. Given

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that $R'(\cdot)$ is decreasing in $\kappa$, this case applies when $\kappa^{SB}(\nu_2) < \kappa_2 < \bar{k}^{FB}$, a condition that can be otherwise written as $\theta \kappa^{SB}(\nu_2) < M_2 < \Delta \theta \bar{k}^{FB}$.

**Case 3:** $\gamma_1 = 0$ ((IC) is slack), $\gamma_2 > 0$ ((RC) is binding)  Substituting $\gamma_1 = 0$ into (19) we have that $\nu_2 = \gamma_2$ and substituting into (21) we have that $R'(\kappa_2) = \theta$ and therefore $\kappa_2 = \bar{k}^{FB}$.

As the participation constraint of an inefficient general partner is binding, $t_2 = \theta \kappa^{FB} + M_2$. This case is satisfied as long as ((IC)) is satisfied, a condition that can be written as $M_2 > \Delta \theta \kappa^{FB}$. For large values of $M_2$ the incentive constraint of an inefficient general partner may be violated. Therefore, we need to impose an upper limit to $M_2$. Carrying out the due substitutions, we have that the incentive constraint of an inefficient general partner is satisfied if $M_2 \leq \Delta \theta \kappa^{FB}$.

In sum, depending on the values of the parameters, the optimum second period renegotiation proof contract requires different thresholds for $M_2$. A contract is renegotiation-proof if and only if $\Delta \theta \kappa^{SB}(\nu_2) \leq M_2 \leq \Delta \theta \kappa^{FB}$. Moreover, (a) if $\Delta \theta \kappa^{SB}(\nu_2) \leq M_2 < \Delta \theta \kappa^{FB}$ then $t_2 = \theta \kappa^{FB} + M_2$, $k_2 = \bar{k}^{FB}$, $\bar{t}_2 = \theta M_2 / \Delta \theta$, $\bar{k}_2 = M_2 / \Delta \theta$ (Cases 1 and 2); (b) if $\Delta \theta \kappa^{FB} \leq M_2 \leq \Delta \theta \kappa^{FB}$ then $t_2 = \theta \kappa^{FB} + \Delta \theta \kappa^{FB}$, $k_2 = \kappa^{FB}$, $\bar{t}_2 = \bar{k}^{FB}$, $\bar{k}_2 = \bar{k}^{FB}$ (Case 3).

With regards to Case 3, observe that for the limited partner at $t_0$ it is never optimal to choose $M_2 > \Delta \theta \kappa^{FB}$ because above the threshold $M_2 = \Delta \theta \kappa^{FB}$ the optimum contract does not change. Raising $M_2$ above $\Delta \theta \kappa^{FB}$ would imply a pure transfer of wealth from the limited partner to the general partner with no benefit for the limited partner. Therefore, the relevant range for $M_2$ in Case 3 degenerates to $M_2 = \Delta \theta \kappa^{FB}$. Given that there is continuity between Case 2 and 3, we conclude that the relevant range for $M_2$ is given by $\Delta \theta \kappa^{SB}(\nu_2) \leq M_2 \leq \Delta \theta \kappa^{FB}$ and the optimum contract is as in (a) above.

As we will show in the proof of Proposition 7, the optimal second-period contract is the one with the lowest rents, $M_2 = \Delta \theta \kappa^{SB}(\nu_2)$. Thus $t_2 = \theta \kappa^{FB} + \Delta \theta \kappa^{SB}(\nu_2)$, $k_2 = \kappa^{FB}$, $\bar{t}_2 = \bar{\theta} \kappa^{SB}(\nu_2)$, $\bar{k}_2 = \bar{k}^{SB}(\nu_2)$, as in contract (2) of Proposition 1 replacing $\nu_1$ with $\nu_2$.

The above renegotiation proof contracts deal with the possible renegotiations that a limited
partner may agree on with an inefficient general partner in the second period. Another reason why the limited partner may renegotiate is to save on the rents that she has agreed to pay to the efficient general partner. To prevent such renegotiation, the default penalties are set equal to the rents of the efficient general partner, i.e. \( P = \Delta \theta k^{SB}(\nu_2) \).

Suppose that the limited partner employs only efficient general partners. The renegotiation proof contract requires the renegotiation-proof condition to bind, i.e. \( t_2 = \theta k_2 - M_2 = 0 \). The optimum contract requires \( k_2 = k^{FB} \) and \( t_2 = \theta k^{FB} + M_2 \). The contract with the lowest rent requires \( M_2 = 0 \) and is as in contract (1) of Proposition 1 replacing \( \nu_1 \) with \( \nu_2 \). Following the same logic as for the other contract, \( P = 0 \).

**Proof of Proposition 6**

For contract (1), the limited partner might choose to continue with an efficient general partner (case (i)), or default (case (ii)). Substituting the optimal contract (1) into the objective function, her expected profits in case (i) are \( R^{FB} - M_2 \) and in case (ii) are \( I - M_2 \) because the default penalties are zero.

For contract (2), the limited partner might choose to continue with both types (case (i)), with the efficient general partner only (case (ii)), or default (case (iii)). Substituting the optimal contract (2) into the objective function, her expected profits are \( \nu_2(R^{FB} - M_2) + (1 - \nu_2)(R(\overline{k}_2) - \theta \overline{k}_2) \) in case (i) and \( \nu_2(R^{FB} - M_2) + (1 - \nu_2) I \) in case (ii). In the latter, the limited partner makes a termination offer of 0 to the general partner, and only the inefficient general partner accepts it. In case (iii), the limited partner pays the default penalty and obtains \( I - M_2 \).

As we will show in the proof of Proposition 7, \( M_2 = 0 \) for contract (1) and \( M_2 = \Delta \theta k^{SB}(\nu_2) \) for contract (2). A comparison across cases yields the statement of the proposition.

**Proof of Proposition 7**

As in Laffont and Tirole (1990), the limited partner should offer a menu of two contracts, such that the efficient general partner is indifferent between telling the truth or lying about
his type. The menu depends on $x$, the probability that an efficient general partner tells the truth. Conditional on $x$, the limited partner’s updated beliefs at $t_2$ that the general partner is efficient are

$$\nu_2 = 1 \text{ and } \nu_2 = \frac{\nu_1(1-x)}{1 - \nu_1 x}.$$  

In the contract designed for the efficient general partner, substituting $\nu_2$ into the contract and thresholds of the proofs of Propositions 5 and 6, the limited partner should allocate no rent to the efficient general partner ($M_2 = 0$) and set $t_2 = \theta k^{FB}$. In the first period, the investment size is $k_1 = k^{FB}$.

In the contract designed for the inefficient general partner, the general partner is generally not fully identified after the first round. From Proposition 5, we obtain $k_2 = k^{FB}$, $\bar{k}_2 = M_2/\Delta \theta$, $t_2 = \theta k^{FB} + \Delta \theta \bar{k}_2$ and $\bar{t}_2 = \theta \bar{k}_2$. We still need to determine $M_2$ or equivalently $\bar{k}_2$. Substituting $M_2$ and $\bar{v}_2$ in the thresholds of the proof of Proposition 6, the limited partner continues with both types if $I \leq \bar{R} \equiv R(\bar{k}_2) - \theta \bar{k}_2$, continues only with an efficient general partner if $\bar{R} < I < R_1 \equiv R^{FB} + (1 - \nu_1)/(\nu_1(1 - x))\Delta \theta \bar{k}_2$, and defaults if $I \geq R_1$.

In the first round, following standard arguments, upon announcement of an efficient general partner, the investment size is $k_1 = k^{FB}$. Second, upon announcement of an inefficient general partner, the limited partner offers no rents $\bar{t}_1 = \theta \bar{k}_1$. Third, an efficient general partner is indifferent between telling the truth or lying because her intertemporal incentive compatibility condition is binding, i.e. $t_1 = \theta k^{FB} + \Delta \theta \bar{k}_1 + \Delta \theta \bar{k}_2$.

It only remains to determine the optimal $\bar{k}_1$, $\bar{k}_2$ and $x$. For a given $x$, substituting all the other terms, the limited partner should maximize the following problem subject to the
renegotiation proof condition (i.e. \( k_2 \geq \bar{k}^{SB}(\nu_2) \))

\[
\max_{\{k_1, k_2\}} \nu_1 x \left[ R^{FB} - \Delta \theta \bar{k}_1 - \Delta \theta \bar{k}_2 \right] + (1 - \nu_1) x \left[ R (\bar{k}_1) - \bar{R} \bar{k}_1 \right] \\
+ \nu_1 x \left[ \int_{0}^{\bar{k}^{FB}} R^{FB} dF(I) + \int_{\bar{k}^{FB}}^{+\infty} 1 \ dF(I) \right] \\
+ \int_{0}^{\bar{k}^{FB}} \left[ \nu_1(1 - x) (R^{FB} - \Delta \theta \bar{k}_2) + (1 - \nu_1) \bar{R} \right] dF(I) \\
+ \int_{\bar{k}^{FB}}^{+\infty} \left[ \nu_1(1 - x) (I - \Delta \theta \bar{k}_2) + (1 - \nu_1) (I - \Delta \theta \bar{k}_2) \right] dF(I)
\]

The first order condition with respect to \( \bar{k}_1 \) gives \( \bar{k}_1 = \bar{k}^{SB}(\nu_1 x) \). To find the optimum level of \( \bar{k}_2 \) observe that profits are higher in line 5 of the objective function than in line 4, and in line 4 than in line 3. The probability of line 5 occurring is decreasing in \( \bar{k}_2 \). Profits in line 5 and in line 4 are decreasing in \( \bar{k}_2 \). Profits in line 3 are increasing in \( \bar{k}_2 \) up to \( \bar{k}_2 = \bar{k}^{SB}(\nu_2) \) and decreasing after this point. Finally, profits in line 1 are decreasing in \( \bar{k}_2 \). Thus, piecemeal maximization starting from line 3 requires \( \bar{k}_2 = \bar{k}^{SB}(\nu_2) \), while in all other lines (1,4 and 5) it requires \( \bar{k}_2 = 0 \). We then conclude that at the optimum it must be \( \bar{k}_2 \leq \bar{k}^{SB}(\nu_2) \), which implies that the LHS of the renegotiation-proof condition is binding. Therefore, the optimum renegotiation-proof contract requires \( \bar{k}_2 = \bar{k}^{SB}(\nu_2) \) and \( M_2 = \Delta \theta \bar{k}^{SB}(\nu_2) \).

Therefore the optimal menu of contracts consists of a first contract \( k_1 = k^{FB}, \ t_1 = \theta k^{FB} + \Delta \theta k^{SB}(\nu_1 x), \ k_2 = k^{FB}, \ t_2 = \theta k^{FB}, \) and \( P = 0 \), which is chosen by the efficient general partner with probability \( x \), and a second contract \( \bar{k}_1 = \bar{k}^{SB}(\nu_1 x), \ \bar{t}_1 = \bar{k}^{SB}(\nu_1 x), \ k_2 = k^{FB}, \ t_2 = \theta k^{FB} + \Delta \theta \bar{k}^{SB}(\nu_2), \ \bar{k}_2 = \bar{k}^{SB}(\nu_2), \ \bar{t}_2 = \bar{k}^{SB}(\nu_2) \) and \( P = \Delta \theta \bar{k}^{SB}(\nu_2) \), which is chosen by the inefficient general partner and by the efficient general partner with probability \( 1 - x \). Substituting for \( x = 1 \), we obtain the contracts in the text.

Proof of Proposition 9
Condition (4) can be written as \( G(\theta, \Delta \theta, \nu_1, F) \equiv \overline{k}^{FB} - \overline{k}^{SB}(\nu_1) - F(\overline{R}^{FB})\overline{k}^{FB} > 0 \).

Rewriting, \( \overline{k}^{SB}(\nu_1) \) is defined as the unique \( k \) that satisfies \( R'(k) = \theta + 1/(1 - \nu_1)\Delta \theta \). Given that \( \overline{k}^{SB} \) is decreasing in \( \Delta \theta \) and \( \nu_1 \), \( G \) is increasing in \( \Delta \theta \) and \( \nu_1 \). \( G \) is also decreasing in \( F(\cdot) \) for a given \( \overline{R}^{FB} \).
Limited Partner offers menu of contracts at time $t_0$.

Limited Partner privately observes by the Limited Partner at time $t_1$.

Limited Partner offers menu of contracts or defaults at time $t_2$.

Payoff of 1st investment round is observed at time $t_3$.

Payoff of 2nd investment round is observed at time $t_4$.

Figure 1: Timing of Contracting
Figure 2. This figure illustrates the payoffs of fully separating contracts without default penalties. The optimal two-period contract consists of a menu \((l_1, k_1)\) and \((l_2, k_2)\) for the first period (the second period terms are irrelevant). The first contract will be chosen by the efficient General Partner and the second contract will be chosen by the inefficient General Partner. If \((l_1, k_1)\) is chosen, termination occurs if \(l > \overline{b}^{ia}\) and continuation occurs otherwise. If the Limited Partner decides to continue it should then offer \((l_2, k_2)\). If \((l_2, k_2)\) is chosen, termination occurs if \(l > \overline{b}^{ia}\). If the Limited Partner decides to continue, she offers a menu of contracts \((l_3, k_3)\) and \((l_4, k_4)\), out of which the inefficient General Partner will always choose the latter.
Figure 3. This figure illustrates the payoffs of fully separating contracts with default penalties. The optimal two-period contract consists of a menu of two contracts: the first is \( l_1 = \bar{\theta}_2^a + \Delta \theta_2^a (v_i) + \Delta \theta_2^a \) and the second is \( l_2 = \theta_2^a \). The first contract will be chosen by the efficient General Partner and the second contract will be chosen by the inefficient General Partner. If the first contract is chosen, termination occurs if \( P = 0 \) and continuation occurs otherwise. If the second contract is chosen, termination occurs if \( l > \bar{\theta}_2^a \).
Figure 4: Comparative Statics

In all panels, the $y$ axis depicts the monetary value of four types of contracts: RED for contracts with default penalties; GREEN for separating contracts without default penalties that satisfy condition (5); YELLOW for separating contracts without default penalties that do not satisfy condition (5); BLACK for pooling contracts without default penalties. In all panels we assume $a=0.4$, $b=1.2$, $\bar{\theta}=1$. Furthermore, in Panel A we assume $\bar{\theta}=1.2$, $\nu_1=0.5$ and let $Z$ vary (depicted on the $x$ axis). In Panel B we assume $\bar{\theta}=1.5$, $\nu_1=0.5$ and let $Z$ vary (depicted on the $x$ axis). In Panel C we assume $Z=0.4$, $\nu_1=0.5$ and let $\bar{\theta}$ vary (depicted on the $x$ axis). In Panel D we assume $Z=0.4$, $\bar{\theta}=1.2$ and let $\nu_1$ vary (depicted on the $x$ axis).