Estimation of Cost and Production Functions

Albert Banal-Estanol

Motivation: Production and Cost Functions

- Objective: Find shape of production/cost functions
- Evaluate efficiency:
  - Increasing returns, economies of scale
  - Complementarity/substitutability between inputs
  - Synergies and mergers
- Productivity analysis:
  - Measurement of productivity across firms or over time
  - Effects of policy (deregulation, tariffs,…)
  - Returns of R&D, returns to adoption of new technology

Marginal/average costs vs cost function

- In some instances,…
  - "Only" need marginal/average cost functions
  - Computed directly from company/industry data
- But, in others,…
  - Need to know if marginal cost varies with quantity
  - Or, if there are economies of scale as size changes

- Economic approach
  - Make assumptions on shape
  - Estimate model's parameters
- Engineering approach:
  - Interview technical expert about effect of costs and scale effects

This chapter

- Accounting and economic costs
- Estimating production functions:
  - Applications: returns to scale and technical change
  - Problems and possible solutions
- Estimating cost functions:
  - Application: efficiency measurement
- Appendix: Index numbers and TFP

Accounting and economic costs

Cost measures from company data

- Need to allocate costs across multiple operations:
  - Important for regulated and non-regulated business
  - Existence of “cost allocation methodologies”
- Cost might not reflect value if vertically integrated:
  - Transfer upstream/downstream for tax or regulatory reasons
- Costs and revenues might not occur at same time
  - Buy plant now, revenues over 30 years. Cost per year?
  - Substantial differences between accounting/economic costs

Accounting versus economic cost of an input

- "Accounting" costs:
  - Out-of-pocket historical costs, appropriately depreciated
- "Economic" costs:
  - Remuneration received in the next best alternative
  - If market available, equal to market price, e.g. price of steel
  - If not, need to find a value, e.g. economic cost of…
    - Investing in new capacity: return of capital used if it had been invested elsewhere (adjusted for risk)
    - Company owner's time, highest income in another occupation
- How to compute economic costs?

Example: opportunity cost of capital

- Cost of capital often reported as:
  - Depreciation charges
  - May also include interest paid if firm needs to borrow

- Assuming constant depreciation, accounting cost:
  $Cost_t = \delta$ Original capital investment

- Instead, economic cost:
  $Cost_t$= Opportunity cost of $capital_t$ + Economic $depreciation_t$, where
  - Opportunity $cost_t = r\ V_t$ where V_t : value of the good and r: “appropriate rate”
  - Economic $depreciation_t = V_t - V_{t+1}$

Or in other words:

$Cost_t$= (r + Depreciation $rate_t$) V_t where Depreciation $rate_t = (V_t – V_{t+1}) / V_t$

Example: company car

- Buying a Volkswagen Passat in Belgium, 2007:
  - Prices: new: €28000 & similar 1-year old: €21000
  - Interest rate: 10%

- Accounting cost:
  - Assuming 5-year life, depreciated at constant rate (20%)
  - Cost_t= 28000 / 5= €5610

- Economic cost:
  - Cost_t= (0.1 + 0.251) 28000 = €9855
  - …because Depreciation rate_t= (28000– 21000) / 28000 = 25.1%

Estimating production functions

Theoretical framework: input substitution effects

- To bake a cake, need fixed proportion of ingredients:
  - 1kg of flower, 6 eggs,….
  - Given I_1 kgs of flower, I_2 eggs, Q cakes can be baked:

$$Q = \min\left\{\frac{I_1}{\beta_1}, \frac{I_2}{\beta_2}, \ldots, \frac{I_n}{\beta_n}\right\}$$

where β_1, β_2,… are the proportions

- But, if we also need capital and labour, one can use:

  (i) a small amount of labour and a cake-mixer or
  (ii) a large amount of labour and a spoon

  - Need general production function, allowing for substitution

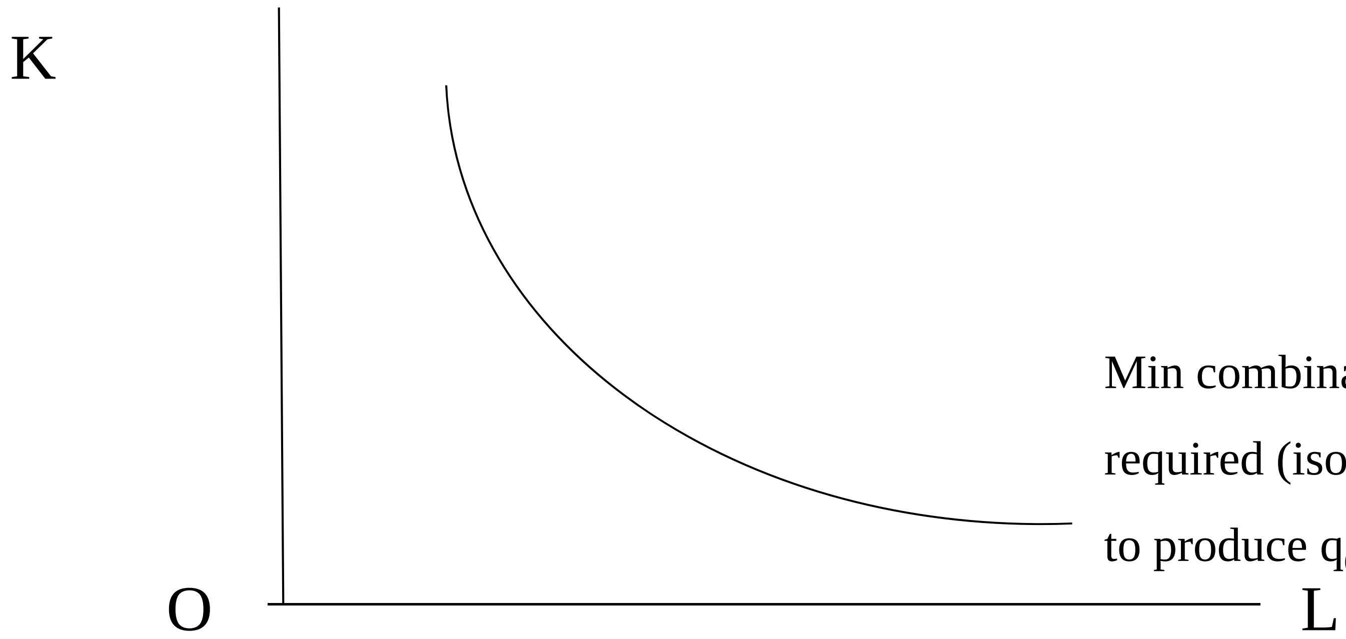
$$Q = f(I_1, I_2, \ldots, I_n, \beta_1, \beta_2, \ldots, \beta_n)$$

Where β_1, β_2,… are the parameters

Example: Cobb-Douglas

- Two inputs: labour and capital and

$$Q = K^{\beta_1} L^{\beta_2}$$



Estimation Strategies

- If firm minimises costs, it solves

$$C(Q,\beta,u) = Min_{I_1,I_2,\ldots,I_n}\ p_1 I_1 + p_1 I_2 + \ldots + p_n I_n$$

$$\text{s.t.}\ Q \le f(I_1, I_2, \ldots, I_n, \beta_1, \beta_2, \ldots, \beta_n, u)$$

where 1, …, n are observed inputs and u an unobserved “input” (e.g. productivity), which will be the econometric error term

- Option 1: Estimate production function (cost indirectly)

$$Q = f(I_1, I_2, \ldots, I_n, \beta_1, \beta_2, \ldots, \beta_n, u)$$

  - need input-output data

- Option 2: Estimate cost function directly

$$C = C(Q, p_1, p_2, \ldots, p_n, u)$$

  - need cost, outputs and input prices

A Model

- Assume production follows a Cobb-Douglas function:

$$Q_i = e^{\alpha} K_i^{\beta_k} L_i^{\beta_l} e^{u_i} \text{ where}$$

Q_i is the output of plant (or firm) I ; L_i is labour input (or more generally variable input); K_i is capital input (or more generally fixed input); u_i is an error term; α, β_l, β_k are parameters to be estimated

- Remarks:
  - We might also include: material, energy, different types of labour
  - Error term includes: technology or management differences, measurement errors, variation in external factors
- Taking logs (y_i=ln (Q_i), l_i=ln (L_i) and k_i=ln (K_i)), we obtain

$$y_i = \alpha + \beta_k k_i + \beta_l l_i + u_i$$

Variables

- Production: If multiproduct firm then…
  - Output typically sales divided by price index (more on this later)
  - Or using a quantity index (more on this later)
- Labour:
  - Usually hours or employees per year
  - May adjust for types of labour
- Other non-capital inputs:
  - E.g. fuel (litres)
  - Other (expenditures divided by price index)
- Capital:
  - Investment at different times computing depreciation

Application 1: Returns to Scale

- Estimating the previous model by OLS one finds estimates of α, β_l and β_k
- β_l and β_k are the elasticities of output wrt input
- Remember that with Cobb-Douglas model:

$$Q_i = AK^{\beta_k} L^{\beta_l} \text{ exhibits...}$$

$$\text{CRS iff } \beta_l + \beta_k = 1$$

$$\text{IRS iff } \beta_l + \beta_k > 1$$

$$\text{DRS iff } \beta_l + \beta_k < 1$$

- We can perform F-tests to determine the case

Application 2: Technical Change Measurement

- With time series (or panel) data, we can estimate technical change:

$$y_t = \alpha + \beta_k k_t + \beta_l l_t + \beta_t t + u_t$$

where t is a time trend (t=1,2,…T)

- Interpretation:
  - β_t provides an estimate of the annual percentage change in output resulting from technical change (one year)

Problem 1: Endogeneity

- What if the true model is

$$y_i = \alpha + \beta_k k_i + \beta_l l_i + \beta_x X_i + u_i$$

but we estimate

$$y_i = \tilde{\alpha} + \tilde{\beta}_k k_i + \tilde{\beta}_l l_i + u_i$$

- Then our estimates are biased if…
  (a) the omitted variable has an effect on the dependent variable and
  (b) the omitted variable is correlated with an included variable

- X may not be observable:
  - Managerial efficiency:
    - Better managers may need less labour to produce same output
    - These firms produce more with less labour and OLS will underestimate β_l
  - Productivity shock:
    - A higher productivity shock may mean more labour
    - OLS attributes all the increase in output to change in labour and overestimate β_l

Solution 1: Instrumental Variables (IV)

- Example: productivity shock
  - unobserved by the econometrician (but the firm observes)
  - when choosing its level of labour, the firm has observed the shock
  - and therefore the input is correlated with the shock
- OLS is inconsistent. Valid instrument: price of labour
  - (a) <u>uncorrelated</u> with the error (if labour market is competitive);
  - (b) <u>correlated</u> with the level of labour
- Intuitively, using a variable as an instrument means that
  - (a) regressing endogenous variable on the instrument (and other variables)
  - (b) use predicted endogenous variable as a regressor in initial model
- Recent research challenges IV analysis:
  - More recent techniques are dynamic panel data
  - Structural estimatation (see Olley and Pakes, 1996; Levinsohn and Petrin, 2003).

Solution 2: Panel Data Analysis

- Suppose that we have panel data :
  - Several firms followed at different points in time

- Assume that:

$$y_{i,t} = \alpha + \beta_k k_{i,t} + \beta_l l_{i,t} + \gamma_i + \varepsilon_{i,t} \text{ where}$$

$$\gamma_i \text{ is firm-specific and}$$

$$\varepsilon_{i,t} \text{ satisfies classical assumptions}$$

Firm Fixed Effects

- Assume that γ_i is predetermined:
  - Not result of random variation but fixed, long standing characteristics
  - For example: managerial ability (can be correlated with labour)

- If this is true then panel data….
  - Allows us to obtain unbiased estimates for the variables included (especially for those that adapt quickly as labour and materials)
  - Correct for unobservable heterogeneity
  - Acts like a dummy shifting the intercept for each individual (impossible to do with cross section)

Time Fixed Effects

- We may also include a time-variant variable:
  - Common effect impacting on all individuals
  - Can account for industry-wide productivity shocks
- We can then use:

$$y_{i,t} = \alpha + \beta_k k_{i,t} + \beta_l l_{i,t} + \gamma_i + \gamma_t + \varepsilon_{i,t} \text{ where}$$

$$\gamma_i \text{ is firm-specific and } \gamma_t \text{ is time-specific and}$$

$$\varepsilon_{i,t} \text{ satisfies classical assumptions}$$

Problem 2: Functional form

- Functional form needs to reflect…
  - Plausible input substitution possibilities
  - Plausible nature of returns to scale
- If unsure,…
  - Use flexible econometric specification
  - Cobb-Douglas imposes same returns to scale over whole output range
- But if overly flexible…
  - Implausible results might be obtained
  - Data might not be able to identify
- As a result,
  - Impose what you know
  - But don't impose what you do not!

Estimating cost functions

Cost Functions

- Similarly one can estimate a cost function:

$$(a)\ \ln c_i = \beta_o + \beta_y \ln y_i + \varepsilon_i \text{ where}$$

c_i are the operating costs of plant (or firm) i

ε_i is an error term (due for example to managerial efficiency)

β_o and β_y are parameters to be estimated

- But, larger firms might obtain raw materials at lower price:
  - This does not reflect managerial efficiency but contained in ε_i
  - Correlation between error term and included variable
  - OLS estimator may be biased. Need to control for it!

Cost Functions

- If one has data on input prices:

$$(b)\ \ln c_i = \beta_o + \beta_w \ln w_i + \beta_r \ln r_i + \beta_y \ln y_i + \varepsilon_i \text{ where}$$

c_i are the operating costs of plant (or firm) i

w_i is the price of the labour input (wages) of plant (or firm) i

r_i is the price of the capital input of plant (or firm) I

ε_i is an error term (due again for example to managerial efficiencies)

β_o,β_w, β_r and β_y are parameters to be estimated

Application: efficiency measurement

- Assume, for simplicity, that we estimate the model

$$(c)\ \ c_i = \beta_o + \beta_y y_i + \varepsilon_i$$

where β_o is the fixed cost and β_1 is the marginal cost of y

- Then

$$c_i = \hat{c}_i + \hat{\varepsilon}_i$$

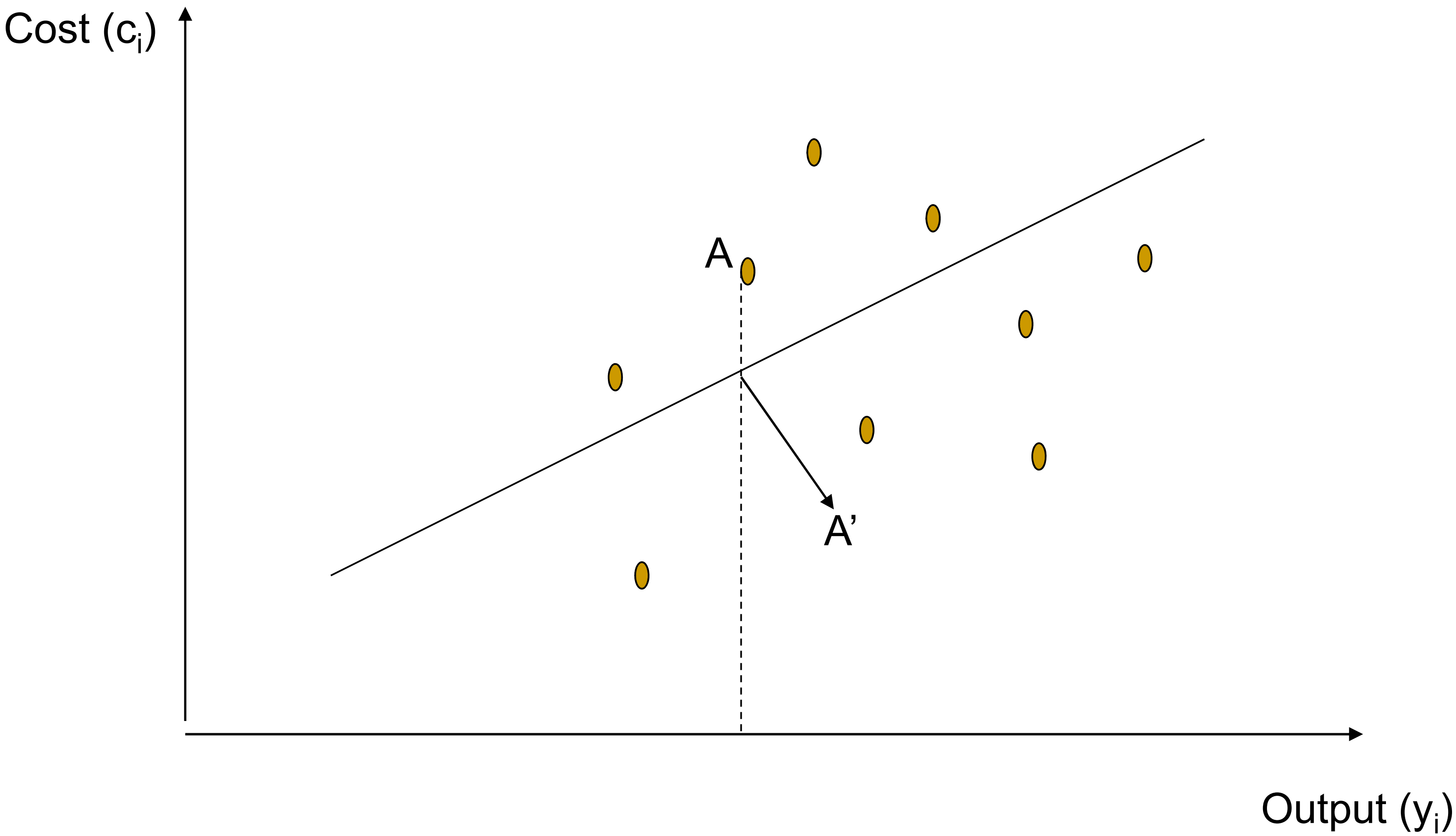
Where $\hat{c}_i$ can be thought as an estimate of the costs of an average efficiency company (*benchmark*) producing the same output

And $\hat{\varepsilon}_i$ estimates the efficiency difference between company i and this average company

The company is less efficient than average if $\hat{\varepsilon}_i > 0$

The company is more efficient than average if $\hat{\varepsilon}_i < 0$

Efficiency measurement, graphically



Extensions

- More than one cost driver:

$$(d)\ \ c_i = \beta_o + \beta_p P_i + \beta_L L_i + \beta_y y_i + \varepsilon_i$$

- More than one year (panel, see Lab):

$$(e)\ \ c_{i,t} = \beta_o + \beta_p P_{i,t} + \beta_L L_{i,t} + \beta_y y_{i,t} + t + \varepsilon_i$$

- Non-linear functional forms? (logs,…)

References

- Davis, P. and E. Garces (2010), “Quantitative Techniques for Competition and Antitrust Analysis”, Princeton and Oxford University Press
- Greene, W., (2003) “Econometric Analysis”, Prentice Hall

Appendix: index numbers

Index Numbers

- Instrument to measure change in levels of various related economic variables (RPI,…)
- Here, some price and quantity index numbers relevant to productivity analysis
- Objectives:
  - Measure changes in Total Factor Productivity (TFP) (see chapter on benchmarking)
  - "Aggregate data" into smaller sets of inputs and outputs

Price Index Numbers

- In a multi-product setting, how do we measure price and quantity changes, across time or individuals? (index number problem)
- Tornqvist price index P^T, e.g., from s to t is defined as:

$$\ln P_{s,t}^{T} = \sum_{i=1}^{N} \left(\frac{\omega_{i,s} + \omega_{i,t}}{2} \right) \left[\ln p_{i,t} - \ln p_{i,s} \right]$$

where $p_{i,s}$ and $q_{i,s}$ are the price and quantity of i product in s-period

$$\omega_{i,s} = \frac{p_{i,s} q_{i,s}}{\sum_{i=1}^{N} p_{i,s} q_{i,s}}$$ is the value of the i product in period s

- Others include Laspeyres, Paasche, Fisher

Quantity Index Numbers & TFP

- Similar to before we have several possible indices
- Tornqvist quantity index Q^T from s to t, as before:

$$\ln Q_{s,t}^{T} = \sum_{i=1}^{N} \left(\frac{\omega_{i,s} + \omega_{i,t}}{2} \right) \left[\ln q_{i,t} - \ln q_{i,s} \right]$$

- With Tornqvist quantity indices, e.g., compute TFP:

$$\ln TFP_{s,t} = \ln \frac{Output_{s,t}^{T}}{Input_{s,t}^{T}} = \ln Output_{s,t}^{T} - \ln Input_{s,t}^{T}$$