Lecture 8: Data Envelopment Analysis

Quantitative Methods for Regulation and Competition

Introduction

- Objective: assess efficient costs of regulated firm
- Benchmarking: compare with other firms (or plants)
- Methods:
  - Regression analysis (Chapter 7)
  - DEA (this chapter)
  - Econometric methods (COLS, SFA) (Chapter 9)
- Applications:
  - Review of BT (see NERA 2004)
  - Review of Royal Mail (see Lecture 10 and LECG 2005)
  - OFWAT 2005

Data Envelopment Analysis

- Introduction
- DEA (CRS input-oriented) for technical efficiency:
  - Technical efficiency graphically
  - DEA graphically and theoretically
  - Numerical example
- Extensions:
  - Constant, variable returns to scale and scale efficiency
  - Input vs output orientation
  - Technical, allocative and cost efficiency
- Dangers of using DEA
- Comparison with Regression Analysis
- Practicalities in using DEA

Technical Efficiency

- If we know that there is a technology that allows firms…
  - Using L labour units and K capital units (inputs)
  - To produce q_0 units of output, according to…

Other things equal = outputs

- Definition of technical efficiency:

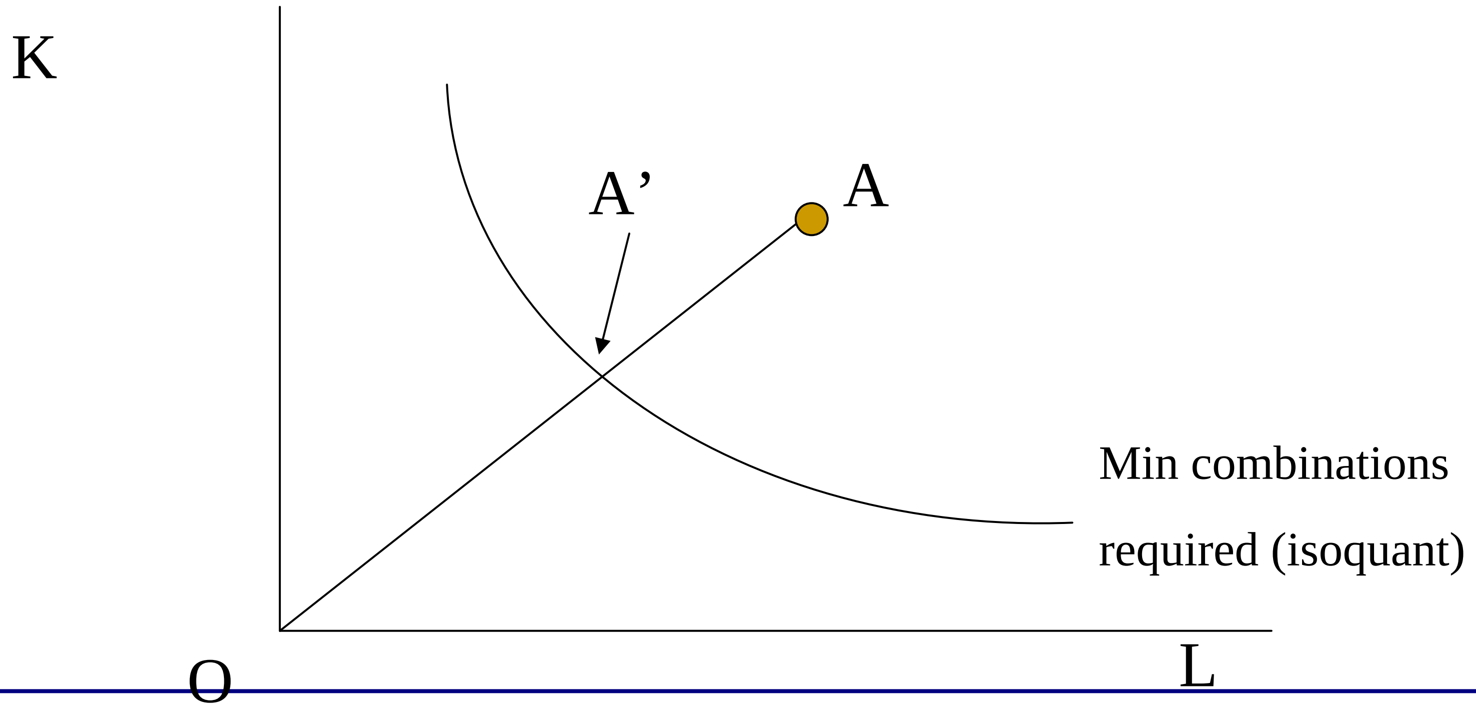
"produce a level of output using the minimum level of inputs"

Technical Efficiency (2)

Then we can measure inefficiency

E.g suppose that firm A is producing q_0. It is not efficient, i.e. it uses more inputs to produce this output

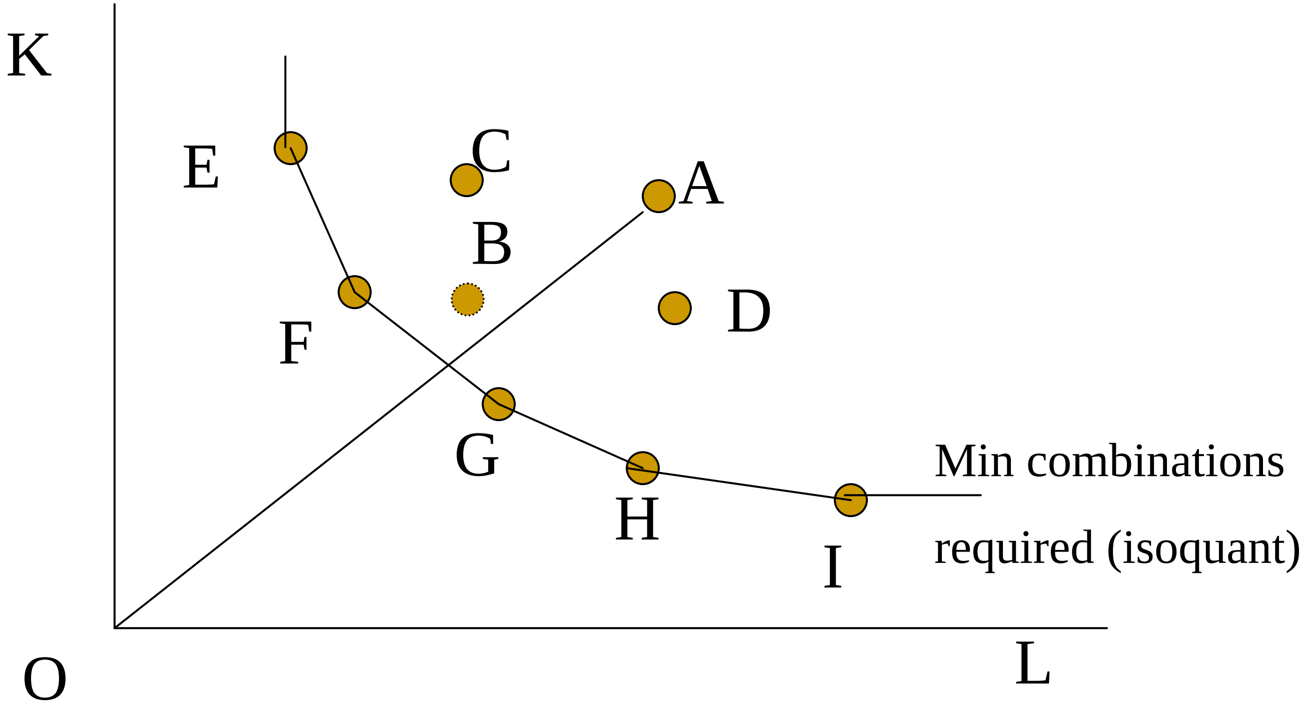
Measuring A's technical efficiency = OA'/OA



DEA Graphically (1)

- However, the isoquant is not known directly
- DEA estimates it from the data, using piecewise interpolation
- Suppose that firms A,B,C,D,E,F,G, H and I, all produce q_0

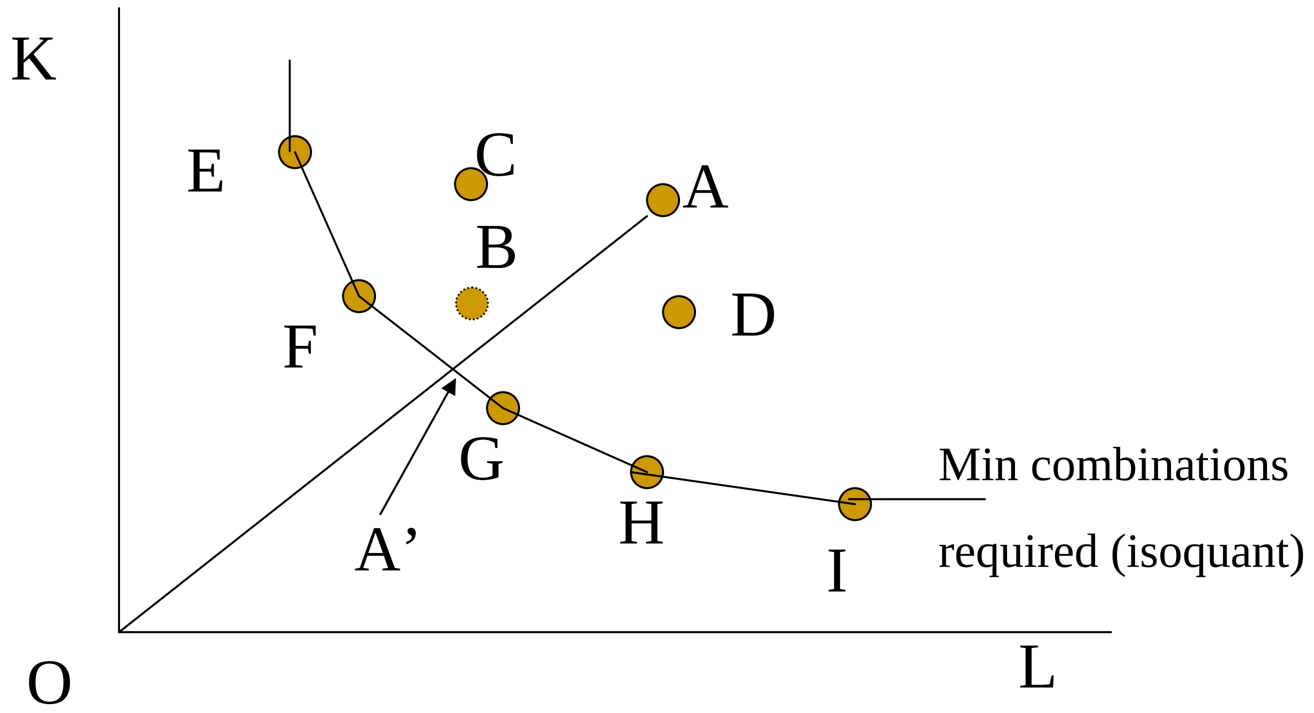
E, F, G, H, I define a “frontier”



DEA Graphically (2)

- DEA defines A’s inefficiency as OA’/OA
- A’ (the target or shadow of A) is a linear combination of F and G (the peers of A)

E, F, G, H, I define a “frontier”



DEA Theoretically

- For each i of the I firms (N inputs and M outputs):

$$\min_{\theta,\lambda} \theta$$

$$\text{subject to } -q_i + Q\lambda \geq 0$$

$$\theta x_i - X\lambda \geq 0$$

$$\lambda \geq 0$$

where X is a $N \times I$ matrix of inputs (x_i is the vector for firm i)

Q is a $M \times I$ matrix of outputs (q_i is the vector for firm i)

θ is a scalar and λ is a vector of constants

- Interpretation:
  - Find a combination of θ (=OA'/OA) and a vector of λ's
  - Creating a shadow firm with inputs Xλ and outputs Qλ
  - Contracting the vector x_i while remaining in input set

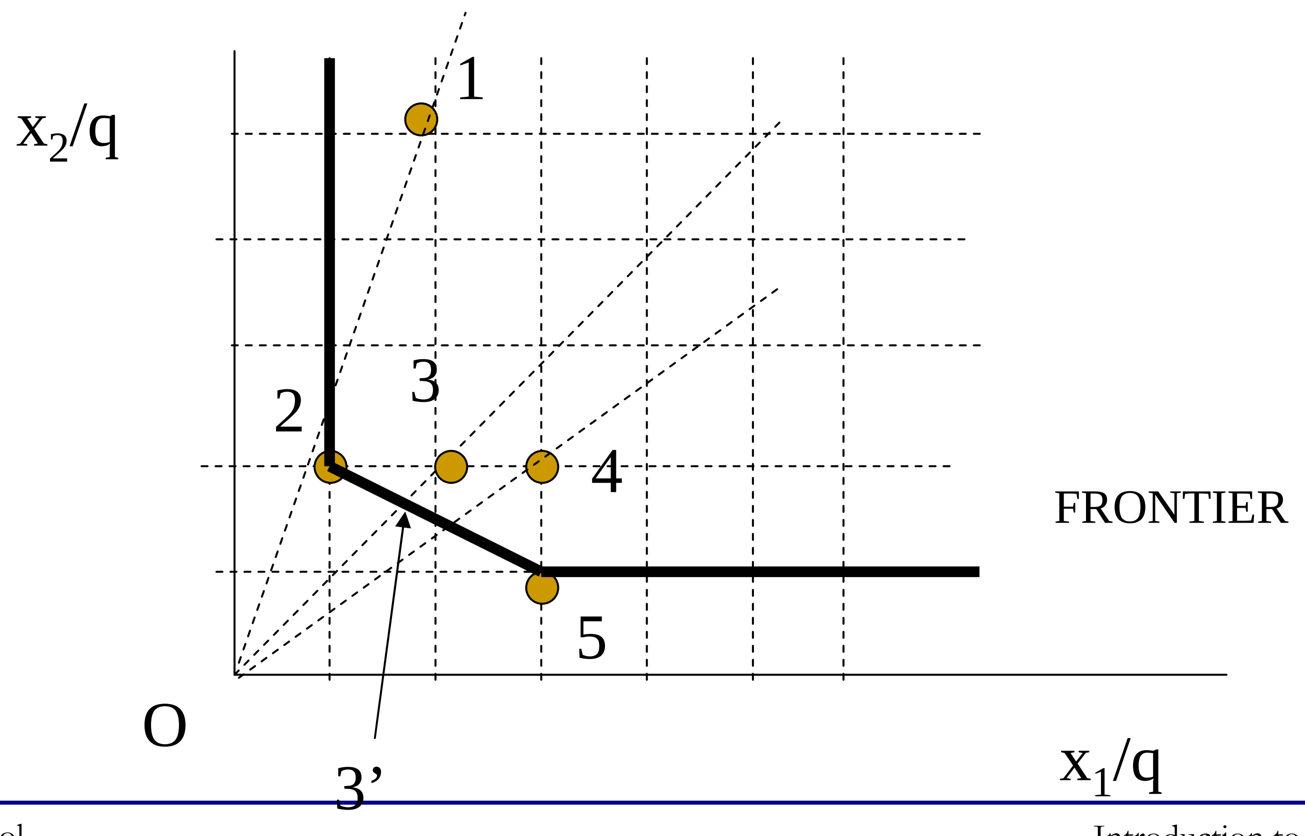
Numerical Example (Coelli et al.)

- 5 firms producing one type of output with two inputs

Firm	Q	x_1	x_2	x_1/q	x_2/q
1	1	2	5	2	5
2	2	2	4	1	2
3	3	6	6	2	2
4	1	3	2	3	2
5	2	6	2	3	1

- Assume a constant returns to scale technology

Numerical Example (Coelli et al.)



Results from Numerical Example

Firm	θ	λ_1	λ_2	λ_3	λ_4	λ_5
1	0.5	-	0.5	-	-	-
2	1	-	1	-	-	-
3	0.83	-	1	-	-	0.5
4	0.71	-	0.21	-	-	0.28
5	1.	-	-	-	-	1

Example: Technical efficiency of Firm 3 = 0.83

Input could be reduced by 16.7% without reducing output

Firms 2 and 5 are its peers (and 3' is a linear combination with weights 1 and 0.5, respectively)

Variable Returns to Scale and Scale Efficiencies

- So far, we have assumed CRS
- This is appropriate when firms operate at the optimal scale… but this may not be true
- Decompose TE scores from CRS into scale inefficiency and “pure” technical inefficiency

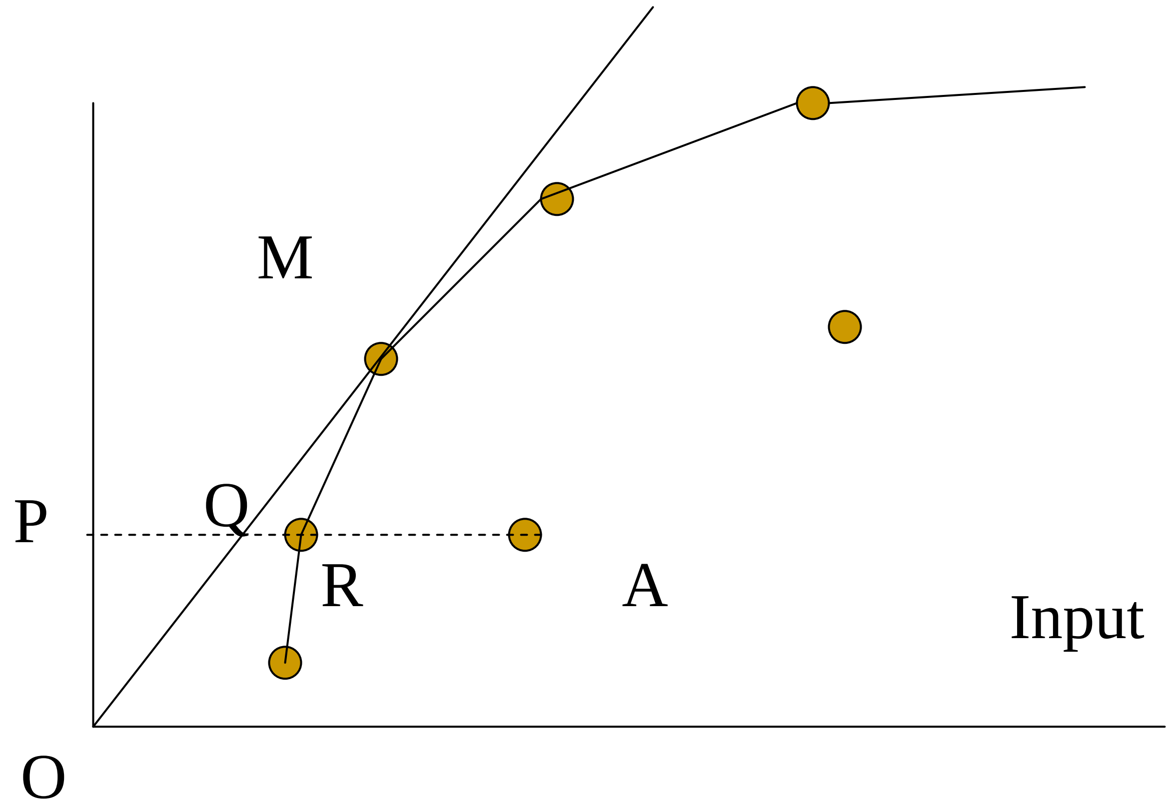
- Minimisation problem needs to be modified

Constant and Variable Returns to Scale

T.& S.E = PQ/PA

T.E. = PR/PA

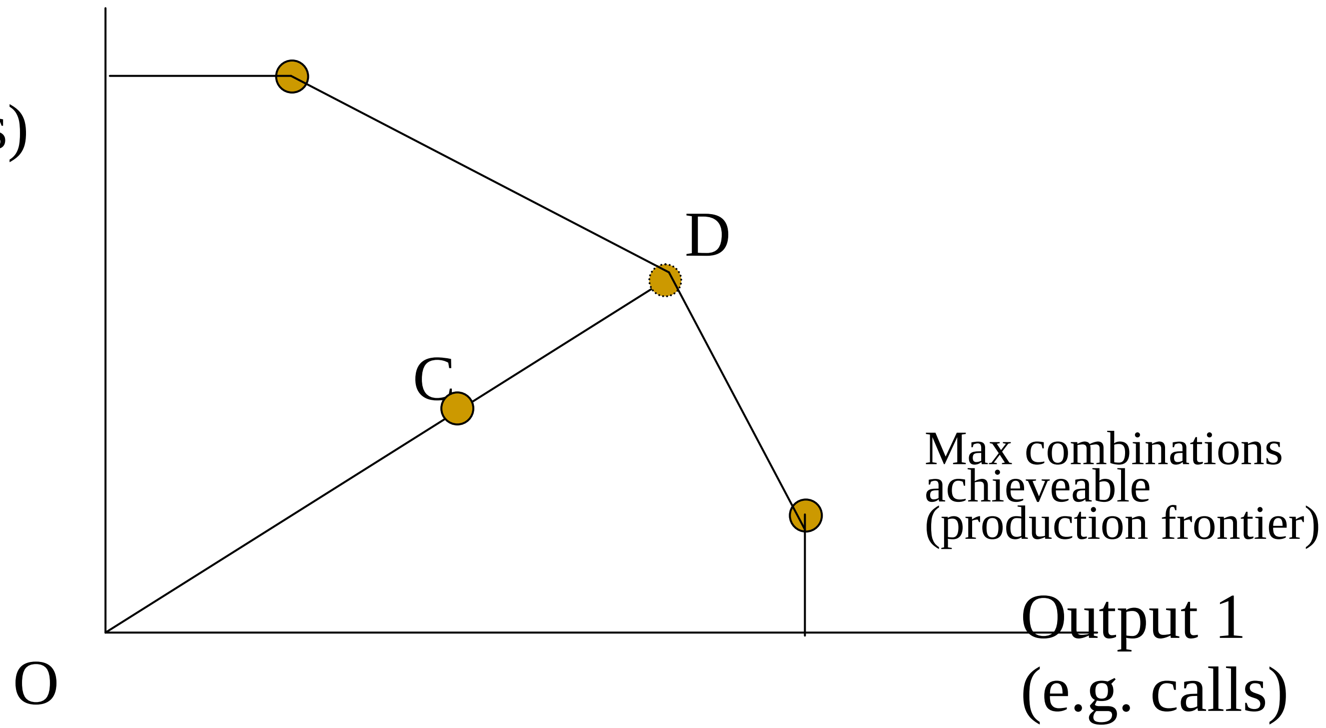
S.E. = PQ/PR



Input vs Output Orientation

- So far, reduction in inputs keeping same output (input orientation):
  - Firms have particular orders to fill (e.g. in electricity generation)
  - Input quantities are the primary decision variables
- But we may look at increasing output keeping same inputs (output orientation):
  - In some other industries, firms have fixed quantity of resources (inputs)
  - Output quantities are the primary decision variables
- Generally, select the orientation in which managers have most control over

Output Orientation



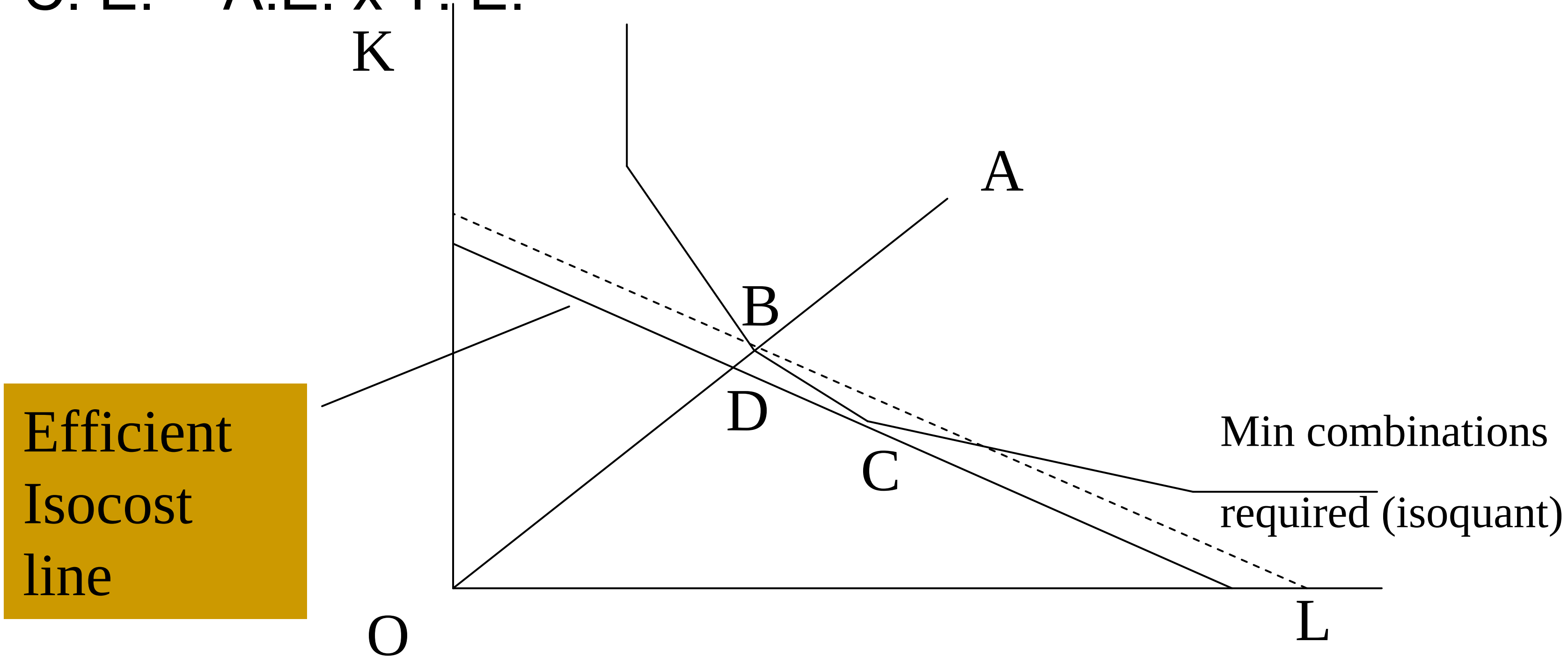
Other things equal = inputs

Technical, Allocative and Cost Efficiency

- Allocative efficiency (AE):
  - Definition: "company (or plant) uses the *right* combination of inputs for given input prices"
  - Examples:
    - Firm using too little capital and too much labour
    - Managers carrying secretarial tasks
  - To measure AE input prices need to be available
- Cost (or economic) efficiency (CE):
  - Definition: "company (or plant) produces at the lowest feasible level of costs for given input prices"
  - TE and AE required to achieve CE

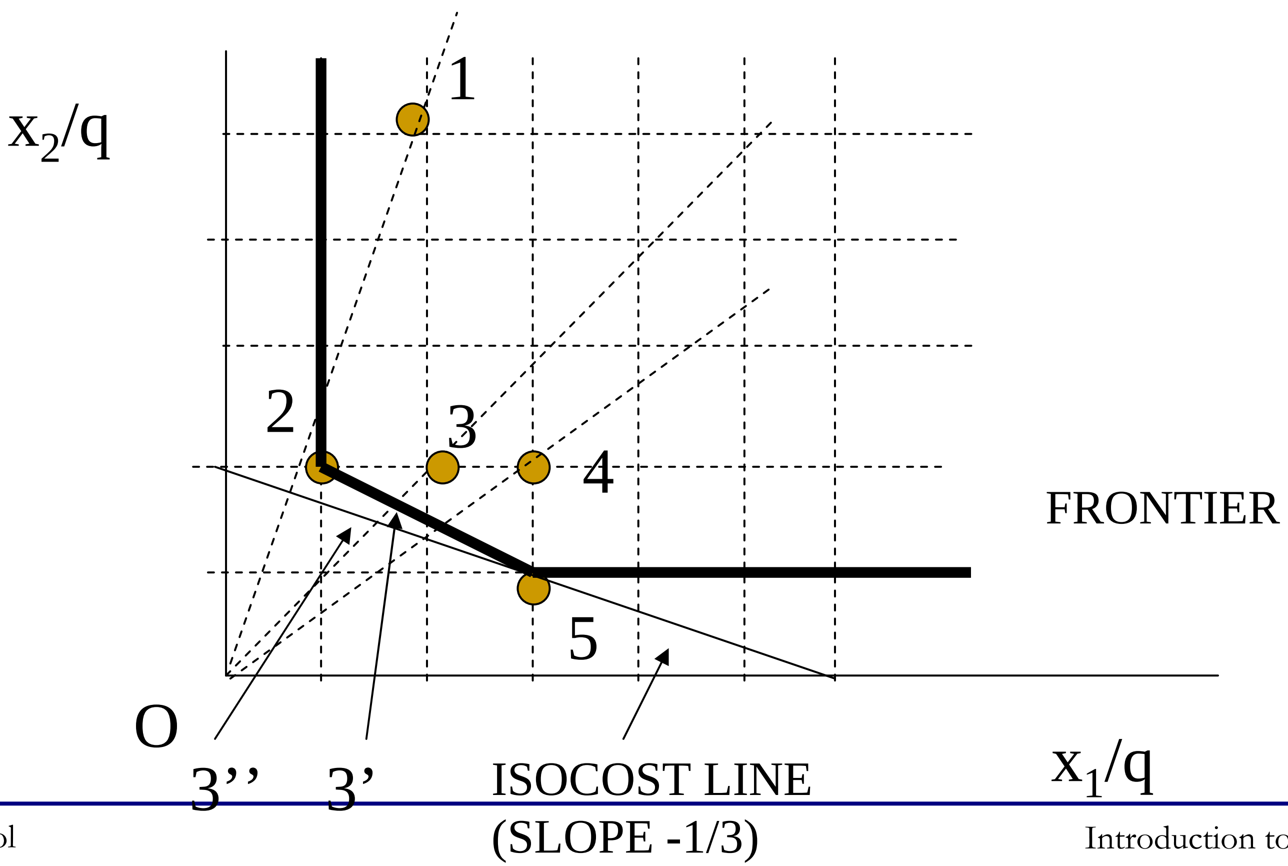
Allocative and Cost Efficiency

T. E. = OB/OA
A. E. = OD/OB
C.E. = OD/OA
C. E. = A.E. x T. E.



Numerical Example (as before)

- Same data as before but now prices available:
  - Price input 1= 1, Price input 2= 3



Results from Numerical Example

Firm	Technical Efficiency	Allocative Efficiency	Cost Efficiency
1	0.5	0.71	0.35
2	1	0.86	0.86
3	0.83	0.9	0.75
4	0.71	0.93	0.66
5	1	1	1
mean	0.81	0.88	0.72

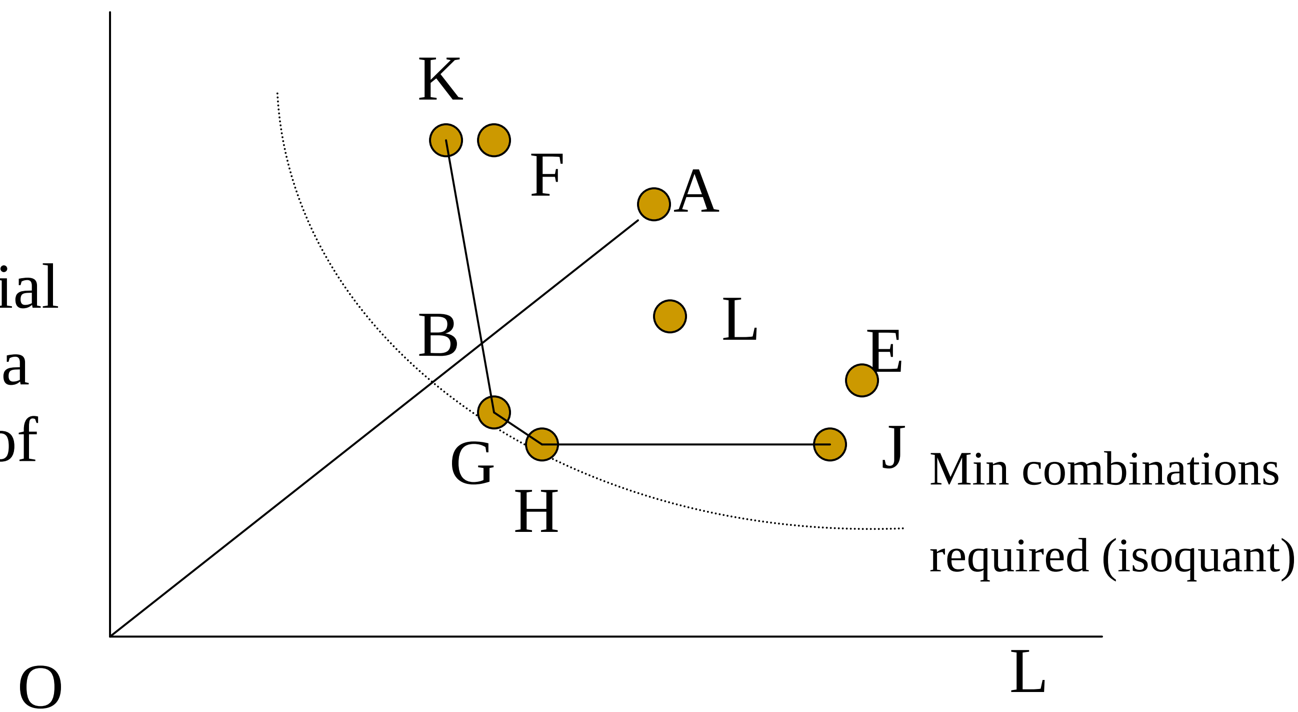
How reliable is DEA?

- Depends on whether frontier can be populated by efficient firms:
  - number of observations
  - number of dimensions
  - closeness to frontier of enough firms
  - distribution of variables

Dangers of DEA (1)

Outliers appear efficient

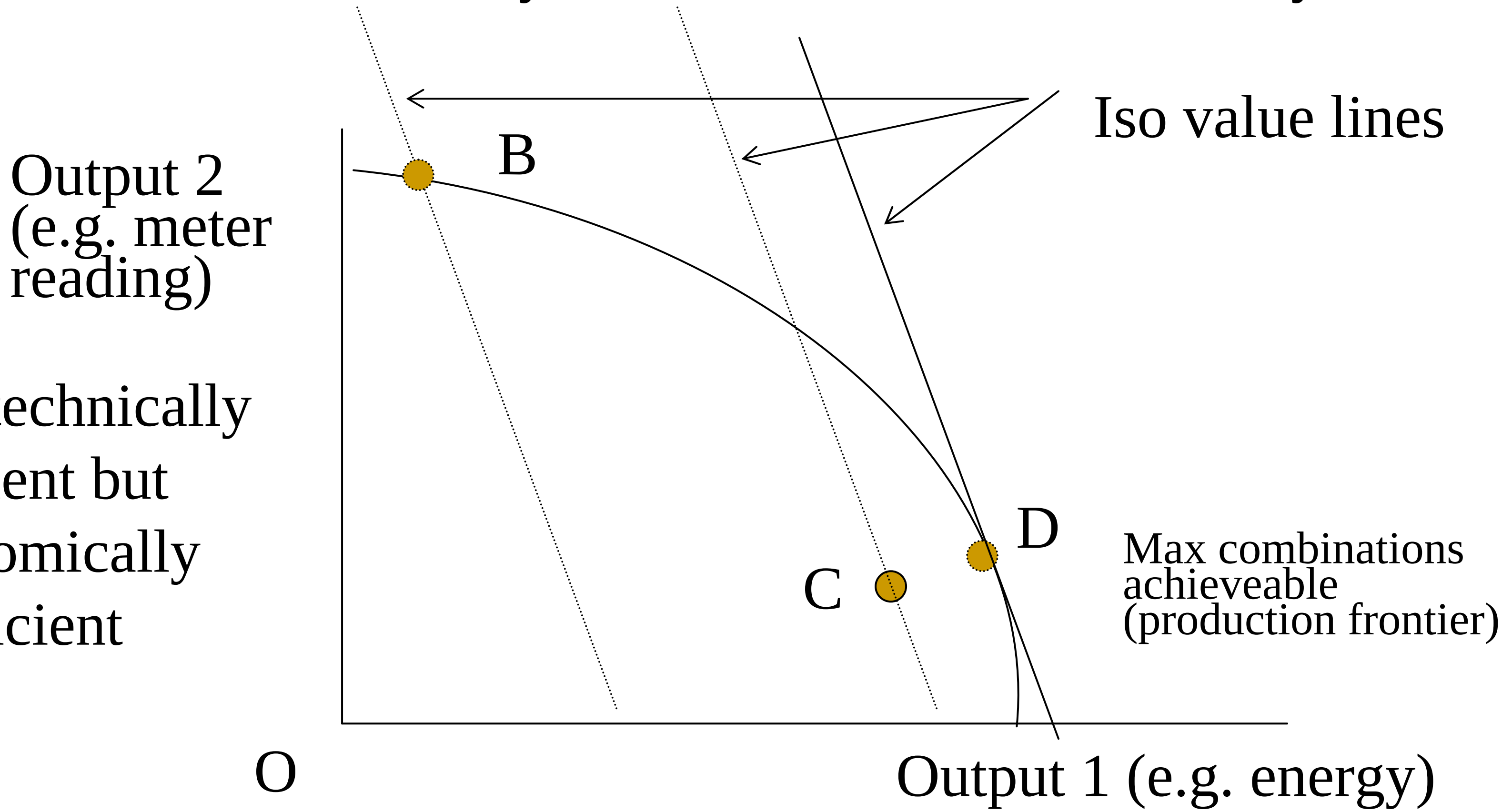
B is an artificial observation - a combination of F and G



Dangers of DEA (2)

- Technical efficiency is not economic efficiency

B is technically efficient but economically inefficient



Dangers of DEA (3)

Dilemma: which variables do we include?

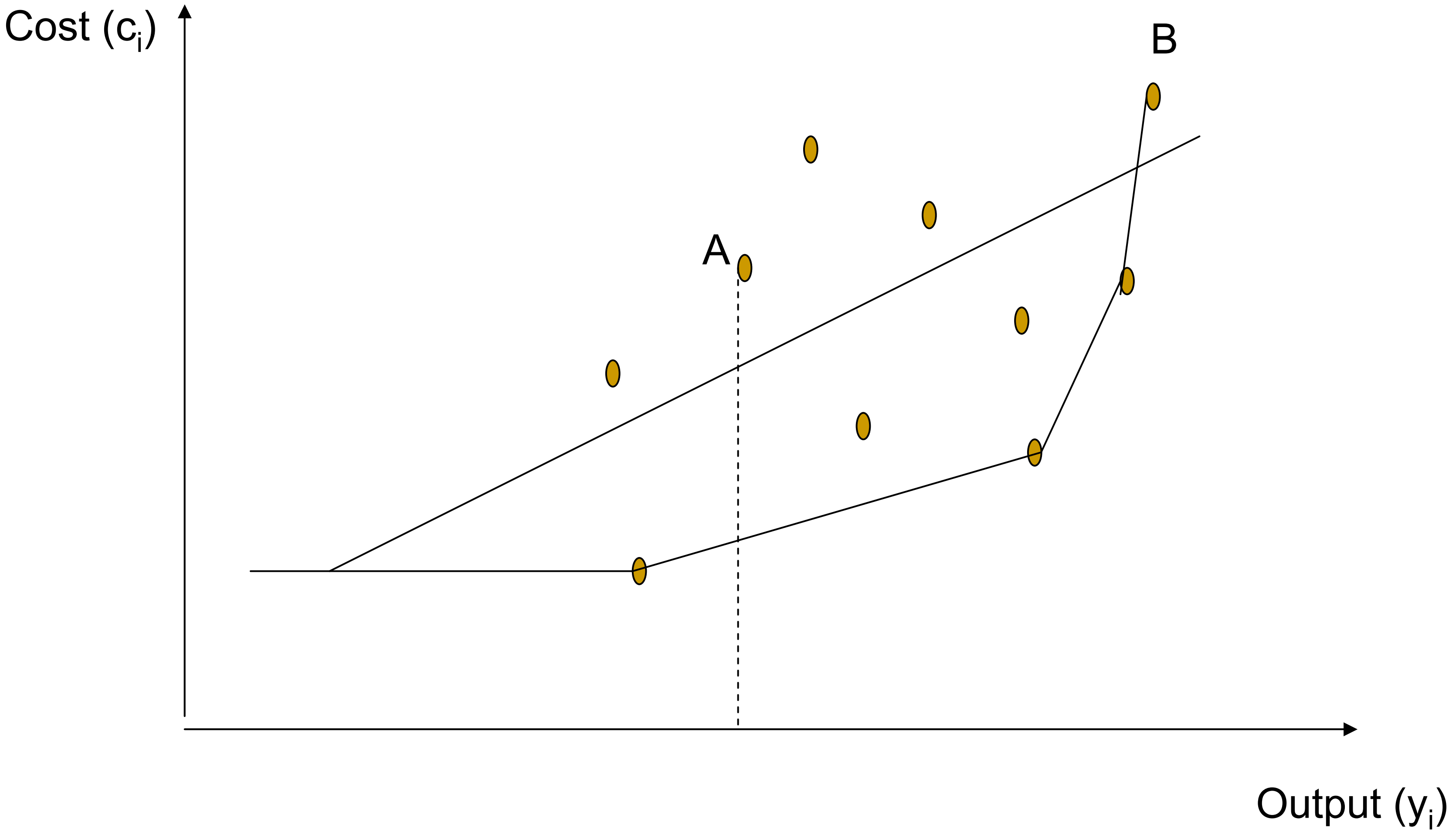
- Include => spuriously efficient
- Exclude => spuriously inefficient

No statistical test for inclusion/ exclusion

DEA vs Regression Analysis

- Fitting the data…
  - RA fits an average efficient line (linear shape)
  - DEA fits a frontier or best practice efficiency line (no assumption on shape)
- Type of technique:
  - RA is a statistical technique
  - DEA uses linear programming
- Inclusion of appropriate cost drivers…
  - With RA one can test which are important
  - With DEA one cannot

RA and DEA



Running DEA

- Purpose - built software
- Excel/Solver macros (see today's lab)
- Identify inputs, outputs and non-controllables
- Organise data for input
- Run
- Interpret