

Quantitative Techniques – Term 2

Laboratory 8

16 March 2006

Overview

The purpose of this lab is to get you familiar with the basic logic of data envelopment analysis (DEA).

Go to www.staff.city.ac.uk/a.banal-estanol/teaching.htm, download Lab8.xls to your directory. This is a DEA model written in Excel by Professor John Cubbin. The spreadsheet is purely for a lab exercise and not to be used for real. The present text is also largely based on Professor Cubbin's lecture notes.

Introduction – Input Minimization DEA with CRS

Data envelopment analysis (DEA) is a linear programming based technique for measuring the relative performance of organisational units where the presence of multiple inputs and outputs makes comparisons difficult.¹

Let's consider an example. Suppose A, B, and C all produce the same output of 100. The table shows the inputs and outputs of the three units:

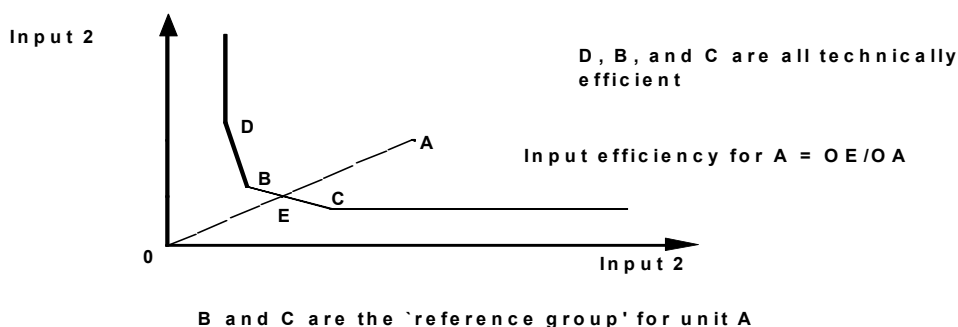
	Input 1	Input 2	Output
A	100	100	100
B	100	50	100
C	50	100	100
Equal combination B and C (Equivalent to point E in figure)	75	75	100

A combination of B and C (E, in our example) could produce the same output with fewer of each input. B and C are said to constitute A's *reference group*. E is said to be A's *efficiency target*. In graphical terms, E is the point of intersection between a radius OA and the envelope curve (see figure below). The frontier is obtained by joining adjacent efficient points together.

Finding the DEA is a matter of defining a linear program, or more generally a constrained optimisation. For this purpose, we will use the algorithm in Lab8.xls.

¹ Source: <http://www.deazone.com/tutorial/index.htm>

**Input Efficiency: A Comparison of Units
Producing the Same Output**



As a reminder from Lecture 8, and to ensure consistency with the notation used by Professor Cubbin, we recap the DEA dual problem with constant returns to scale (Coelli, Rao and Battese, 1998).

$$\text{Min}_{\theta, \lambda} \theta$$

$$\text{s.t.} \quad (1) \quad \sum_i \lambda_i \cdot x_i \leq \theta \cdot x_j, \forall \text{input}$$

$$(2) \quad \sum_i \lambda_i \cdot y_i \geq y_j, \forall \text{output}$$

$$(3) \quad \lambda_i \geq 0, \forall \text{firm}$$

The value of θ is the efficiency score for firm j and is the objective of the minimization problem (cell I2 – highlighted in yellow). If $\theta = 1$, the firm is on the frontier and therefore it is efficient.

Note: In the spreadsheet, we need to minimize cell I3, where the objective function has been adapted to try to avoid a problem with “local stationarities” which were not necessarily optimal. This is embodied in the objective function in I3 rather than the normal one in I2.

The interpretation of the above problem is as follows (Coelli, Rao and Battese, 1998): ‘The problem takes the j -th firm and then seeks to radially contract the input vector, x_j , as much as possible, while still remaining within the feasible input set.’ The boundary of this set is given by the observed data points and is the piece-wise linear isoquant we have in the figure above.² ‘The contraction of the input vector, x_j , produces a projected point’ on the isoquant.

Remember that the minimization problem has to be solved once for each firm.

² The radial contraction in our figure is from A to E, on the isoquant.

Task 1 – Inspecting the Model

When you open the spreadsheet Lab8.xls, a window will pop up about allowing macros. Click on ‘Enable macros’.

In Excel, check that you have ‘Solver’ in your Tools Menu. If not, then go to Tools... Add-Ins and click on Solver.

1. *Open the file and go to the ‘Analysis’ worksheet.*

In the top-left area of the worksheet, the data refers to inputs and outputs of 8 “units” - for our purposes they are companies. We are trying to look at their technical efficiency.

- The choice of the unit to be analysed can be done in Cell I25 – highlighted in pink. We start with ‘unit 1’;
- You can select input minimisation or output maximisation (Cell I24 – highlighted in green). For the moment, choose 0 (input minimization);
- Every time you run the algorithm for a given firm, you will obtain an efficiency score. This is at the top of your ‘Analysis’ worksheet. For instance, for input minimization, our objective is cell I3 – the one below theta. See note above on the difference between I2 and I3.

Further variables required for the solution of the minimization problem:

- The quantities to which constraints (1) and (2) refer are calculated in columns M to P, rows 6 – 13;
- The lambdas – constraint (3) - are in columns C and D, rows 16 – 23.

2. *Focus on unit 1*

- Go to cell I3 and, from the Menu, click on Tools... Solver;
- What are the variables with respect to which the function is minimized?
- Go through the constraints listed in the window that pops up and make sure that you understand whether they correspond to constraints (1), (2) and (3) listed above.

Constraints can be easily changed, removed or added. Try and modify one of them, but then remember to change it back to the correct one for our problem.

Task 2 – Run the Solver

1. Run the solver for firm 1 (i.e. click on ‘Solve’ in the ‘Solver Parameters’ window);
2. If you get a window asking whether you want to ‘Keep solver solution’ or ‘Restore initial values’, chose the first one. Do not request any reports;
3. The solution is given by the efficiency measure (I3) and the lambdas (C16:C23).

For firm 1, given the data on inputs and outputs, we obtain $\theta = 0.70$. This means that firm 1 could reduce the consumption of all inputs by 30% without reducing output.

Observe the values in M7 - M9. These indicate the quantities of inputs that firm 1 could use while keeping the output constant. Therefore, DEA gives an indication on which factors the

firm should reduce to improve its efficiency.

4. Change firm number in cell I25 (pink) to see what efficiency scores you obtain for the other firms. Are your results in line with expectations, on the basis of the input and output quantities?
5. You can copy the results for a given firm in another worksheet ('Results') by clicking on the button 'Copy current analysis to results sheet';
6. Look at the summary of the analysis at the bottom of the worksheet: can you interpret the output?
7. In particular, look at the "reference unit weights". These define the reference group for the unit we are analyzing.

If the solver cannot find a feasible solution, then we use the macro 'Re-initialize lamda' by clicking on the button. What the macro does is assigning a starting value of 1 to the lamda corresponding to the firm we are analyzing.

Caution: Solver does not reliably get the minimum solution: so a) do not be surprised if you can find a "better" outcome; b) this spreadsheet is purely for a lab exercise. Do not use it for real!

Further exercise:

- Try and adapt the algorithm to the case of variable returns to scale. In practice, you should add the constraint that the sum of the λ_i s is equal to 1 to the 'Solver'.

Task 3 – Change from Input Orientation to Output Orientation

We summarize here the linear programming problem for an output-oriented DEA:

Max $_{\phi, \lambda}$ ϕ

s.t. (1) $\sum_i \lambda_i \cdot x_i \leq x_j, \forall input$

(2) $\sum_i \lambda_i \cdot y_i \geq \phi \cdot y_j, \forall output$

(3) $\lambda_i \geq 0, \forall firm$

Note also that $1/\phi$ defines a technical efficiency (TE) score which varies between zero and one.

1. Make a copy of the 'Analysis' worksheet in the same Excel file;
2. Try changing to output maximisation: the quantity to maximize is the same as before, which is phi adjusted to avoid problems of "local stationarities" (cell I3). Technical efficiency is defined as $1/\phi$ in cell L3;
3. You will have to change the objective in Solver from "min" to "max", but also remember to change the constraints, when needed.

Do you get similar results with output maximisation? What is the interpretation of phi?

NB: some error messages or !DIV#0 etc. may be due to the fact that you have not switched correctly between maximisation and minimisation in the Solver dialog.

Final caution message

(from Professor Cubbin's notes)

There are some problems with using solver to do DEA – for example in one model I tried to develop for today's lab Solver was not finding the true minimum. How can you check the model we are actually using?

If the model is working, you can create macros *and run* with Solver. See the Microsoft website <http://support.microsoft.com/kb/843304>. **However, the key thing to do is as follows:**

"To use the Microsoft Excel Solver add-in functions in a VBA macro, you must reference the add-in from the VBA project of the workbook that contains the macros. If you do not reference the Microsoft Excel Solver add-in, you will receive the following compile error when you try to run the macro:

Compile Error: Sub or Function not defined.

To reference the Microsoft Excel Solver add-in for macros in your workbook, use the following steps:

1. Open your workbook.
2. On the **Tools** menu, point to **Macro**, and then click **Visual Basic Editor**.
3. On the **Tools** menu, click **References**.
4. In the **Available References** list, click to select the **Solver.xls** check box, and then click **OK**.
Note If you do not see Solver.xls in the **Available References** list, click **Browse**. In the **Add Reference** dialog box, locate and select the Solver.xla file, and then click **Open**. The Solver.xla file is typically found in the C:\Program Files\Microsoft Office\Office\Library\Solver subfolder.

You are now ready to use the Microsoft Excel Solver functions in a VBA macro."

If you do not do this you will get an error when you try to run the macro, as the Solver software will not be found.