Econ 602b-Albert Banal-Estanol Due date: March 30, 2005

Problem Set 9

1.- Exercise 9.C.7 in Mas-Colell et al. [Hint: assume that you have a WPBE and consider, in turn, that in this WPBE player 1 plays B, T and a pure mixed strategy over this two strategies]. Printing mistake in the book: the WPBE is not unique.

2.- For the game represented in Figure 1:

a) Find the Nash equilibria in pure strategies.

b) Find the weak sequential equilibria in which each player's strategy is pure.

3.- Find the set of weak sequential equilibria of the game in Figure 2.

4.- (Evolutionary Game by Maynard Smith (1991)) Some young animals expend energy begging for food from their parents- they squawk and bleat and scream, sometimes extravagantly. Can we expect these demands to signal their needs accurately? To answer this question, consider the following game.

A hungry parent has a piece of food that it may give to its offspring or keep for itself. It does not detect whether its offspring is hungry. In either case, the offspring may signal that it is hungry to its parent (by squawking, for example). An animal is stronger and thus produces more offspring (i.e. has a higher biological fitness) if it gets the food than if it does not. Normalize the parent's strength if it keeps the food to be 1, and denote its strength if it gives the food to its offspring by $S < 1$. If the offspring does not squawk, its strength is 1 if it gets the food, $V < 1$ if it is not hungry and does not get the food, and 0 if it is hungry and does not get the food. If the offspring squawks, its strength is multiplied by the factor $1-t$, where $0 \leq t \leq 1$ (i.e. squawking may be costly). Denote the degree to which the parent and offspring are related by r, and take each player's payoff to be its strength plus r times the other player's strength. Evolutionary pressure will lead to behavior for each player that maximizes that player's payoff, given the other player's behavior.

a) Represent this game of incomplete information as a game of imperfect information (Bayesian game).

b) Find the conditions on r, in terms of S, V and t, under which the game has the following weak perfect Bayesian equilibrium ("separating equilibrium"): the offspring squawks if and only if it is hungry and the parent gives it the food if and only if it squawks.

c) Show that if the offspring's payoff from obtaining the food exceeds her payoff from not obtaining it, regardless of whether she is hungry (which means that $r < \frac{1-V}{1-S}$), then the game has such an equilibrium only if $t > 0$. That is, in this case an equilibrium exists in which the signal is accurate only if the signal is costly.

d) Show that if $r < \frac{1-S}{1-(1-\alpha)V}$, then the game has the following weak perfect Bayesian equilibrium ("pooling equilibrium"): the offspring is always quiet and the parent always keep the food. (For other parameter values, the game has a pooling equilibrium in which the offspring is always quiet and the parent always gives the food.)

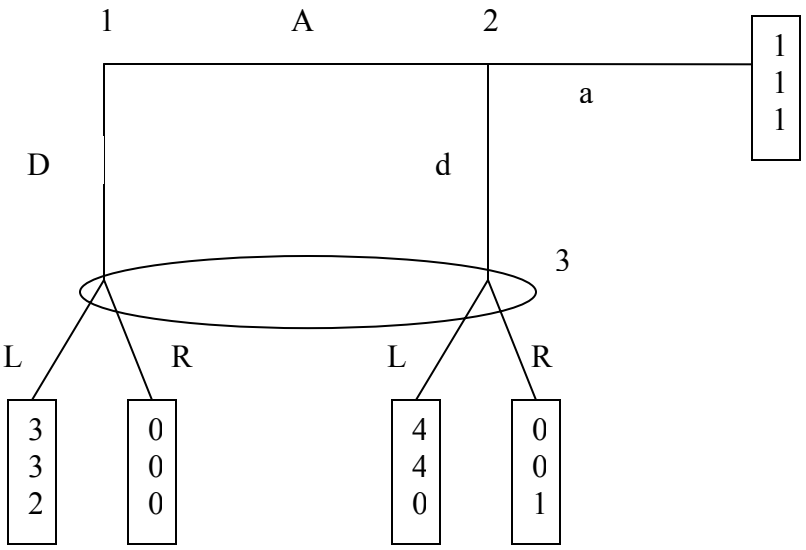

Figure 1

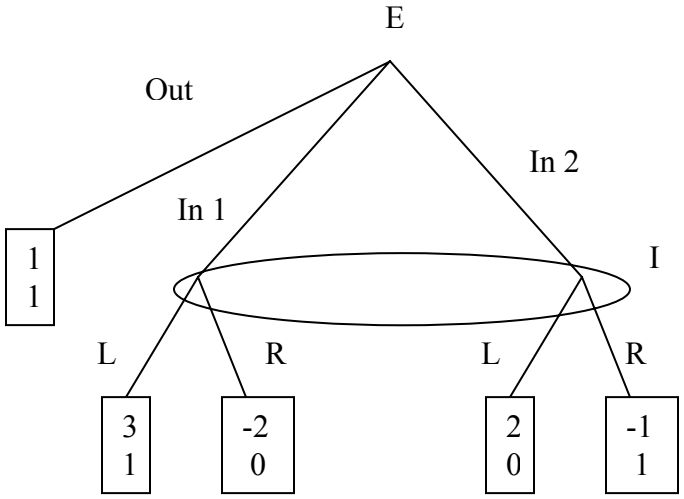

Figure 2