Econ 602b-Albert Banal-Estanol   Due date: March 23, 2005

Problem Set 8

1.- Suppose that Michael and John are playing the following game of incomplete information. Michael (the row player) is perfectly aware of the payoffs but John (the column player) does not know if they are as in G1 or as in G2.

$G1$	L	R		$G2$	L	R
U	$1,1$	$0,0$		U	$0,0$	$0,0$
D	$0,0$	$0,0$		D	$0,0$	$2,2$

a) Model this situation as a Bayesian game.

b) Assuming that is common knowledge that payoffs are as in G1 or as in G2 with equal probabilities, find all Bayesian-Nash equilibria of the game.

2.- Consider the following strategic situation. Two opposed armies are poised to seize an island. Each army's general can choose either "attack" or "not attack." In addition, each army is either "strong" or "weak" with equal probability (the draws for each army are independent), and an army's type is known only to its general. Payoffs are as follows: The island is worth M if captured. An army can capture the island either by attacking when its opponent does not or by attacking when its rival does if it is strong and its rival is weak. If two armies of equal strength both attack, neither captures the island. An army also has a "cost" of fighting, which is s if it is strong and w if it is weak, where $s < w$. There is no cost of attacking if its rival does not. Identify all pure strategy Bayesisn Nash equilibria of this game.

3.- Two firms simultaneously decide whether to enter a market. Firm i's entry cost is $\theta_i \in (0, \infty)$. Firms' entry costs are private information and are independently drawn from the distribution P with strictly positive density p. Firm i's payoff is $\Pi^m - \theta_i$ if it enters but the other firm does not enter, $\Pi^d - \theta_i$ if both enter and 0 if it does not enter. Π^m and Π^d are the monopoly and duopoly profits gross of entry costs and are common knowledge. Assuming that $\Pi^m > \Pi^d$, compute a Bayesian Nash equilibrium.

4.- Imagine an economy consisting of two consumers, labeled 1 and 2. Consumer i chooses x_i in $[0, 1]$. Consumer i's preferences are given by:

$$u_i(x_i, x_j) = x_i - (x_j)^2\theta_i/2 + t_i \text{ (for } j \neq i),$$

where ti is a transfer from the government. Nature selects each θ_i randomly and independently from the interval $[1, 2]$. The distribution of each θ_i over the interval $[1, 2]$ is given by some CDF, F.

a) Suppose that consumers choose the x_i simultaneously and independently. Assume there are no transfers. Solve for a Bayesian Nash equilibrium. Is it unique?

b) Suppose that the government can observe the preference parameters θ_i $(i = 1, 2)$, and that it seeks to maximize $u_1 + u_2$. Setting transfers equal to zero, compute the first best allocation as a function of these parameters.

c) Now suppose that the government cannot observe the preference parameters. Design a mechanism (wherein each consumer announces its preference parameter, and the government determines actions and transfers as functions of these announcements) satisfying the following two properties: (1) the mechanism implements the first-best allocation in dominant strategies, and (2) the mechanism always results in a balanced budget ($t_1 = -t_2$) for all possible announcements. [Note: Do not give a general formula; give the specific mechanism that works for this particular problem.]