Econ 602b-Albert Banal-Estanol Due date: March 2, 2005

Problem Set 5

1.- Obtain the pure strategy Nash equilibria of the games described in figures 1, 2 and 3 of problem 4 of Problem Set 2.

2.- Consider the game in extensive form of figure 4 of problem 4 of Problem Set 2.

(a) Find the set of Nash equilibria obtained by backwards induction.

(b) Find all Nash equilibria (pure and mixed) and verify that the equilibria obtained in (a) is a subset of the set of equilibria.

3.- Consider the three player extensive form game with imperfect information depicted in Figure 1.

a) Show that (A,a) is not the play of a Nash equilibrium.

b) Find a Nash equilibrium of the game. Is it a subgame perfect Nash equlibrium?

4.- (Hotelling) Suppose that we have two firms located on a line of length 1. The unit costs of the good for each store is c. Consumers incur a transportation cost tx^2 for a length of x. Consumers have unit demands; each consumer derives a utility of $\overline{v}$ for consumption. Firm 1 is located at point $a \geq 0$ and firm 2 at point $1-b$, where $b \geq 0$ and without loss of generality, $1-a-b \geq 0$ (firm 1 is to the left of firm 2; $a = b = 0$ corresponds to maximal differentiation and $a+b = 1$ corresponds to minimal differentiation, i.e. perfect substitutes). Assume that the market is covered and firms sell positive quantities.

a) Show that given a and b, firm 1 will charge

$$p_1 = c + t(1-a-b)(1+\frac{a-b}{3})$$

whereas firm 2 will charge

$$p_2 = c + t(1-a-b)(1+\frac{b-a}{3}).$$

b) Find firms' market shares of the market.

c) Now consider a first stage to this game where the two firms choose their locations, knowing that prices will be chosen in the second stage as in a). Where do they locate?

d) What would be the socially optimal location of the two firms? Compare with the market outcome.

5.- Consider the N-player version of the (Rubinstein's) bargaining game presented in class. At dates 1, $N+1$, $2N+1$, ..., player 1 offers a division $(x_1, ..., x_N)$

of the pie with $x_i \geq 0$ for all i, and $\sum_{i=1}^{N} x_i \leq 1$. At dates 2, $N+2$, $2N+2$, ..., player 2 offers a division, and so on. When player i offers a division, the other players simultaneously accept or veto the division. If all accept, the pie is divided; if at least one vetoes, player $i+1$ (mod N) offers a division in the following period. Assuming that the players have common discount factor δ, show that, for all i, player i offering a division

$$\left(\frac{v}{1+\cdots+\delta^{N-1}}, \frac{v\delta}{1+\cdots+\delta^{N-1}}, ..., \frac{v\delta^{N-1}}{1+\cdots+\delta^{N-1}}\right)$$

for players i, $i+1$, ...,$i+N-1$ (mod N) at each date $(kN+i)$ and the other players's accepting any offer equal or higher than those amounts is a subgame-perfect Nash equilibrium.

6.- (Application to Bilateral Monopoly) Consider a market for some intermediate good with a single seller S and a single buyer B. If the buyer buys a quantity q at a price p, then the profits to the players are as follows,

$$\begin{aligned} \Pi_B(p,q) &= R(q) - pq \\ \Pi_S(p,q) &= pq - C(q) \end{aligned}$$

where $R(q)$ is the revenue obtained by the buyer by transforming the quantity q of the input into some output and then selling the output on some final good market, and $C(q)$ is the cost to the seller of producing the quantity q of the input. Assume that $R(0) = 0$, $R'(q) > 0$, $R''(q) < 0$, $C(0) = 0$, $C'(q) > 0$, $C''(q) > 0$ and $R'(0) > C'(0)$.

Players bargain over the price and quantity of trade according to the (infinite) model presented in class. However, here, an offer is a pair (p,q) where $p \geq 0$ and $q \geq 0$. If they reach an agreement at time $t\Delta$ on a pair (p,q), then player i's payoff is $\Pi_i(p,q)\exp(-r_i t\Delta)$

We look for a SPNE such that, **in equilibrium**, players play stationary strategies (stationary for each player) and *any offer is accepted*. Let (p_i^*, q_i^*) denote the equilibrium offer that player i makes whenever she has to make an offer. Let $V_i^* \equiv \Pi_i(p_i^*, q_i^*)$.

a) Start with seller's offer. Argue that $\Pi_B(p_S^*, q_S^*) = \delta_B V_B^*$ and that optimality requires the seller to offer $q_S^* = q^e$ where q^e is the unique solution to $R'(q) = C'(q)$. Notice that the equilibrium quantity traded maximizes total surplus or gains from trade $(R'(q) = C'(q))$ and since it is reached in the first period, the SPNE is Pareto efficient. Denote $\pi = R(q^e) - C(q^e)$.

b) Using a) and a symmetric argument for buyer's offer, find a system of two equations in (V_B^*, V_S^*). Show that there is a SPNE where players obtain $(V_B^*, \delta_S V_S^*)$ if the buyer is the first in making an offer and $(V_S^*, \delta_B V_B^*)$, where

$$V_B^* = \frac{\pi(1-\delta_S)}{1-\delta_B\delta_S} \text{ and } V_S^* = \frac{\pi(1-\delta_B)}{1-\delta_B\delta_S}.$$

c) Show that

$$\begin{aligned} p_B^* &= \frac{\delta_S(1-\delta_B)}{1-\delta_B\delta_S}\frac{R(q^e)}{q^e} + \frac{1-\delta_S}{1-\delta_B\delta_S}\frac{C(q^e)}{q^e} \\ p_S^* &= \frac{1-\delta_B}{1-\delta_B\delta_S}\frac{R(q^e)}{q^e} + \frac{\delta_B(1-\delta_S)}{1-\delta_B\delta_S}\frac{C(q^e)}{q^e} \end{aligned}$$

d) Finally show that, when the time intervals tend to 0, the equilibrium trade price is, no matter who makes the offer at time 0, p^*, a convex combination of the equilibrium average revenue and average cost, where

$$p^* = \frac{r_B}{r_B + r_S}\frac{R(q^e)}{q^e} + \frac{r_S}{r_B + r_S}\frac{C(q^e)}{q^e}.$$