Econ 602-Albert Banal-Estanol Due date: February 9, 2004

Problem Set 4

1.- (Mixed strategies) Show that the two-player game illustrated below has a unique equilibrium in pure strategies and that there is no additional equilibrium in mixed strategies.

	L	M	R
U	1,-2	-2,1	0,0
M	-2,1	1,-2	0,0
D	0,0	0,0	1,1

2.- (Guessing the number) Two players, 1 and 2, choose simultaneously a positive integer up to K. If they choose the same number then player 2 pays 1 dollar to player 1; otherwise, no payment is made. Players' preferences are represented by their expected monetary payoff. Show that this game has a unique mixed strategy Nash equilibrium (and find it).

3.- (Mergers) There are three identical firms in the industry. The demand is $P(Q)=1-Q$. The marginal costs are zero.
a) Compute the Nash Equilibrium.
b) Show that if two of the three firms merge (transforming the industry into a duopoly), the profit of these firms decreases. Explain [What would be the changes in production and in price following the merger? Would outsider firms benefit from the merger?]
c) What happens if all three firms merge?
d) If the firms were competing in prices and sold differentiated products, would a merger between two of them be profitable? (Work at an intuitive level deriving the best reply functions).

4.- (Example by R.Gibbons based on a discussion by Hume, 1739) Suppose that there are I farmers, each of whom has the right to graze cows on the village common. The amount of milk a cow produces depends on the total number of cows, N, grazing on the green. The revenue produced by n_i cows is $n_i v(N)$ for $N<M$ and $v(N)=0$ for $N>M$, where $v(0)>0$, $v'<0$, and $v''\leq 0$. Each cow costs c, and cows are perfectly divisible. Suppose $v(0)>c$. Farmers simultaneously decide how many cows to purchase; all purchased cows will graze on the common.
a) Write this as a game in normal form (do not represent).
b) Find the Nash equilibrium, and compare against the social optimum.
c) Discuss the relationship between this game and the Cournot oligopoly model.

5.- (Edgeworth duopoly) There are two identical firms producing a homogeneous good whose demand curve is $q=100-p$. Firms simultaneously choose prices. Each firm has a capacity constraint of K. If the firms choose the same price they share the market equally. If the prices are unequal, $pi<pj$, the

low-price firm, i, sells min(100-pi,K) and the high-price firm, j, sells $\min[\max(0,100-p_i-K,K)]$. The cost of production is 10 per unit.

a) Find firm i's payoff function.

b) Does this game satisfy the assumptions of the theorem of the existence of Nash equilibrium in pure strategies?

c) Suppose that $30<K<45$. Show that this game does not have any pure-strategy Nash equilibrium. [Hint: Show it by contradiction. Firstly, show that in any Nash equilibrium firms should select the same price, p. Secondly, show that $10<p\leq 100-2K$ and then that $p=100-2K$. Finally, show that this is not a Nash equilibrium because any firm has an incentive to raise its price.