Econ 602-Albert Banal-Estanol Due date: February 2, 2005

Problem Set 3

1.- Find the Nash equilibria of the game obtained in part (b) of Exercise 1 in Problem set 2.

2.- Find the Nash equilibria of the games (in normal form) obtained from Figures 1,2 and 3 of Exercise 4 in problem set 2.

3.- Two players have $10 to divide between themselves. To do so, they use the following procedure: Each player names a number of dollars (a non-negative integer), at most equal to 10. If the sum of the two numbers is at most 10 then each player receives the amount of money she names (and the remainder is destroyed). If the sum of the two numbers exceeds 10 and the two numbers are different then the player who names the smaller number receives that amount and the other player receives the remaining money. If the sum of the two numbers exceeds 10 and the two numbers are equal each players receives $5. Determine the best-reply correspondence of each player, plot them in a diagram, and find the Nash equilibria of the game.

4.- Each of I people chooses whether to become or not a political candidate, and if so which position to take. There is a continuum of citizens, each of whom has a favorite position; the distribution of favorite positions is given by a density function f on [0,1] with f(x)>0 for all x in [0,1]. A candidate attracts the votes of those citizens whose favorite positions are closer to his position than to the position of any other candidate; if k candidates choose the same position then each receives the fraction 1/k of the votes that the position attracts. The winner of the competition is the candidate who receives the most votes. Each person prefers to be the unique winning candidate than tie for the first place, prefers to tie for the first place than to stay out of the competition, and prefers to stay out of the competition than to enter and lose. Formulate this situation as a static game, find the set of Nash Equilibria when I=2, and show that there is no Nash Equilibrium when I=3.

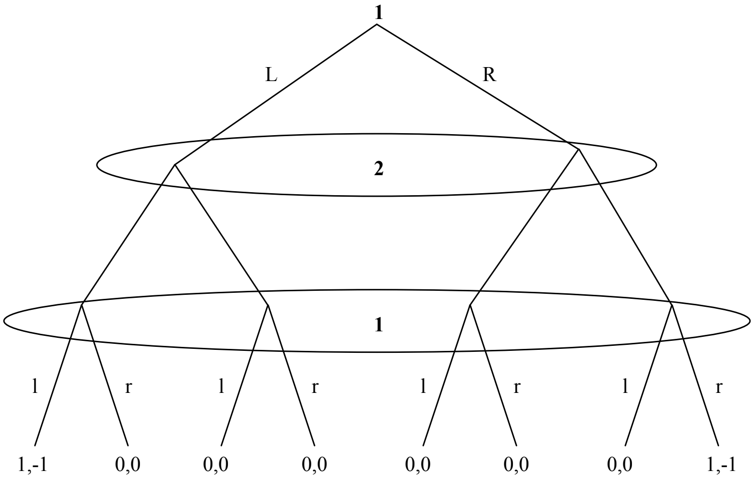
5.- An object is to be assigned to a player in the set {1,...,I} in exchange for a payment. Player i's valuation of the object is v_i and $v_1 > v_2 > ... > v_I > 0$. The mechanism used to assign the object is a (sealed-bid) auction: the players simultaneously submit bids (non-negative numbers), and the object is given to the player with the lowest index among those who submit the highest bid, in exchange for payment.
(a) In a first price auction the payment that the winner makes is the price she bids. Formulate a first price auction as a static game and analyze its Nash equilibria. In particular, show that in all equilibria player 1 obtains the object
(b) In a second price auction the payment that the wins makes is the highest bid among those submitted by the players who do not win (so if only one agent submits

the highest bid then the price paid is the second highest bid). Show that in a second price auction the bid v_i of any player i is a weakly dominant strategy.

6.- Two players are involved in a dispute over an object. The value of the object to player i is $v_i>0$. Time is modeled as a continuous variable that starts at 0 and runs indefinitely. Each player chooses when to concede the object to the other player; if the player to concede does so at time t, the other player obtains the object at that time. If both players concede simultaneously, the object is split equally between them, player i recieving a payoff of $v_i/2$. Time is valuable: until the first concession each player loses one unit of payoff per unit of time. Formulate this situation as a strategic game and show that in all Nash equilibria one of the players concedes immediately.

7.- Exercise in class ("M_2" is never a best response but is not strictly dominated).

8.- Consider the following game in extensive-form with imperfect recall. Show that player 1's best behavioral strategy assures him a payoff of 1 with probability ¼, while there is a mixed strategy that assures him the payoff 1 with probability 1/2.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