Chapter 2: Consumer Theory

Albert Banal-Estanol

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2.- Consumer Theory

2.1 Preference and Utility

- Preferences of rational individuals?
- Representation of preferences: utility
- Indifference curve maps
- Marginal rate of substitution concept
- Examples of utility functions

“Rationality”

- Definition: An individual is rational if for any “situation” A, B and C, she...
  prefers A to B, B to A or she is indifferent between A and B (completeness)
  If she prefers A to B and B to C then she prefers A to C (transitivity)
  If she strictly prefers A to B then she prefers a C close to A to B (continuity)

"Representation" of Preferences

- Proposition: A rational's individual preferences can be ranked from least to most desirable, according to a utility function $u(\cdot) \in \mathbb{R}$:

$$A \text{ is preferred to } B \text{ if and only if } u(A) \geq u(B)$$

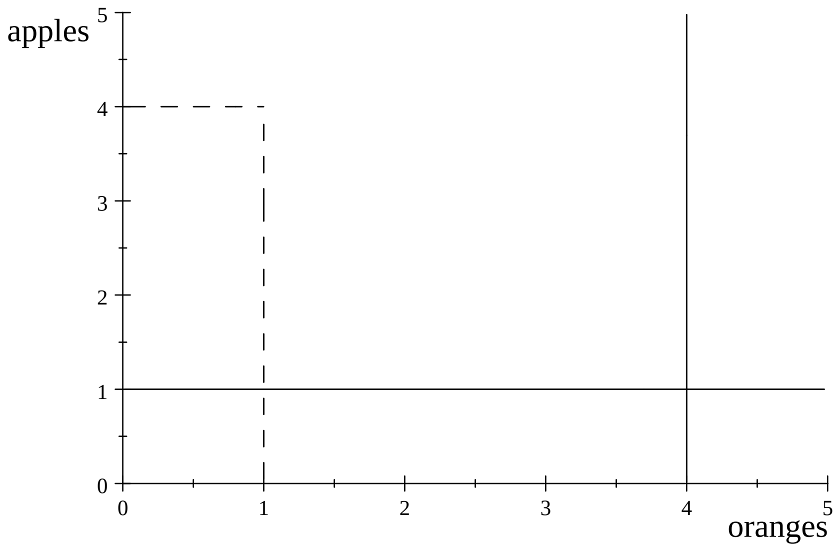
- Example: prefer one orange to one apple: $u(1 \text{ orange}) = 10 > u(1 \text{ apple}) = 5$

- Proposition: Utility function is unique up to an order preserving transformation

- Previous preferences are the same as: $u(1 \text{ orange}) = 1 > u(1 \text{ apple}) = -1$ (only rank matters)

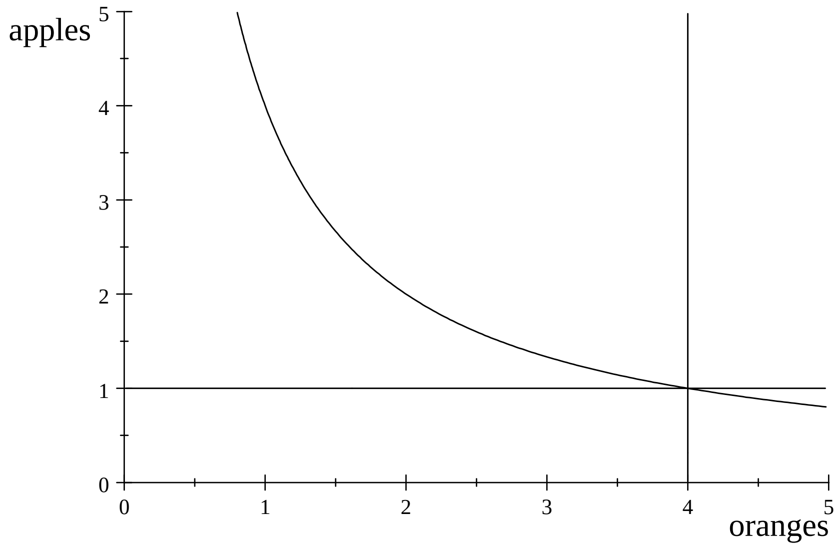
Preferences over Bundles

- Do you prefer 4 apples and 1 orange or 1 apple and 4 oranges?
- What should be always preferred to a bundle of 4 oranges and 1 apple?



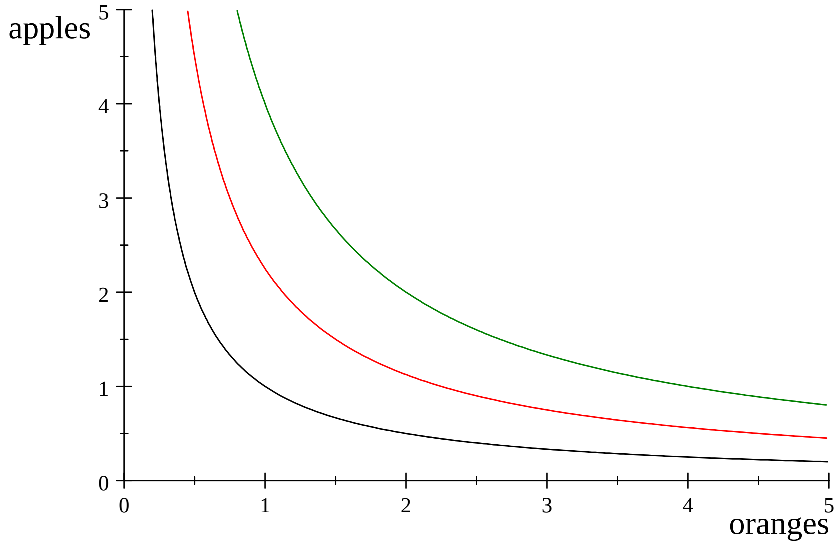
Indifference Curve

- Definition: set of consumption bundles among which an individual is indifferent



- Indifference curve decreasing. Would you expect it to be convex or concave?

Indifference Curve Map of one Individual



- U(green)>U(red)>U(black). Attach a value to each: $U(x_1, x_2) = \sqrt{x_1 x_2}$
- Indifference curves as a contour map (with altitudes). Can they cross?

Marginal Rate of Substitution (MRS)

- More generally, preferences over bundles of any number of goods: $U(x_1, ..., x_n)$
- MRS: negative of the slope of indifference curve (willingness to trade x_j for x_i)

$$MRS_{i,j}(x_1, ..., x_n) \equiv \frac{\frac{\partial U(x_1,...,x_n)}{\partial x_i}}{\frac{\partial U(x_1,...,x_n)}{\partial x_j}}$$

- Example: $U(x_1, x_2) = \sqrt{x_1 x_2}$

$$MRS_{i,j}(x_1, x_2) = MRS(x_1, x_2) = \frac{\frac{x_2}{2\sqrt{x_1 x_2}}}{\frac{x_1}{2\sqrt{x_1 x_2}}} = \frac{x_2}{x_1}$$

(a) $MRS(1, 4) = 4$, (b) $MRS(1, 1) = 1$ and (c) $MRS(4, 1) = 1/4$

- Interpretation for (a): for each extra orange, she is willing to give up 4 apples

Examples of utility functions (1)

- Cobb-Douglas

$$U(x_1, x_2) = x_1^{\alpha} x_2^{\beta}$$

where α,$\beta > 0$. α and β represent the relative importance of x_1 and x_2

- Same preferences as

$$\widehat{U}(x_1, x_2) \equiv U(x_1, x_2)^{1/(\alpha+\beta)} = x_1^{\widehat{\alpha}} x_2^{\widehat{\beta}}$$

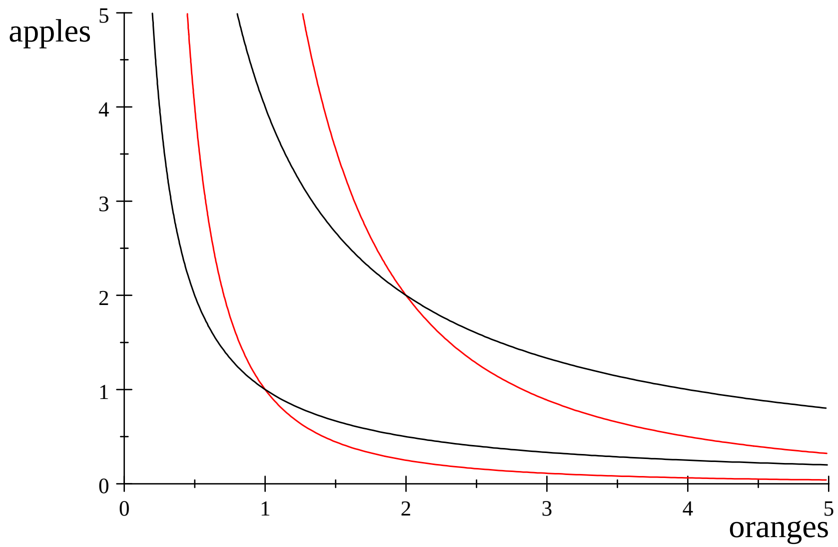
where $\widehat{\alpha} = \alpha/(\alpha + \beta)$ and $\widehat{\beta} = \beta/(\alpha + \beta)$ and therefore $\widehat{\alpha} + \widehat{\beta} = 1$

- Use α,β such that $\alpha + \beta = 1$ from now on

- $MRS(x_1, x_2) = \frac{\alpha}{\beta}\frac{x_2}{x_1}$. Interpretation with respect to α and β?

Preferences of two individuals

- Cobb-Douglas with $\alpha = 2/3$ & $\beta = 1/3$ (red) and $\alpha = 1/2$ & $\beta = 1/2$ (black)



- Red individual: more willing to trade apples for one orange (higher MRS)

Examples of utility functions (2)

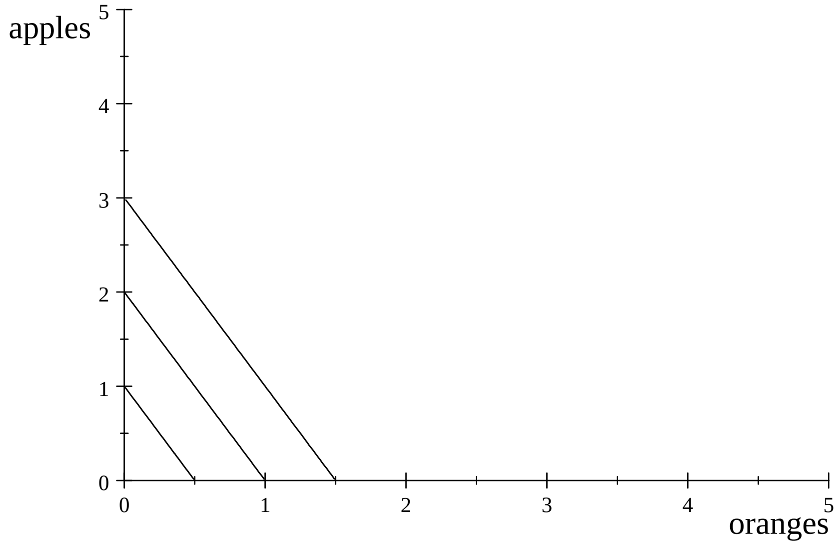
- Perfect substitutes

$$U(x_1, x_2) = \alpha x_1 + \beta x_2$$

where α,$\beta > 0$

- $MRS(x_1, x_2) =$?
- Reasonable? Examples?

Perfect Substitutes with $\alpha = 2$ and $\beta = 1$



Examples of utility functions (3)

- Perfect complements

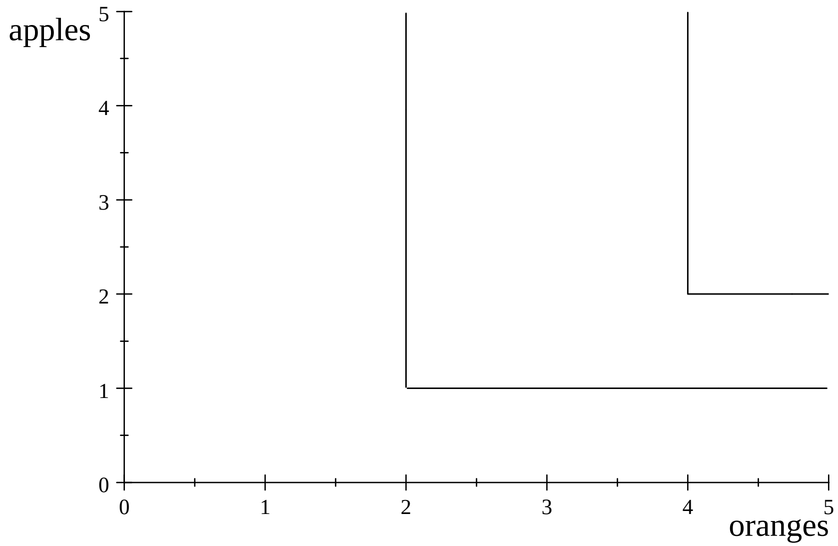
$$U(x_1, x_2) = \min\{\alpha x_1, \beta x_2\}$$

for α,$\beta > 0$.

- $MRS(x_1, x_2) =$?

- Examples of this utility function?

Perfect Complements with $\alpha = 1$ and $\beta = 2$



2.2.- Utility Maximisation and Choice

- Different goods are available (orange, apples,...): goods $1, 2, ..., n$
- Individual can buy any combination of them subject to a maximum budget
- Choice will depend on individual preferences
- Dual problem of expenditure minimisation

- People do not solve maximisation problems when buying in the supermarket
- But, we will argue that they act as if they were making these calculations
- As starting point, we will assume selfishness

What can you afford to buy?

- If price of each good i is p_i and the individual has I euros to allocate between these goods, budget constraint is

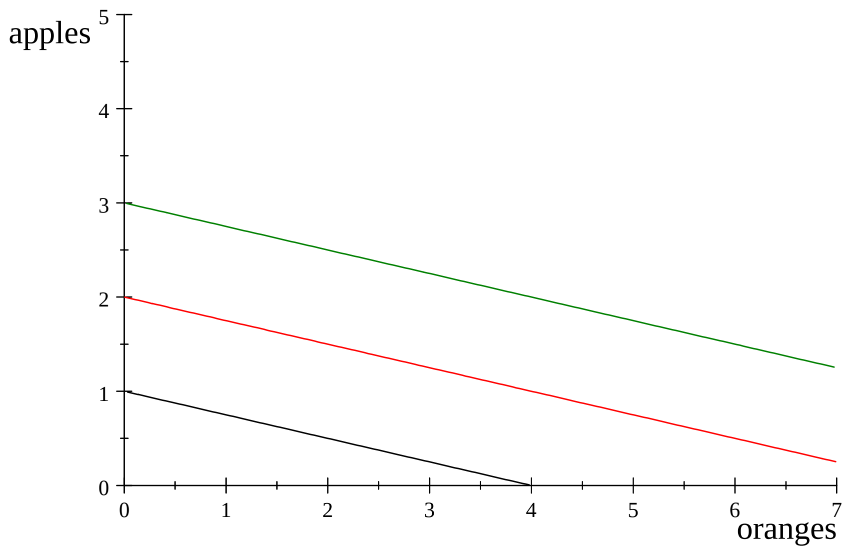
$$p_1 x_1 + p_2 x_2 + ... + p_n x_n \leq I$$

- For $n = 2$, graphical representation. Budget frontier can be represented as

$$x_2 = \frac{I}{p_2} - \frac{p_1}{p_2} x_1$$

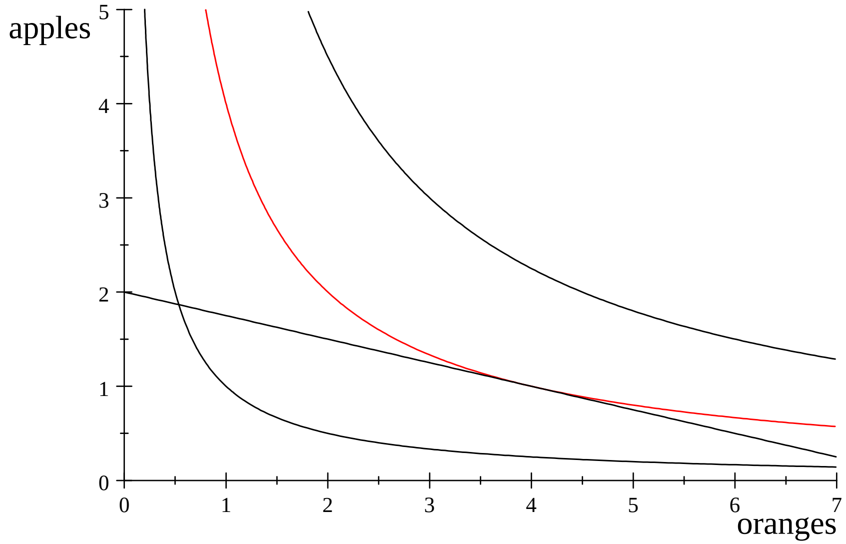
- Straight line from $\frac{I}{p_2}$ (vertical) to $\frac{I}{p_1}$ (horizontal axis) and slope $-\frac{p_1}{p_2}$

Budget Constraint



- Here $p_1 = 0.25$ and $p_2 = 1$ with $I = 1$ (black), 2 (red) and 3 (green)

What do you want to buy?



- Bundle s.t. (a) spend all money and (b) utility tangent to budget constraint

Mathematically

- Problem: Find $x_1^*, ..., x_n^*$ that solve

$$Max\ U(x_1, ..., x_n)$$
$$subject \text{ to } p_1 x_1 + ... + p_n x_n = I$$

which can be rewritten as

$$Max_{x_1,...,x_n,\lambda}\ \mathcal{L} = \ U(x_1, ..., x_n) + \lambda(I - p_1 x_1 - ... - p_n x_n)$$

or

$$\frac{\partial \mathcal{L}}{\partial x_i} = \frac{\partial U(x_1^*, ..., x_n^*)}{\partial x_i} - \lambda^* p_i = 0 \text{ for all } i$$
$$\frac{\partial \mathcal{L}}{\partial \lambda} = I - p_1 x_1^* - ... - p_n x_n^* = 0$$

Shadow Value and MRS

- Notice that

$$\lambda^* = \frac{\frac{\partial U(x_1^*,...,x_n^*)}{\partial x_1}}{p_1} = ... = \frac{\frac{\partial U(x_1^*,...,x_n^*)}{\partial x_n}}{p_n}$$

- Interpretation: at the maximum, each good gives same marginal utility (1 extra euro, $1/p_i$ extra units, $\frac{\partial U}{\partial x_i}$ marginal utility)

- Also,

$$MRS_{i,j}(x_1^*, ..., x_n^*) = \frac{\frac{\partial U(x_1^*,...,x_n^*)}{\partial x_i}}{\frac{\partial U(x_1^*,...,x_n^*)}{\partial x_j}} = \frac{p_i}{p_j} \text{ for any } i \text{ and } j$$

Example

- Utility: $U(x_1, x_2) = x_1^{0.5} x_2^{0.5}$ prices $p_1 = 0.25$, $p_2 = 1$ and $I = 2$

$$MRS(x_1^*, x_2^*) = \frac{x_2^*}{x_1^*} = \frac{0.25}{1} \text{ and } 2 - 0.25x_1^* - 1x_2^* = 0$$

- Solving $x_1^* = 4$ and $x_2^* = 1$, $U(x_1^*, x_2^*) = 2$ and $\lambda^* = 1$

Cobb-Douglas

- More generally, for any Cobb-Douglas utility function $U(x_1, x_2) = x_1^{\alpha} x_2^{\beta}$,

$$x_1^* = \frac{\alpha I}{p_1} \text{ and } x_2^* = \frac{\beta I}{p_2}$$

- In this case, fraction of money spent in each good is

$$\frac{p_1 x_1^*}{I} = \alpha \text{ and } \frac{p_2 x_2^*}{I} = \beta$$

Demand and Indirect Utility Function

- For any utility function we have a demand function:

$$x_i^* \equiv d_i(p_1, ..., p_n, I)$$

- Substituting into the utility function we have

$$U(x_1^*, ..., x_n^*) = U(d_1(p_1, ..., p_n, I), ..., d_n(p_1, ..., p_n, I)) \equiv v(p_1, ..., p_n, I)$$

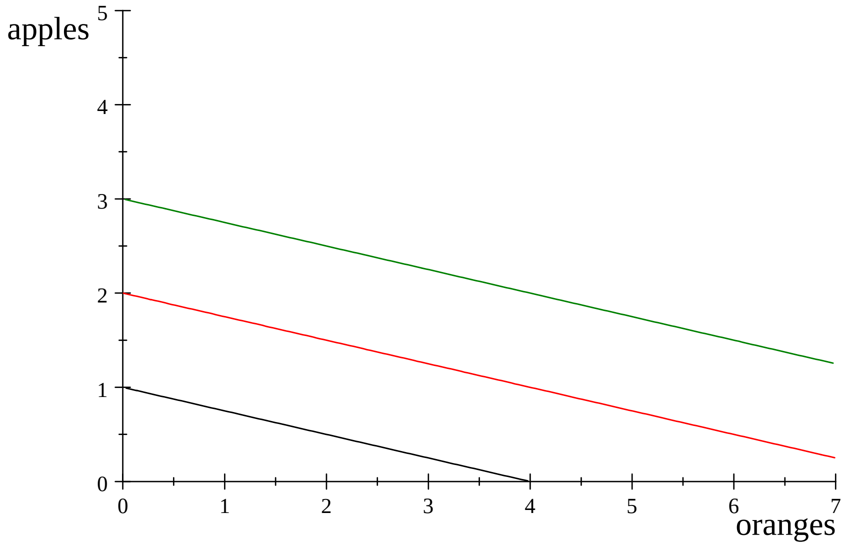
- In the Cobb-Douglas example

$$v(p_1, p_2, I) = U(x_1^*, x_2^*) = \left(\frac{\alpha I}{p_1}\right)^\alpha \left(\frac{\beta I}{p_2}\right)^\beta = \alpha^\alpha \beta^\beta \frac{I}{p_1^\alpha p_2^\beta}$$

- Indirect utility increasing in income and decreasing in prices

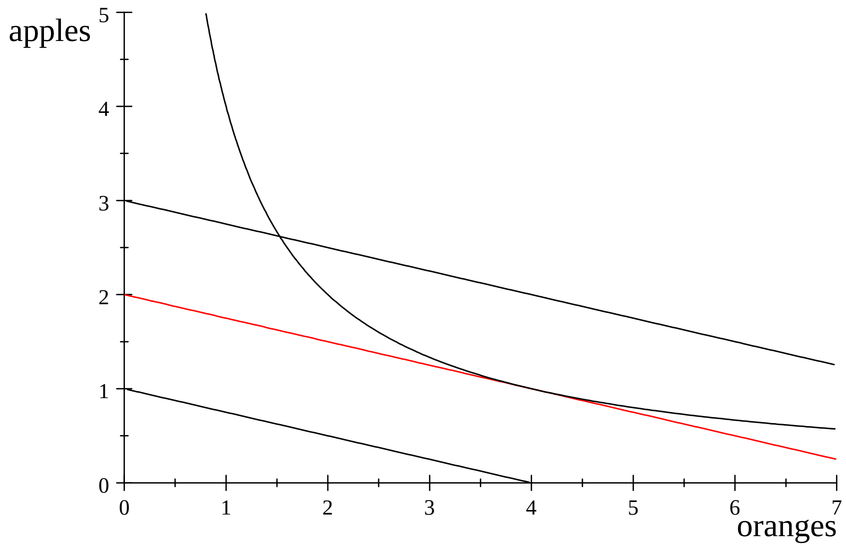
Expenditure Minimisation

- “Dual problem” of utility maximisation
- Expenditures increasing when moving “Northeast”: E(green)>E(red)>E(black)



Utility Constraint

- Instead of maximise utility given income, minimise expenditures given utility level



Mathematically

- Find $x_1^*, ..., x_n^*$, where $x_i^* \equiv h_i(p_1, ..., p_n, \overline{U})$, s.t.

$$Min\ p_1 x_1 + ... + p_n x_n$$
$$\text{subject to } \overline{U} = U(x_1, ..., x_n)$$

- Minimum expenditures: $E(p_1, ..., p_n, \overline{U}) = p_1 x_1^* + ... + p_n x_n^*$

Cobb-Douglas example

$$Min_{x_1,x_2}\; p_1x_1 + p_2x_2$$
$$\text{subject to } x_1^{\alpha}x_2^{\beta} = \overline{U}$$

and therefore

$$Min_{x_1,x_2,\lambda}\; \mathcal{L} = \; p_1x_1 + p_2x_2 + \lambda(\overline{U} - x_1^{\alpha}x_2^{\beta}) \quad \text{or}$$

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial x_1} &= p_1 - \lambda^* \alpha \left(x_1^*\right)^{\alpha-1} \left(x_2^*\right)^{\beta} = 0 \text{ and} \\
\frac{\partial \mathcal{L}}{\partial x_2} &= p_2 - \lambda^* \alpha \left(x_1^*\right)^{\alpha} \left(x_2^*\right)^{\beta-1} = 0 \\
\frac{\partial \mathcal{L}}{\partial \lambda} &= \overline{U} - \left(x_1^*\right)^{\alpha} \left(x_2^*\right)^{\beta} = 0
\end{aligned}$$

- Solving

$$x_1^* = \overline{U}\left(\frac{\alpha}{\beta}\frac{p_2}{p_1}\right)^{\beta} \text{ and } x_2^* = \overline{U}\left(\frac{\beta}{\alpha}\frac{p_1}{p_2}\right)^{\alpha} \quad \text{and}$$

$$E(p_1, p_2, \overline{U}) = p_1 x_1^* + p_2 x_2^* = \overline{U} p_1^{\alpha} p_2^{\beta}\left[\left(\frac{\alpha}{\beta}\right)^{\beta} + \left(\frac{\beta}{\alpha}\right)^{\alpha}\right]$$

- If $\alpha = \beta = 0.5$, $p_1 = 0.25$, $p_2 = 1$, $\overline{U} = 2$, then $x_1^* = 4$, $x_2^* = 1$ and $E = 2$

2.3.- "Comparative Statics"

- Demand with respect to income
- Demand with respect to price
- Decomposition into income and substitution effects

Normal and inferior goods

- Remember that for any utility function we have a demand function:

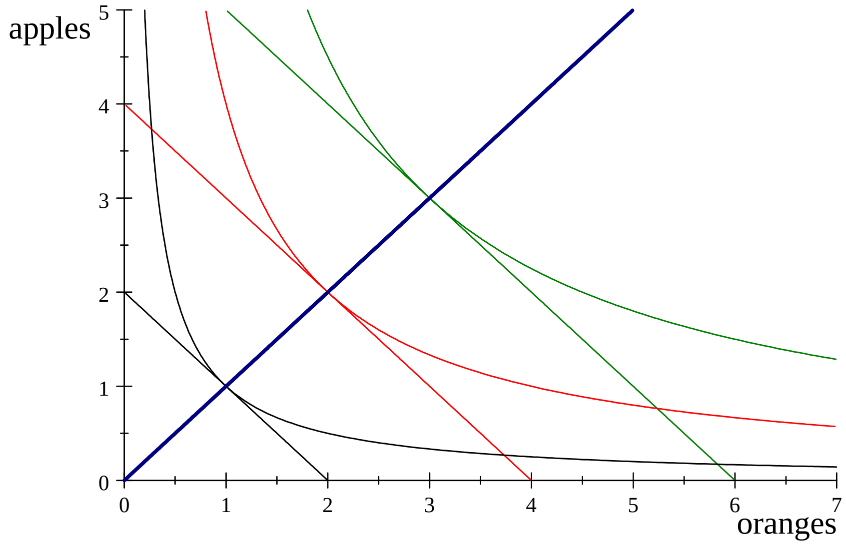
$$x_i^* = d_i(p_1, ..., p_n, I)$$

- In Cobb-Douglas, higher income, choose more of both goods

$$x_1^* = \frac{\alpha I}{p_1} \text{ and } x_2^* = \frac{\beta I}{p_2}$$

- Normal good: "A good for which demand increases with income"
- Inferior good: "A good for which demand decreases with income"
- A good can be normal over some range and inferior over another
- Can you think of an example of an inferior good?

Engel Curve of a Normal Good (blue)



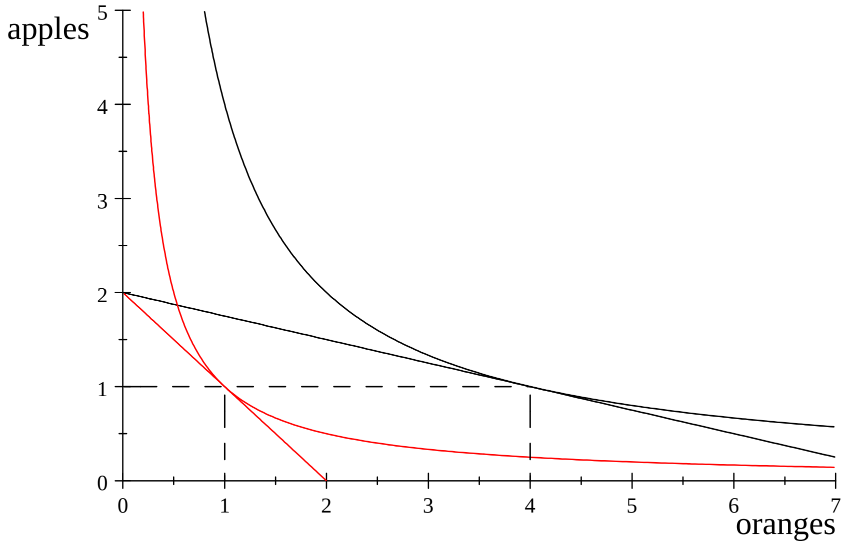
- Higher income levels (black, red, green) allows attainability of higher utility levels
- MRS at optimum does not change (relative prices constant)

What about price changes?

- $x_1^* = 4$ if Cobb-Douglas $\alpha = \beta = 0.5$, $p_1 = 0.25$, $p_2 = 1$ and $I = 2$
- $x_1^* = 1$ if p_1 increases to $p_1 = 1$
- Price increase changes relative prices (substitution effect) and makes one poorer (income effect)

What about price changes?

- Price change affects both slope and intercept of the budget constraint



Decomposing

- Quantity if we keep utility ("purchasing power") constant?

$$x_1^* = \overline{U}\left(\frac{\alpha}{\beta}\frac{p_2}{p_1}\right)^{\beta} \text{ and therefore } x_1^* = 2\left(\frac{1}{1}\right)^{0.5} = 2$$

since

$$U(x_1^*, x_2^*) = U(1, 4) = 2$$

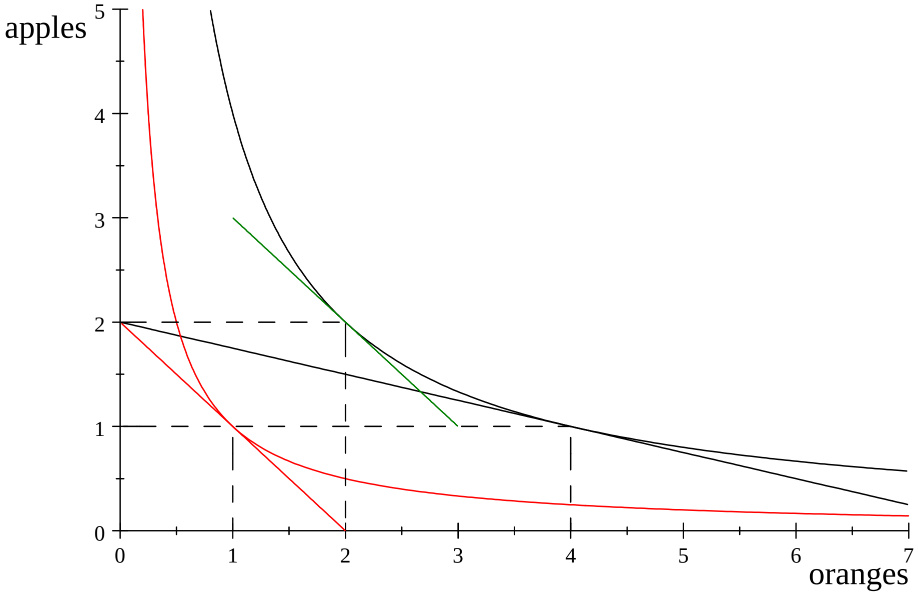
- Similarly, quantity if income is increased to compensate purchasing power loss?

$$x_1^* = \frac{\alpha I}{p_1} \text{ and therefore } x_1^* = \frac{0.5 * 4}{1} = 2$$

given that

$$E(p_1, p_2, \overline{U}) = \overline{U} p_1^{\alpha} p_2^{\beta}\left[\left(\frac{\alpha}{\beta}\right)^{\beta} + \left(\frac{\beta}{\alpha}\right)^{\alpha}\right] \text{ and } E(p_1, p_2, \overline{U}) = 2 * 2 = 4$$

Graphically



- From black to green (change in relative prices) and from green to red (change in income)

Slutsky equation

- Formally, duality problem:

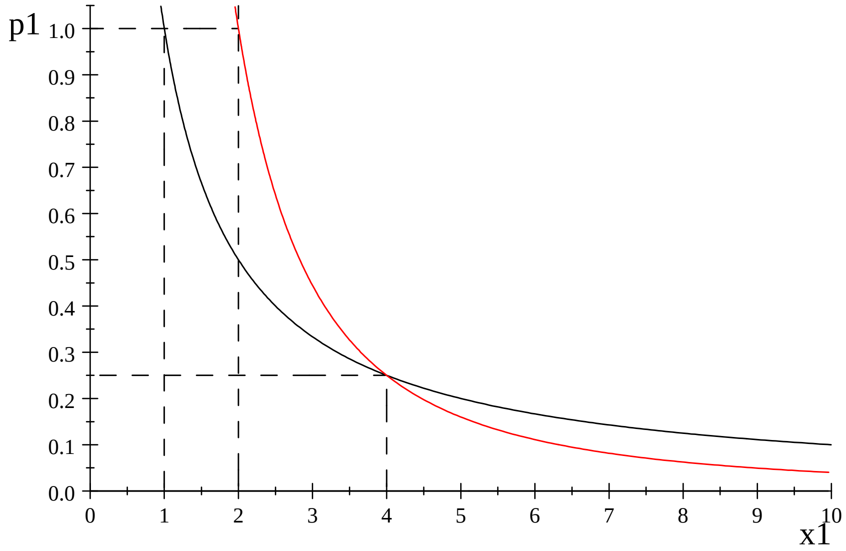
$$h_i(p_1, ..., p_n, \overline{U}) = d_i(p_1, ..., p_i, ..., p_n, E(p_1, ..., p_n, \overline{U}))$$

- Deriving,

$$\frac{\partial h_i}{\partial p_i} = \frac{\partial d_i}{\partial p_i} + \frac{\partial d_i}{\partial E}\frac{\partial E}{\partial p_i} \quad \text{and} \quad \frac{\partial d_i}{\partial p_i} = \frac{\partial h_i}{\partial p_i} - \frac{\partial E}{\partial p_i}\frac{\partial d_i}{\partial E}$$

- Right hand side of last equation:
first term (substitution): change in demand while keeping utility constant
second and third term (income): necessary change in expenditure to keep utility constant and how this change in income affects demand
- In a price increase, first effect always negative (provided MRS diminishing)
- Second negative (positive) if good is normal (inferior)
"Giffen" good: so inferior to compensate first effect (potatoes in Ireland, 1900s)

"Demands" for a Normal Good



- Compensated demand (red) changes less rapidly than demand (black) because income is compensated
- Demand holds income constant and compensated demand holds purchasing power constant

2.4.- Market Demand and Elasticity

- From individual to market demands
- Price elasticity of demand concept
- Income and cross-price elasticities
- Examples of types of market demands

Market Demand

- m individuals ($j = 1, ..., m$) and n goods ($i = 1,...,n$). For each good i, each individual j demands

$$x^*_{i,j} = d_{i,j}(p_1, ..., p_n, I_j)$$

- Market demand for good i

$$D_i(p_1, ..., p_n, I_1, ..., I_m) \equiv \sum_{j=1}^{m} d_{i,j}(p_1, ..., p_n, I_j)$$

- Market demand curve: vary p_i holding p_{-i} and $I_1, ..., I_m$ constant
- Example:

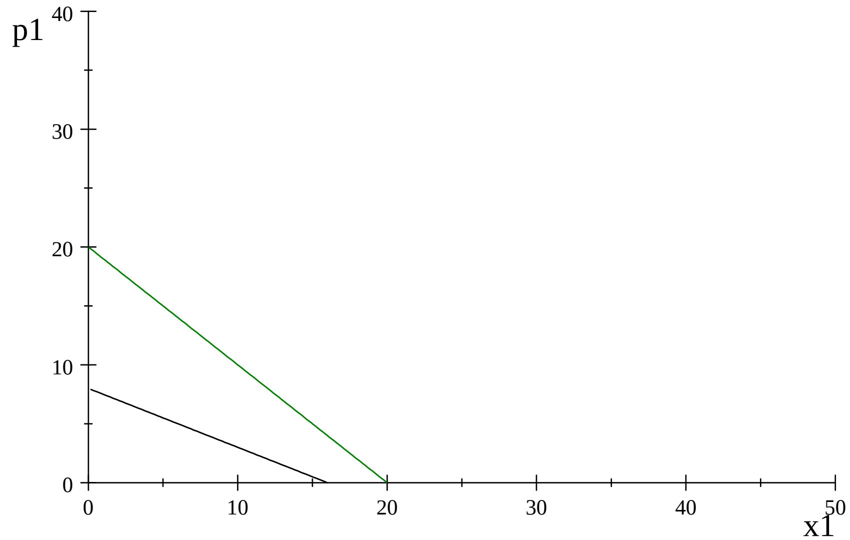
$$x^*_{1,1} = 10 - 2p_1 + 0.5p_2 + 0.1I_1 \text{ and } x^*_{1,2} = 17 - p_1 + 0.5p_2 + 0.05I_2$$

and

$$D_i(p_1, p_2, I_1, I_2) = 27 - 3p_1 + p_2 + 0.1I_1 + 0.05I_2$$

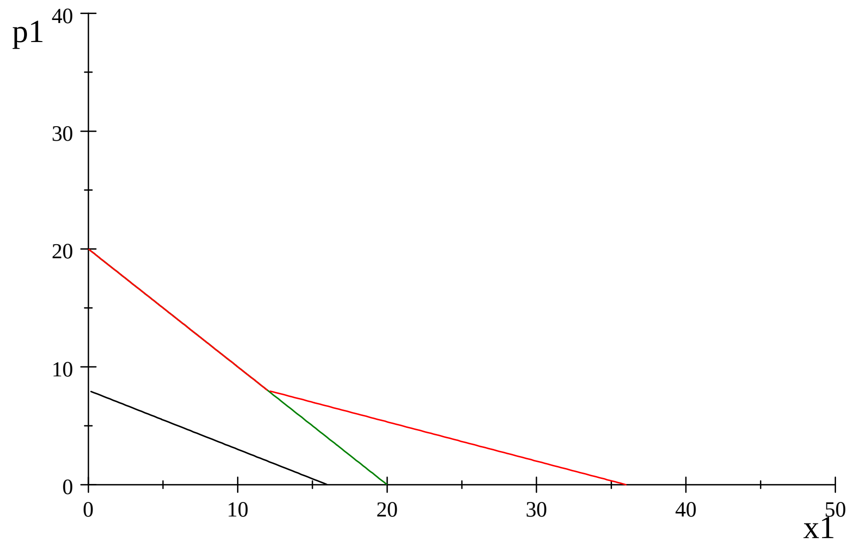
Individual Demand Curves

- For (constant) $p_2 = 4$ and $I_1 = 40$ and $I_2 = 20$



Market Demand Curve

- For (constant) $p_2 = 4$ and $I_1 = 40$ and $I_2 = 20$



Price Elasticity of Demand

- For any demand function $Q(P)$ (either individual or market)
  Derivative: marginal change in demand when price changes marginally
  Elasticity: marginal percentage change if price changes by marginal percentage

$$e_{Q,P} = \frac{\partial Q(P)}{\partial P}\frac{P}{Q(P)}$$

- Example 1: Cobb-Douglas individual demand

$$x_i^* = \frac{\alpha I}{p_i} \text{ and therefore } e_{x_i,p_i} = -\frac{\alpha I}{p_i^2}\frac{p_i}{\frac{\alpha I}{p_i}} = -1$$

- Example 2: Linear market demand

$$Q(P) = 10 - P \text{ and therefore } e_{Q,P} = -\frac{P}{10 - P}$$

Classification

value of elasticity	terminology
$e_{Q,P} < -1$	elastic
$e_{Q,P} = -1$	unit elastic
$e_{Q,P} > -1$	inelastic

Table: Selected Estimates of Demand Elasticities

	Short Run	Long Run
Cigarettes	_	0.35
Water	_	0.4
Beer	_	0.8
Physicians' Services	0.6	_
Gasoline	0.2	0.5-1.5
Automobiles	_	1.5
Chevrolets	_	4.0
Electricity	0.1	1.9
Air Travel	0.1	2.4

Source: Browning and Mark Zupan, *Microeconomics and Applications*. Hendrik Houthakker and Lester Taylor, *Consumer Demand in the United States, 1929-1970*. Kenneth Etzinga, "The Beer Industry", in *The Structure of American Industry*, edited by Walter Adams. James Sweeney, "The Response of Energy Demand to Higher Prices: What Have We Learned?", *American Economic Review*, 74, #2, May 1984, pp.31-37.

Price Elasticity and Total Expenditure

- Expenditure in a given good: $TE = PQ(P)$. Then

$$sign\left(\frac{\partial TE}{\partial P}\right) = sign\left(\frac{\frac{\partial TE}{\partial P}}{Q(P)}\right) = sign(1 + e_{Q,P})$$

- Expenditures increase (decrease) with price if demand is inelastic (elastic)

- Examples of each?

Income Elasticity

- Income elasticity of demand

$$e_{Q,I} = \frac{\partial Q(I)}{\partial I} \frac{I}{Q(I)}$$

- Positive if normal good
- A good such that $e_{Q,I} > 1$ is denoted a "luxury good"

Cross-Price Elasticity

- Cross-price elasticity of demand (denote price of another good as P')

$$e_{Q,P'} = \frac{\partial Q(P')}{\partial P'} \frac{P'}{Q(P')}$$

- Substitutes if $e_{Q,P'} > 0$, complements if $e_{Q,P'} < 0$

- Cobb-Douglas:

$$e_{Q,P'} = \frac{\alpha}{p_i} \frac{I}{\frac{\alpha I}{p_i}} = 1 \text{ and } e_{Q,P'} = 0$$

Own and Cross-Price Elasticities

	Sentra	Escort	LS400	735i
Sentra	-6.528	0.454	0.000	0.000
Escort	0.078	-6.031	0.001	0.000
LS400	0.000	0.001	-3.085	0.032
735i	0.000	0.001	0.093	-3.515

Source: Berry, Levinsohn and Pakes, "Automobile Price in Market Equilibrium," Econometrica 63 (July 1995), 841-890

Elasticities

Elasticity	Coke	Pepsi
Price elasticity of demand	-1.47	-1.55
Cross-price elasticity of demand	0.52	0.64
Income elasticity of demand	0.58	1.38

Source: Gasmi, Laffont and Vuong, "Econometric Analysis of Collusive Behavior in a Soft Drink Market," Journal of Economics and Management Strategy 1 (Summer, 1992) 278-311

Examples of Demand Functions (1)

- Example 1: “Linear” market demand:

$$Q(P, P', I) = a + bP + dP' + cI$$

where:

$b \leq 0$, except for “Giffen goods”

$c \geq 0$, except for “inferior goods”

$d \gtrless 0$, depending on whether goods are substitutes or complements

- Elasticities can be computed as before

- Remark: elasticity changes along the demand curve

Examples of Demand Functions (2)

- Example 2: "Constant elasticity" market demand

$$Q(P, P', I) = aP^b P'^d I^c$$

where $a > 0$, $b \leq 0$, $c \geq 0$, $d \gtrless 0$ and

$$e_{Q,P} = b \text{ and } e_{Q,P'} = d \text{ and } e_{Q,I} = c$$

- Remark: this function is linear in logarithms and coefficients can be interpreted as elasticities (widely used in econometrics)