Chapter 1: Methodolgy of Microeconomics

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Theoretical Models

- Real world is extremely complex
- Need to simplify or abstract: build economic models
- Verification:
  (a) Check assumptions
  (b) Verify conclusions
- Example: Do firms maximise profits?

General Features

- Ceteris paribus: other things that affect are kept constant
- Optimisation: Each agent pursue a goal
  Firms minimise costs
  Consumers maximise welfare
  Governments maximise public welfare
- Positive/Normative distinction:
  positive: explain what happens
  normative: what should be done

Constrained Maximisation

- In many economic problems, we need to maximise objective function with a restriction

$$Max\ f(x_1, ..., x_n)$$
$$subject \text{ to } g(x_1, ..., x_n) = 0$$

- Method: Maximise "Lagrangian" $\mathcal{L}$ where

$$\mathcal{L} \equiv \ f(x_1, ..., x_n) + \lambda g(x_1, ..., x_n)$$

  where λ is the Lagrange multiplier

- Results in a system of $n + 1$ equations with $n + 1$ unknowns

$$\frac{\partial \mathcal{L}}{\partial x_i} = \frac{\partial f}{\partial x_i}(x_1, ..., x_n) + \lambda \frac{\partial g}{\partial x_i}(x_1, ..., x_n) = 0 \text{ for all } i$$
$$\frac{\partial \mathcal{L}}{\partial \lambda} = \frac{\partial g}{\partial x_i}(x_1, ..., x_n) = 0$$

Example

$$Max\ f(x_1, x_2) \equiv -x_1^2 + 2x_1 - x_2^2 + 4x_2 + 5$$
$$s.t.\ x_1 + x_2 = 1$$

- Maximising Lagrangian $\mathcal{L}$

$$\mathcal{L} \equiv -x_1^2 + 2x_1 - x_2^2 + 4x_2 + 5 + \lambda(1 - x_1 - x_2)$$

results in a system of 3 equations with 3 unknowns

$$\begin{aligned} -2x_1 + 2 - \lambda &= 0 \\ -2x_2 + 4 - \lambda &= 0 \\ 1 - x_1 - x_2 &= 0 \end{aligned}$$

- Solving $x_1 = 0$, $x_2 = 1$ and $\lambda = 2$ and $f(0, 1) = 8$

Lagrange Multiplier

- Lagrange multiplier or shadow value λ satisfies

$$\lambda = \frac{\frac{\partial f}{\partial x_i}(x_1, ..., x_n)}{-\frac{\partial g}{\partial x_i}(x_1, ..., x_n)} \text{ for all } i$$

- Marginal change in objective function for a marginal “increase” in the constraint
- High λ: constraint is “very” binding, $\lambda = 0$: not binding at all
- In our example:

$$\lambda = \left|\frac{-2x_1 + 2}{1}\right|_{(0,1)} = \left|\frac{-2x_2 + 4}{1}\right|_{(0,1)} = 2$$

- Need to check second order conditions! Same for constrained minimisation