

### Suggested Supervisions 3

- 1.** (A parlor game) Represent the following parlor as an extensive-form game with imperfect information. First player 1 receives a card that is either H or L with equal probabilities. Player 2 does not see the card. Player 1 may announce that her card is L and pay 1 dollar to player 2, or may claim that her card is H. In the latter, player 2 may choose to concede or to insist on seeing player 1's card. If player 2 concedes then he must pay 1 dollar to player 1. If he insists on seeing player 1's card then player 1 must pay him 4 dollars if her card is L and he must pay her 4 dollars if her card is H.
- 2.** (Voting game) A committee with three members,  $\{1, 2, 3\}$ , has to choose a new member of a club among a set of four candidates,  $\{a, b, c, d\}$ . Each member of the committee has veto power which is used in a successive way, starting by member 1, and finishing with member 3. Each member of the committee has to veto one and only one of the candidates that have not been vetoed yet.

  - (a)** Draw the extensive form of the game, writing in the terminal nodes the name of the elected candidate.
  - (b)** How many strategies does each player have? Do not try to write them (player 3 should have a lot).
- 3.** (Labour market) There are two firms ( $i=1,2$ ) in an industry. Each of them has a job opening. Firm  $i$  offers a wage  $w_i$ . There are two workers that decide simultaneously to which firm they apply (they can only apply to one). If only one worker applies to a given firm that worker gets the job; if the two workers apply to the same firm, the firm hires one of them randomly and the other remains unemployed (that has payoff zero).

  - (a)** Represent this game in extensive form.
  - (b)** Represent this game in normal form.
- 4.** (Burning a bridge, exercise 173.4 in Osborne (2004)) Army 1, of country 1, must decide whether to attack army 2, of country 2, which is occupying an island between the two countries. In the event of an attack, army 2 may fight, or retreat over a bridge to its mainland. Each army prefers to occupy the island than not to occupy it; a fight is the worst outcome for both armies. Model this situation as an extensive game with perfect information (find also a set of payoffs consistent with these preferences) and show that 2 can increase its subgame perfect equilibrium payoff (and reduce army 1's payoff) by burning the bridge to its mainland (assume that this act entails no cost), eliminating its option to retreat if attacked.