Paper 1 Microeconomics (Game Theory and Industrial Organisation)
Albert Banal-Estanol

Suggested Supervisions 2

1. (Dividing a pie) Two players have \$10 to divide between themselves. To do so, they use the following procedure: Each player names a number of dollars (a nonnegative integer), at most equal to 10. If the sum of the two numbers is at most 10 then each player receives the amount of money she names (and the remainder is destroyed). If the sum of the two numbers exceeds 10 and the two numbers are different then the player who names the smaller number receives that amount and the other player receives the remaining money. If the sum of the two numbers exceeds 10 and the two numbers are equal each players receives \$5. Determine the best-reply functions (correspondences) of each player, plot them in a diagram, and find the Nash equilibria of the game.

2. (War of attrition, see Section 3.4 and exercise 79.1 in Osborne (2004)) Two players are involved in a dispute over an object. The value of the object to player i is $v_i > 0$. Time is modeled as a continuous variable that starts at 0 and runs indefinitely. Each player chooses when to concede the object to the other player; if the player to concede (i) does so at time t_i, the other player (j) obtains the object at that time. If both players concede simultaneously, the object is split equally between them, player i receiving a payoff of $\frac{v_i}{2}$. Time is valuable: until the first concession each player loses one unit of payoff per unit of time.

(a) Identify the elements of the game.

(b) Find the Nash equilibria of the game proceeding as follows.

(b.1) Argue that if $t_1 = t_2$, $0 < t_i < t_j$, or $0 = t_i < t_j < v_i$ (for $i = 1$ and $j = 2$ or $i = 2$ and $j = 1$), then the pair (t_1, t_2) is not a Nash equilibrium.

(b.2) Argue that any remaining pair is a Nash equilibrium.

3. (Auctions, see Section 3.5 and exercises 84.1, 86.2 and 87.1 in Osborne (2004)) An object is to be assigned to a player in the set {1,...,I} in exchange for a payment. Player i's valuation of the object is v_i and $v_1 > v_2 > ... > v_I > 0$. The mechanism used to assign the object is a (sealed-bid) auction: the players simultaneously submit bids (non-negative numbers), and the object is given to the player with the lowest index among those who submit the highest bid, in exchange for payment.

(a) In a second price auction the payment that the winner makes is the highest bid among those submitted by the players who do not win (so if only one agent submits the highest bid then the price paid is the second highest bid).

(a.1) Show that in a second price auction the bid v_i of any player i is a weakly dominant strategy.

(a.2) Find a Nash equilibrium in which player I obtains the object.

(b) In a first price auction the payment that the winner makes is the price she bids. Formulate a first price auction as a static game and analyze its Nash equilibria proceeding as follows.

(b.1) Show that $(b_1, ..., b_I) = (v_2, v_2, v_3..., v_I)$ is a Nash equilibrium.

(b.2) Show that in a Nash equilibrium the two highest bids are the same, one of these bids submitted by player 1, and the highest bid is at least v_2 and at most v_1. Show also that any action profile satisfying these conditions is a Nash equilibrium.

4. (Nash equilibrium and weakly dominated actions - exercise 47.2 in Osborne (2004)) Give an example of a two-player strategic game in which each player has finitely many actions and in the only Nash equilibrium both players' strategies are weakly dominated.