Lecture 8: Dynamic Games Applications

Albert Banal-Estanol

March 2006

Today's Lecture

- Two dynamic games in Industrial Organisation
- Stackelberg competition: sequential quantity competition
  Model, representation and backwards induction
  Outcome and comparison with Cournot
- Another model of entry in a monopolised industry
  Model, representation and SPNE

Stackelberg Competition: Model

- As in the Cournot model (Lecture 5), two firms select quantities and... an auctioneer chooses the price according to $P()$ where

$$P(q_1 + q_2) = \begin{cases} 1 - (q_1 + q_2) & \text{if } q_1 + q_2 \leq 1 \\ 0 & \text{if } q_1 + q_2 > 1 \end{cases}$$

  and firms' unit costs are c_i (≤ 1)

- But now... Firm 1 sets its output before Firm 2 does
  Firm 2 observes Firm 1's output, q_1, before choosing q_2

Stackelberg Competition: Elements

- Players: Firms 1 and 2. Payoffs:

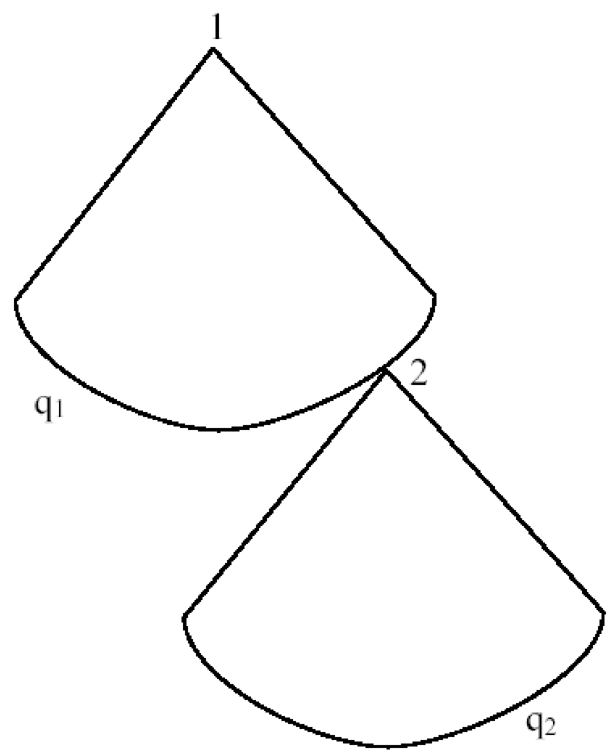
$$\Pi_i(q_i, q_j) = \left(\max\{\left[1 - (q_i + q_j)\right], 0\}\right) q_i - c_i q_i$$

- Strategy for 1: an output $q_1 \in [0, \infty)$
  Strategy for 2: a function $q_2(q_1)$, i.e. an output ($[0, \infty)$) for each possible q_1

- Examples of strategies:

$$\begin{aligned} q_1 &= 0.5 \\ q_2(q_1) &= 3q_1 \text{ for any } q_1 \end{aligned}$$

- (Approximate) representation:



Stackelberg Competition: Backwards Induction (BI)

- Solving by backwards induction, Firm 2's best reply (FOC) is:

$$q_2^*(q_1) = B_2(q_1) = \begin{cases} \frac{1-q_1-c_2}{2} & \text{if } q_1 \leq 1 - c_2 \\ 0 & \text{if } q_1 > 1 - c_2 \end{cases}$$

- Anticipating this, Firm 1 maximises

$$\Pi_1(q_1, B_2(q_1)) = \left[1 - \left(q_1 + \frac{1 - q_1 - c_2}{2}\right)\right] q_1 - c_1 q_1$$

- Solving the FOC (check that the second order conditions hold):

$$q_1^* = \frac{1 - 2c_1 + c_2}{2}$$

- The NE obtained by BI is given by $(q_1^*, q_2^*(q_1))$

Stackelberg Competition: Outcome

- Firm 1 produces

$$q_1^S = q_1^* = \frac{1 - 2c_1 + c_2}{2}$$

whereas Firm 2 produces

$$q_2^S = q_2^*(q_1^S) = \frac{1 + 2c_1 - 3c_2}{4}$$

- Total quantity and prices are given by

$$q_1^S + q_2^S = \frac{3 - 2c_1 - c_2}{4} \text{ and } P\left(q_1^S + q_2^S\right) = \frac{1 + 2c_1 + c_2}{4}$$

and the profits for each firm are given by

$$\Pi_1\left(q_1^S, q_2^S\right) = \frac{(1 - 2c_1 + c_2)^2}{8}, \quad \Pi_2\left(q_1^S, q_2^S\right) = \frac{(1 + 2c_1 - 3c_2)^2}{16}$$

Stackelberg vs Cournot (Symmetric Case)

- Assume for simplicity that $c_1 = c_2 \equiv c$. Firm 1 produces more than in Cournot

$$q_1^S = \frac{1-c}{2} > \frac{1-c}{3} = q_1^C$$

whereas Firm 2 produces less

$$q_2^S = \frac{1-c}{4} < \frac{1-c}{3} = q_2^C$$

- Firm 1 earns more than in Cournot

$$\Pi_1^S = \frac{(1-c)^2}{8} > \frac{(1-c)^2}{9} = \Pi_1^C$$

whereas Firm 2 earns less

$$\Pi_2^S = \frac{(1-c)^2}{16} < \frac{(1-c)^2}{9} = \Pi_2^C$$

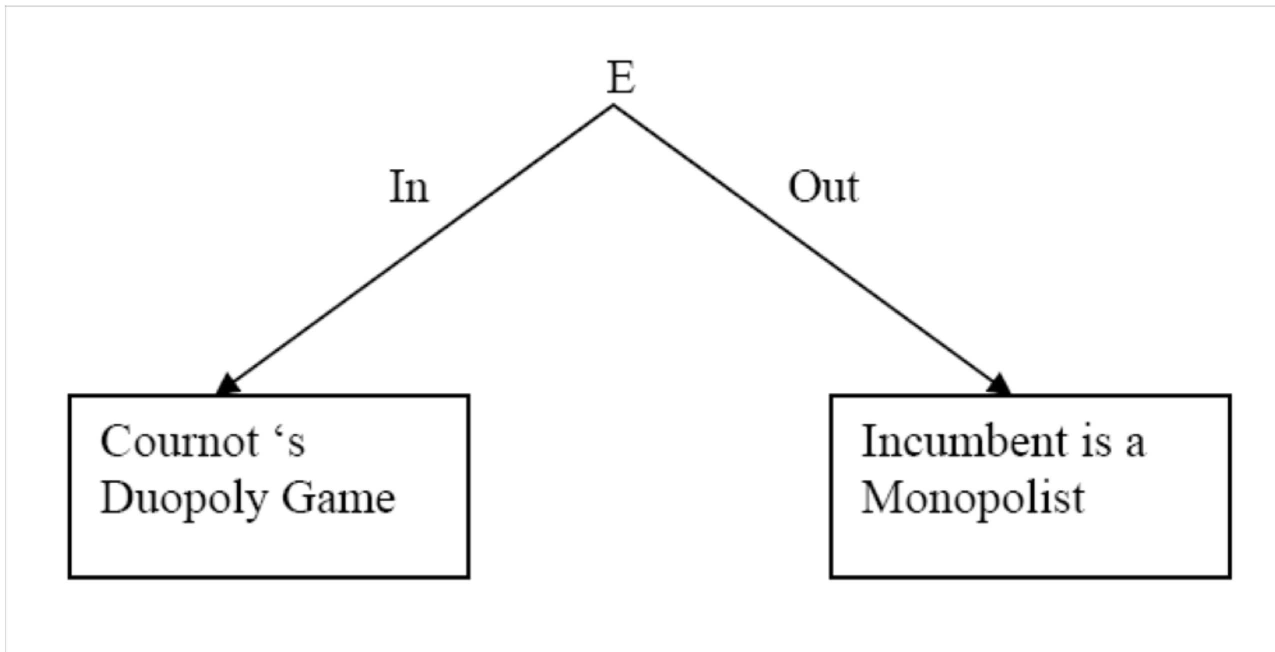
Another Model of Entry

- Consider again an incumbent monopolist facing a potential entrant
- Now, more explicit model:
  New entry entails a positive fixed cost f
  If entrant does not enter, it earns 0 and the incumbent is a monopolist
  If entrant does enter, the two firms compete a la Cournot
  Again demand is given by

$$P(q_I + q_E) = \begin{cases} 1 - (q_I + q_E) & \text{if } q_I + q_E \leq 1 \\ 0 & \text{if } q_I + q_E > 1 \end{cases}$$

and firms' unit costs are c (≤ 1)

Entry Model



Entry Model: SPNE

- Following In there is a subgame, the Cournot game (Lecture 5), except that the payoff of the entrant is reduced by f. The output of each firm in a SPNE is

$$q_i^D = \frac{1-c}{3} \text{ for } i = E, I$$

and the profits would be

$$\Pi_I^D = \left(\frac{1-c}{3}\right)^2 \text{ and } \Pi_E^D = \left(\frac{1-c}{3}\right)^2 - f$$

- Following Out there is a subgame, the monopoly case (Lecture 1). The output of the incumbent in a SPNE is (and the profits would be)

$$q_I^M = \frac{1-c}{2} \text{ and } \Pi_I^M = \left(\frac{1-c}{2}\right)^2 \text{ and } \Pi_E^M = 0$$

- Anticipating this, in a SPNE the potential entrant enters whenever

$$\Pi_E^D = \left(\frac{1-c}{3}\right)^2 - f \geq 0 = \Pi_E^M$$

- Summarising, the *outcome* of the SPNE of the game depend on f and c:
  If $f < \left(\frac{1-c}{3}\right)^2$ then In and $q_i = q_i^C = \frac{1-c}{3}$ for $i = E, I$ (one SPNE)
  If $f > \left(\frac{1-c}{3}\right)^2$ then Out and $q_I = q_I^M = \frac{1-c}{2}$ (one SPNE)
  If $f = \left(\frac{1-c}{3}\right)^2$ then either of the two previous outcomes may arise (two SPNE)

- Notice that, as in Lecture 7, there is a *NE (not SPNE)* in which the incumbent floods the market and the potential entrant does not enter