Lecture 3: Simultaneous Move Games Theory (2)

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Best Response Functions

- So far, we have examined all combination profiles to find whether they were NE
- If there are many possible actions and many players, this might be difficult
- Best response: find best action for a player to any given list of others' actions
  BoS game: for 1, Bach is a best response if 2 chooses Bach and Stravinsky is a best response if 2 chooses Stravinsky
- Best response to a given action of the others may not be unique. Example: for 1, T and B are best responses if 2 chooses L

	L	M	R
T	1,1	1,0	0,1
B	1,0	0,1	1,0

- Formally, denote best response for i to the actions of the others a_{-i} as $B_i(a_{-i})$
  e.g. $B_1(Bach) = Bach$ and $B_1(Stravinsky) = Stravinsky$ in Bos
  e.g. $B_1(L) = \{T, B\}$, $B_1(M) = \{T\}, B_1(R) = \{B\}$ in previous game

Best Response Functions and Nash

- Remember than in a NE, no player can do better than playing her NE strategy if the others play their NE strategies
- Hence, we can redefine the concept of NE in terms of best response functions
- Definition: $(s_1, ..., s_I)$ is a NE if and only if every player's action is a best response to the other players' actions, i.e. s_i is in $B_i(s_{-i})$ for every i
- Method to find NE: (a) Find best response function and (b) Find strategy profiles that are mutually best responses:

	L	C	R
T	1,2	2,1	1,0
M	2,1	0,1	0,0
B	0,1	0,0	1,2

- If best-responses have 1 element: $(s_1, ., s_I)$ is a NE iff $B_i(s_{-i}) = s_i$ for every i
- This is a system of I equations with I unknowns (s_i's)

Example: A Synergistic Relationship

- Two individuals in synergistic relationship: if both devote more effort, both better off
- Suppose that the relationship value for i ($i = 1, 2$) is given by $a_i(c + a_j - a_i)$, where a_i and a_j are own and other's efforts
- Players: $1, 2$. Strategies: $a_i > 0$ and payoffs: $a_i(c + a_j - a_i)$ (for $i = 1, 2$)
- Best response: suppose player j plays a_j, what is my best response?
- Compute (partial) derivative and equate to 0 (check second order condition)

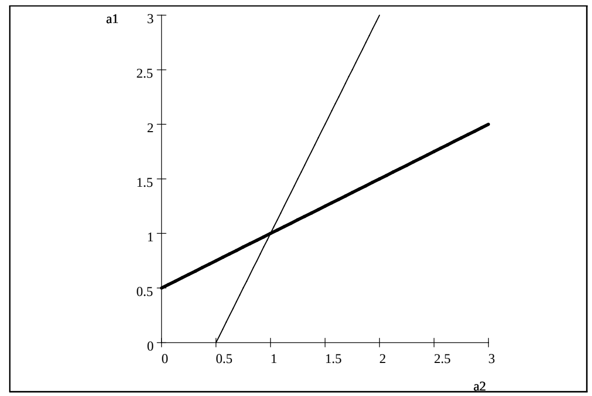
$$c + a_j - 2a_i^* = 0; \quad B_i(a_j) \equiv a_i^* = \frac{c + a_j}{2}$$

- Nash equilibrium: intersection of best reply functions, i.e. solve for

$$a_1^* = \frac{c + a_2^*}{2} \text{ and } a_2^* = \frac{c + a_1^*}{2}; \text{ and therefore } a_1^* = a_2^* = c$$

Example: A Synergistic Relationship (2)

- Suppose that $c = 1$. Best responses are $B_1(a_2) = \frac{1+a_2}{2}$ and $B_2(a_1) = \frac{1+a_1}{2}$. NE is: $a_1^* = a_2^* = 1$. Representation:



Best response functions: $B_1(a_2)$ (thick line, from horizontal to vertical axis) and $B_2(a_1)$ (thin line, from vertical to horizontal axis)
Nash equilibrium: $(1, 1)$ (the intersection of the best response functions)

Nash Equilibrium and Dominance

- A strictly dominated action is not a best response to any action
- Therefore, a strictly dominated action is not used in any Nash equilibrium
- One can eliminate strictly dominated actions when looking for NE
  e.g. in PD, DC is never part of a NE: only possible NE is (C,C) (indeed, it is!)
- Can an action of a NE be weakly dominated? Yes!. Examples:

	B	C
B	1,1	0,0
C	0,0	0,0

	B	C
B	1,1	2,0
C	0,2	2,2

- C is weakly dominated in both games but (C,C) is a NE in both games
- (B,B) is a NE in both games. In the left game, this NE is better for both than the previous NE whereas in the right game, it is worse