Chapter 6: Further Dynamic Games

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This Chapter's Plan

- Stackelberg competition:
  Model, representation and backwards induction
  Outcome and comparison with Cournot

- Another model of entry:
  Model, representation and SPNE

Stackelberg Competition: Model

- As in the Cournot model, two firms select quantities and...
  an auctioneer chooses the price according to $P()$ where

$$P(q_1+q_2) = \begin{cases} 1-(q_1+q_2) & \text{if } q_1+q_2 \leq 1 \\ 0 & \text{if } q_1+q_2 > 1 \end{cases}$$

  and each firm's unit costs are c (≤ 1)

- But now... Firm 1 sets its output before Firm 2 does
  Firm 2 observes Firm 1's output, q_1, before choosing q_2

Stackelberg Competition: Elements

- Players: Firms 1 and 2. Payoffs:

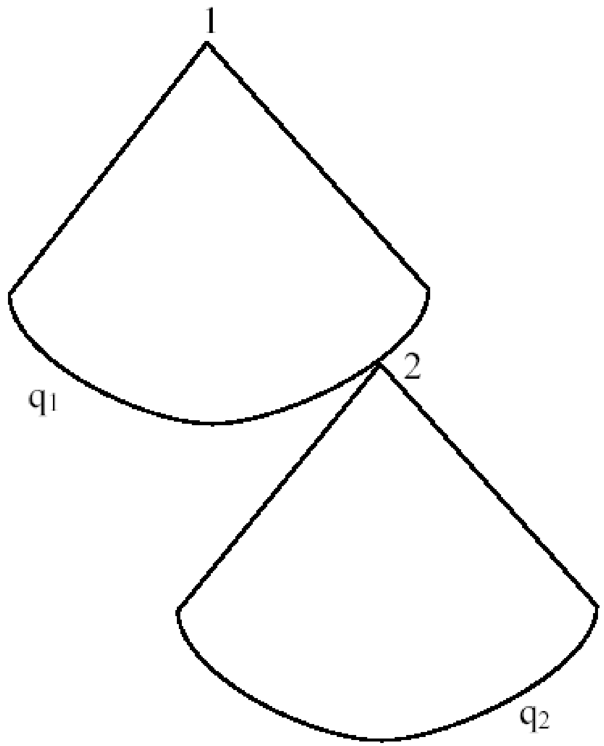
$$\Pi_i(q_i, q_j) = \left(\max\{\left[1 - (q_i + q_j)\right], 0\}\right) q_i - cq_i$$

- Strategy for 1: an output $q_1 \in [0, \infty)$
  Strategy for 2: a function $q_2(q_1)$, i.e. an output ($[0, \infty)$) for each possible q_1

- Examples of strategies:

$$\begin{aligned} q_1 &= 0.5 \\ q_2(q_1) &= 3q_1 \text{ for any } q_1 \end{aligned}$$

- (Approximate) representation:



Stackelberg Competition: Backwards Induction (BI)

- Solving by backwards induction, Firm 2's best reply (FOC) is:

$$q_2^*(q_1) = B_2(q_1) = \begin{cases} \frac{1-q_1-c}{2} & \text{if } q_1 \leq \frac{1-c}{2} \\ 0 & \text{if } q_1 > \frac{1-c}{2} \end{cases}$$

- Anticipating this, Firm 1 maximises (clearly $q_1^* \leq \frac{1-c}{2}$)

$$\Pi_1(q_1, B_2(q_1)) = \left[1 - \left(q_1 + \frac{1 - q_1 - c}{2}\right)\right] q_1 - cq_1$$

- Solving the FOC:

$$q_1^* = \frac{1-c}{2}$$

- The NE obtained by BI is given by $(q_1^*, q_2^*(q_1))$

Stackelberg Competition: Outcome

- Firm 1 produces

$$q_1^S = \frac{1-c}{2}$$

whereas Firm 2 produces

$$q_2^S = q_2^*(q_1^S) = \frac{1-c}{4}$$

- Total quantity and prices are given by

$$q_1^S + q_2^S = \frac{3(1-c)}{4} \text{ and } P\left(q_1^S + q_2^S\right) = \frac{1+3c}{4}$$

and the profits for each firm are given by

$$\Pi_1\left(q_1^S, q_2^S\right) = \frac{(1-c)^2}{8}, \;\; \Pi_2\left(q_1^S, q_2^S\right) = \left(\frac{1-c}{4}\right)^2$$

Stackelberg vs Cournot

- Firm 1 produces more than in Cournot

$$q_1^S = \frac{1-c}{2} > \frac{1-c}{3} = q_1^C$$

whereas Firm 2 produces less

$$q_2^S = \frac{1-c}{4} < \frac{1-c}{3} = q_2^C$$

- Firm 1 earns more than in Cournot

$$\Pi_1^S = \frac{(1-c)^2}{8} > \frac{(1-c)^2}{9} = \Pi_1^C$$

whereas Firm 2 earns less

$$\Pi_2^S = \frac{(1-c)^2}{16} > \frac{(1-c)^2}{9} = \Pi_2^C$$

Another Model of Entry

- Consider again an incumbent monopolist facing a potential entrant

- Now, more explicit model:
  New entry entails a positive fixed cost f
  If entrant does not enter, it earns 0 and the incumbent is a monopolist
  If entrant does enter, the two firms compete a la Cournot
  Again demand is given by

$$P(q_I + q_E) = \begin{cases} 1 - q_I + q_E & \text{if } q_I + q_E \leq 1 \\ 0 & \text{if } q_I + q_E > 1 \end{cases}$$

  and firms' unit costs are c (≤ 1)

Entry Model: Elements

- Players: Firms I and E
- Strategies: for E: $\{In$ or Out, $q_E\}$ and for I: $\{q_I\}$ where $q_E, q_I \in [0, \infty)$
- Payoffs (if the entrant enters):

$$\Pi_I(In, q_E, q_I) = (\max\{1 - (q_E + q_I), 0\})\, q_I - cq_I$$
$$\Pi_E(In, q_E, q_I) = (\max\{1 - (q_E + q_I), 0\})\, q_E - cq_E - f$$

- Payoffs (if the entrant does not enter):

$$\Pi_I(Out, q_I) = (\max\{1 - q_I, 0\})\, q_I - cq_I$$
$$\Pi_E(Out, q_I) = 0$$

Entry Model: SPNE

- Following In there is a subgame, the Cournot game, except that the payoff of the entrant is reduced by f. The output of each firm in a SPNE is

$$q_i^C = \frac{1-c}{3}$$

and the profits would be

$$\Pi_I(In, q_I^C, q_E^C) = \left(\frac{1-c}{3}\right)^2 \text{and } \ \Pi_E(In, q_I^C, q_E^C) = \left(\frac{1-c}{3}\right)^2 - f$$

- Following Out there is a subgame, the monopoly case. The output of the incumbent in a SPNE is (and the profits would be)

$$q_I^M = \frac{1-c}{2} \text{ and } \Pi_I(Out, q_I^M) = \left(\frac{1-c}{2}\right)^2 \text{and } \ \Pi_E(Out, q_I^M) = 0$$

- Anticipating this, in a SPNE the potential entrant enters whenever

$$\Pi_E(In, q_I^C, q_E^C) = \left(\frac{1-c}{3}\right)^2 - f \geq 0 = \Pi_E(Out, q_I^M)$$

- Summarising, the *outcome* of the SPNE of the game depend on f and c:
  If $f < \left(\frac{1-c}{3}\right)^2$: (one SPNE) In and $q_i = q_i^C = \frac{1-c}{3}$ for $i = E, I$
  If $f > \left(\frac{1-c}{3}\right)^2$: (one SPNE) Out and $q_I = q_I^M = \frac{1-c}{2}$
  If $f = \left(\frac{1-c}{3}\right)^2$: (two SPNE) the two previous outcomes may arise

- Notice that, as in Chapter 5, there is a *NE (not SPNE)* in which the incumbent floods the market and the potential entrant does not enter