Lecture 2: Predicting outcomes

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In the Previous Chapter

- Game Theory: set of tools to analyse behaviour in the presence of strategic interdependence
- For example, firms in an oligopoly market
- Static games: players play simultaneously and only once
- Further examples: "prisoner's dilemma", "battle of the sexes", "matching pennies", "stag hunt",...
- Representation in "normal form"

This chapter's Plan

- What should we expect players to play?
- Looking for reasonable concepts in simple predictable games and apply these concepts in other settings
- In this chapter, concentrate in static ("simultaneous move" or "strategic") games
- Solution concepts:
  Use dominant strategies
  Don't use dominated strategies
  Play Nash equilibrium strategies
- Assume that players are rational:
  each chooses her best action according to her preferences

Dominant Strategies

- "Prisoner's dilemma":

$1\backslash 2$	DC	C
DC	-2,-2	-10,-1
C	-1,-10	-5,-5

- What action would you choose?

- No matter what the other does, it is better to play "Confess":
  If she plays DC, one obtains -1 by playing C and -2 by playing DC
  If she plays C, one obtains -5 by playing C and -10 by playing DC

- Formally, "Confess" strictly dominates "Don't Confess"

- "Confess" is a dominant strategy or action

- Both confessing is the outcome! Conflict with Pareto-optimality

Introducing Notation

- $i = 1, ..., I$ the players of a game (e.g. in PD $i = 1, 2$)
- s_i a strategy or action for player i (e.g. $s_1 = C$ or $s_1 = DC$)
- $s = (s_1, ..., s_I)$ a strategy profile: one strategy for each player (e.g. $s = (C, DC)$ or $s = (CD, DC)$)
- $s = (s_i, s_{-i})$ again a strategy profile, where $-i$ means all the players except i
- $u_i(s) = u_i(s_1, ..., s_I)$ the payoff for player i if s is played (e.g. $u_1(C, DC) = -1$, $u_2(C, DC) = -10$,...)

Dominant Strategies

- Definition: Player i's strategy s_i'' strictly dominates strategy s_i' if

  $u_i(s_i'', s_{-i}) > u_i(s_i', s_{-i})$ for every possible list s_{-i} of the other player's actions

- Accordingly, we say that strategy s_i' is strictly dominated

- Example: strategy "Don't Confess" is strictly dominated by strategy "Confess"

- Definition: A strictly dominant strategy for player i is a strategy that strictly dominates all her other strategies

- Example: strategy "Don't Confess" is strictly dominant for both players

- Rational players play dominant strategies

Strictly Dominated Strategies

- Problem: strictly dominant strategies rarely exist. Examples:

(a)

1\2	L	R
U	1,-1	-1,-1
M	-1,1	1,-1
D	-2,5	-3,2

(b)

1\2	L	R
U	5,1	4,0
M	6,0	3,1
D	6,4	4,4

- However, a strictly dominated strategy may still exist.
- Example: strategy D in game (a)
- Rational players do not play dominated strategies

Weakly Dominated Strategies

- Definition: Player i's strategy s_i'' weakly dominates strategy s_i' if

$$u_i(s_i'', s_{-i}) \geq u_i(s_i', s_{-i}) \text{ for every possible list } s_{-i} \text{ of other player's actions}$$
$$\text{with strict inequality for some } s_{-i}$$

- Example: U and M in (b) are weakly dominated
- Should we rule out weakly dominated strategies as well?
- No! Playing M can be as good as playing D if 1 *believes* that 2 will play L

Nash Equilibrium

- Problem: strictly dominated strategies may not exist. Example: BoS game

1\2	Bach	Stravinsky
Bach	2,1	0,0
Stravinsky	0,0	1,2

- Definition: A strategy profile $(s_1, s_2, ..., s_I)$ constitutes a Nash equilibrium if for every player i

$$u_i(s_i, s_{-i}) \geq u_i(s_i', s_{-i}) \text{ for every action } s_i'$$

- Here, we are assuming that...
(1) Players are rational (given the belief about others' actions)
(2) Their beliefs about the actions of the others are correct

Example 1: Prisoner's Dilemma

1\2	Don't Confess	Confess
Don't Confess	-2,-2	-10,-1
Confess	-1,-10	-5,-5

- (C, C) is a NE:
  given that 2 plays C, then playing C is better than DC for 1 (-5 instead of -10)
  given that 1 plays C, then playing C is better than DC for 2 (-5 instead of -10)
- (C, DC) is a not a NE:
  given that 1 plays C, then playing C is better than DC for 2 (-5 instead of -10)
- (DC, C) is a not a NE:
  given that 2 plays C, then playing C is better than DC for 1 (-5 instead of -10)
- (DC, DC) is a not a NE:
  given that 2 plays DC, then playing C is better than DC for 1 (-1 instead of -2)

Remarks

1. NE may not be unique!! Example 2: coordination Game (or BoS)

1\2	Bach	Stravinsky
Bach	2,1	0,0
Stravinsky	0,0	1,2

- NE: (Bach, Bach) and (Stravinsky, Stravinsky)

2. NE may not exist! Example 3: Matching Pennies

1\2	Head	Tail
Head	1,-1	-1,1
Tail	-1,1	1,-1

.

- NE: none!

Exercise

- Exercise: what are the NE in the Stag Hunt game (example 4)?

$1\backslash 2$	Stag	Hare
Stag	2,2	0,1
Hare	1,0	1,1

- Find them intuitively first and show it formally afterwards (using the previous definition)