

Suggested Supervisions Chapter 3

1. (NE, Backwards Induction and SPNE)
 - (a) Obtain the pure strategy Subgame Perfect Nash equilibria of the games described in Figures 1, 2 and 3 below. Can you think of a way of representing them in Normal form?
 - (b) Find the set of Nash equilibria obtained by backwards induction in Figure 4.
 - (c) Find all Nash equilibria (pure and mixed) and verify that the set of equilibria obtained in (b) is a subset of the set of equilibria.
 - (d) Show that (A,a) is not the play of a Nash equilibrium in the extensive form game of imperfect information depicted in Figure 5.
 - (e) Find a Nash equilibrium of the game of part (d). Is it a subgame perfect Nash equilibrium?

2. (Mixed strategies) Show that the two-player game illustrated below has a unique equilibrium in pure strategies and that there is no additional equilibrium in mixed strategies.

	<i>L</i>	<i>M</i>	<i>R</i>
<i>U</i>	1, -2	-2, 1	0, 0
<i>M</i>	-2, 1	1, -2	0, 0
<i>D</i>	0, 0	0, 0	1, 1

3. (Cournot and Stackelberg) There are two firms in a market producing quantities q_1 and q_2 respectively of a homogeneous good, with a market (inverse) demand function $p = f(q_1 + q_2)$ which is positive valued, strictly downward sloping, concave and twice differentiable (over the relevant range of outputs). Firm i has a cost function $c_i(q_i)$ which is strictly increasing, strictly convex, twice differentiable and satisfies $c'_i(0) = 0$.

First, suppose that firms move simultaneously.

- a) Describe the elements of the normal form representation of this game.
- b) Describe necessary first order conditions that must be satisfied by any Cournot-Nash equilibrium. Are these sufficient as well?
- c) Show that the reaction function of firm 2 to firm 1's output, $\hat{q}_2(q_1)$ must be downward sloping.
- d) Among firm 2's strategies, which are strictly dominated?

Now suppose firm 1 is a leader, setting its output q_1 before firm 2 does. So firm 2 observes firm 1's output q_1 and then decides its own output q_2 .

- e) Draw an extensive form tree that represents this game.
- f) Show that in any subgame perfect equilibrium of the resulting (Stackelberg) game, firm 1 produces more and firm 2 produces less compared to the Cournot-Nash equilibrium.

g) Provide an intuitive argument for why firm 1 must earn more profit in the SPNE Stackelberg equilibrium than in the Cournot-Nash equilibrium.

h) What is the highest profit firm 2 can achieve in a Nash Equilibrium of the Stackelberg game? Describe one of the NE in which firm 2 attains it.

4. (Hotelling) Suppose that we have two firms-stores located on a line of length 1 that produce the same good. The unit cost of the good for each store is c . Consumers are uniformly located in the line and they incur a transportation cost tx^2 for travelling a length of x . Consumers have unit demands; each consumer derives a utility of \bar{v} for consumption.

Suppose that firm 1 is located at point $a \geq 0$ and firm 2 at point $1 - b$, where $b \geq 0$ and without loss of generality, $1 - a - b \geq 0$ (firm 1 is to the left of firm 2; $a = b = 0$ can be interpreted as maximal differentiation and $a + b = 1$ can be interpreted as minimal differentiation). Assume that the market is covered and firms sell positive quantities. Firms compete in prices.

a) For any pair of prices, (p_1, p_2) . Which consumer is indifferent between buying from firm 1 or firm 2? Which consumers will buy from firm 1? Find the demand for each firm as a function of the prices.

b) Show that in a NE, given a and b , firm 1 will charge

$$p_1 = c + t(1 - a - b)\left(1 + \frac{a - b}{3}\right)$$

whereas firm 2 will charge

$$p_2 = c + t(1 - a - b)\left(1 + \frac{b - a}{3}\right).$$

c) Find firms' market shares of the market.

d) Now consider a game where prior to price competition and knowing the prices that will be chosen given their locations, the two firms choose their locations simultaneously. Where do they locate?

e) What would be the socially optimal location of the two firms? Compare with the market outcome.

5. (Mergers) There are three identical firms in the industry. The total market demand is $P(Q) = 1 - Q$. The marginal costs are zero.

a) Compute the Nash Equilibrium.

b) Show that if two of the three firms merge (transforming the industry into a duopoly), the profit of these firms decreases. Explain [What would be the changes in production and in price following the merger? Would outsider firms benefit from the merger?]

c) What happens if all three firms merge?

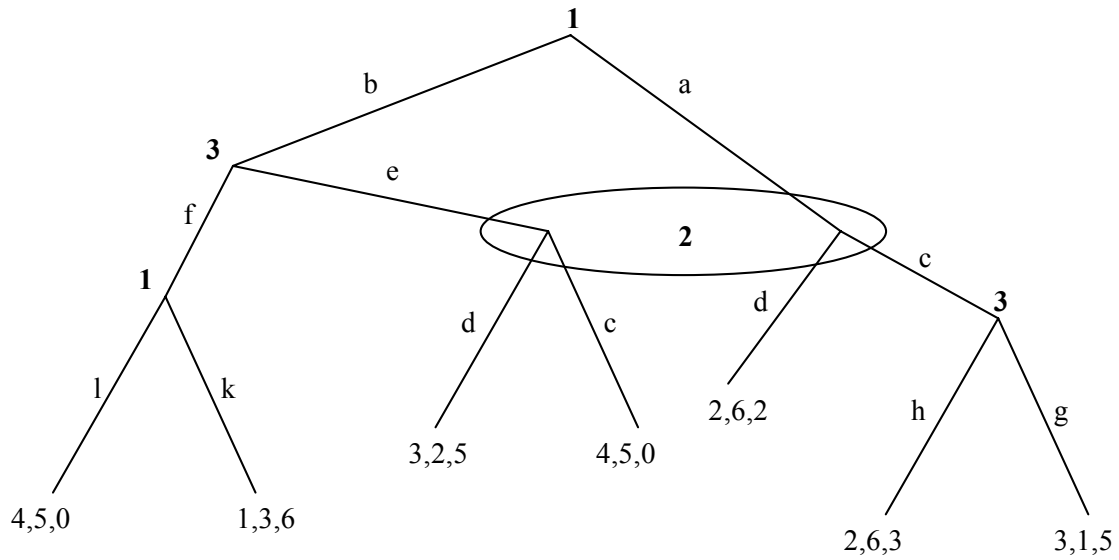


Figure 1

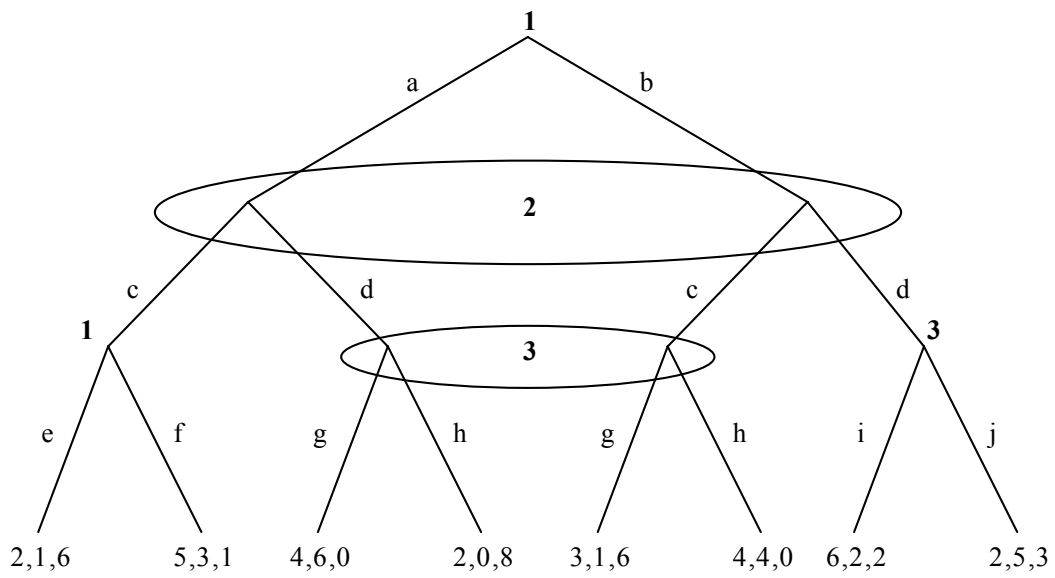


Figure 2

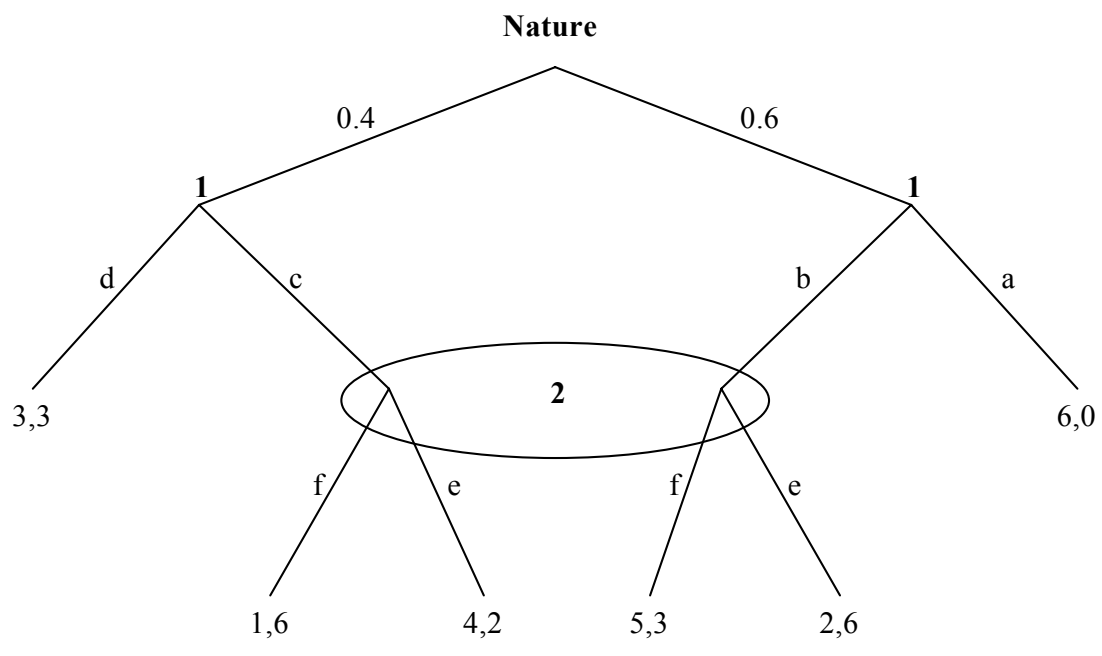


Figure 3

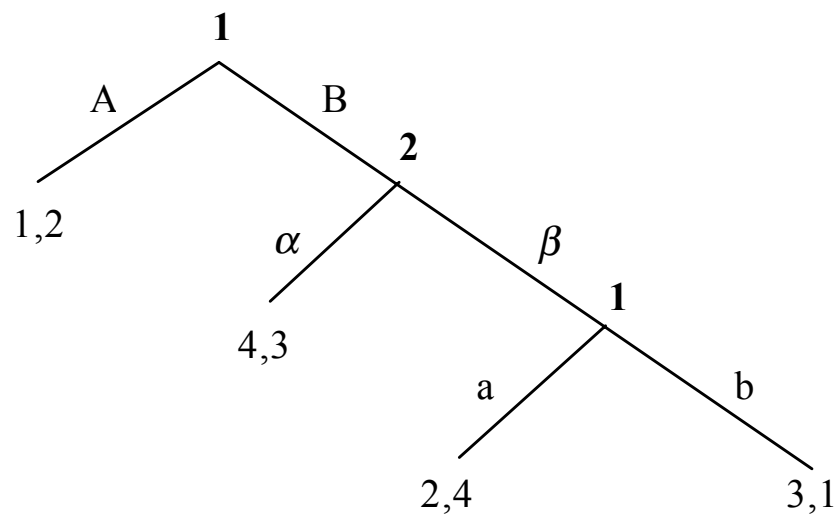


Figure 4

