Lecture 1: Introduction and Elements
Extensive Form Representation

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Introduction: Monopoly Pricing

- In the competitive model, there are many sellers that act as price takers
- In most markets, however, companies can profitably increase prices
- Suppose first that there is only one seller. It solves

$$Max_{q \geq 0} P(q)q - C(q)$$

where $P(\cdot)$ and $C(\cdot)$ are twice-continuous differentiable, $P'(q) < 0$, $P(0) > C'(0)$ and there is a unique q^o such that $P(q^o) = C'(q^o)$

- FOC:

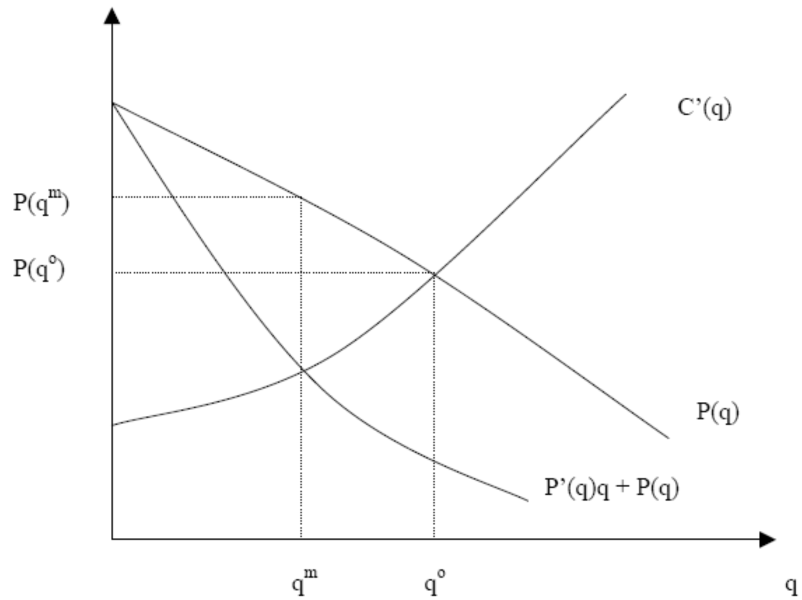
$$P'(q^m)q^m + P(q^m) - C'(q^m) + \lambda = 0,\ \lambda q^m = 0,\ \lambda \geq 0, \text{ and } q^m \geq 0$$

i.e.

$$P'(q^m)q^m + P(q^m) \leq C'(q^m) \text{ with equality if } q^m > 0$$

Introduction: Monopoly Pricing

- But, since $P(0) > C'(0)$, then $q^m \neq 0$ and $P'(q^m)q^m + P(q^m) = C'(q^m)$
- Since $P'(q) < 0$ then $P(q^m) > C'(q^m)$ and $q^m < q^o$



Motivation: Multiple Sellers

- What happens when there are more than one firm (but not many)?
- The demand (and therefore profits) of each firm also depends on the quantity placed (or the price set) by the others
- More generally, the decision-maker well-being also depends on the actions of the others
- As a consequence, the optimal decision depends on the decisions of the others
- Other examples include: Political candidate choosing a policy platform, bidder in an auction, negotiator in a purchase,...
- Game Theory: framework to analyse decisions in the presence of strategic inter-dependence

Today's Lecture

- Motivation
- Elements
- Further examples
- Extensive representation of a game
- Formal definition of the extensive form

Elements of a "Game"

- Players: who is involved?
- Rules: when do you play?
- Outcomes: for each set of moves, what happens?
- Payoffs: what are the preferences over these outcomes?

Representation in Extensive Form

- Example: "Sequential Matching Pennies"
- 2 players: 1 and 2. Player 1 puts first a penny down, heads up or heads down, and then Player 2, after seeing the move of Player 1, puts her penny down, heads up or heads down. If they match, Player 1 pays $1 to Player 2 and if they do not match , Player 2 pays $1 to Player 1.
- Tree structure representation: decision nodes, branches (actions), payoffs,...
- Strategic interdependence: a player's payoff is not independent of the actions of the others
- Additional examples

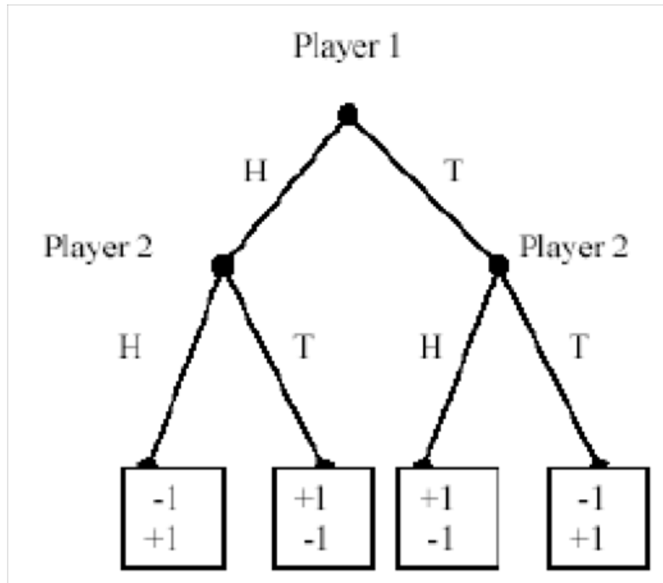


Figure 1:

Simultaneous Moves

- Modified example: suppose that player 1 now hides the penny once put down
- Player 2 does not know what Player 1 played
- Strategically equivalent to "Standard Matching Pennis" (represented equally)
- Representation of an information set

- Game of perfect information: all information sets are singletons
- Game of imperfect information: at least one information set is not a singleton

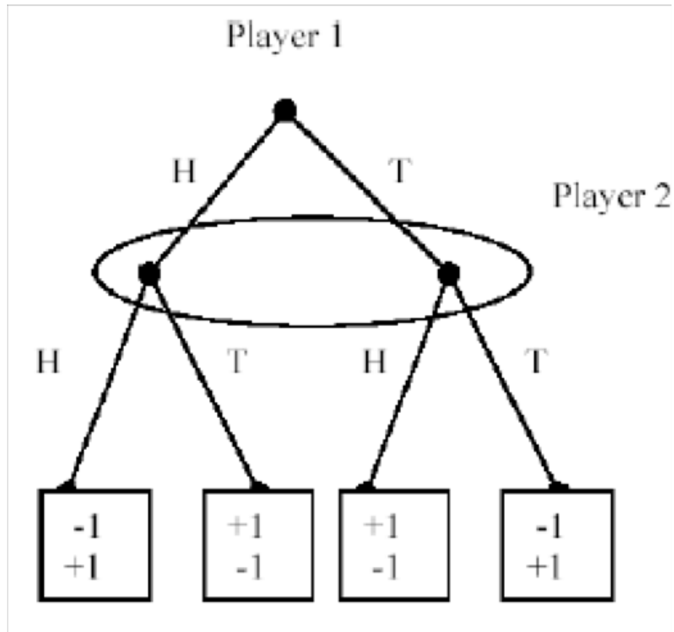
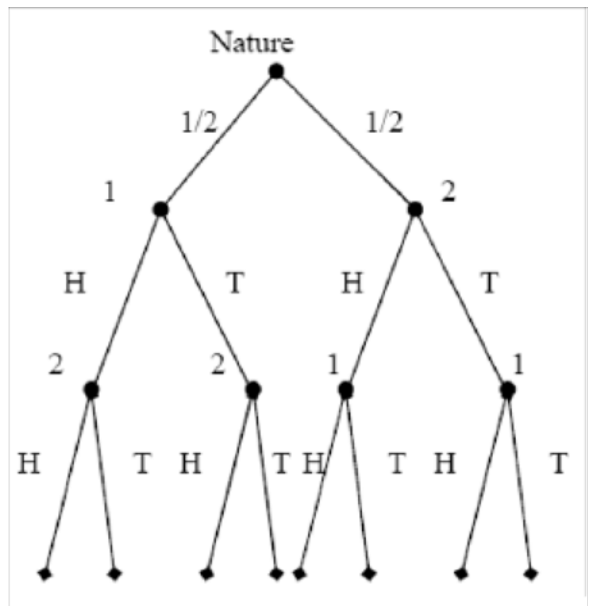


Figure 2:

Randomness

- We can introduce randomness by adding a "Nature" player
- Nature has probabilities to choose each branch



Formal Definition (Finite Game)

- A game in extensive form can be defined as

$$\Gamma_E = [\chi, \Lambda, I, p(), \alpha(), \Xi, H(), \iota(), \rho(), u] \quad \text{where...}$$

- χ is a finite set of nodes, Λ a finite set of actions, and I a finite set of players

- $p : \chi \rightarrow \{\chi \cup \varnothing\}$ determines the predecessor, and $p(x) \neq \varnothing$ for all $x \in \chi$ except for x_o, $p(x_o) = \varnothing$

Define $s(x) = \{x' \in \chi; p(x') = x\}$ as the successors of x (the set of successors and predecessors of a node are assumed to be disjoint)

Define $T = \{x \in \chi; s(x) = \varnothing\}$ as the set of terminal nodes

- $\alpha : \chi \backslash \{x_o\} \rightarrow \Lambda$ determines the action leading to the node (if $x', x'' \in s(x), x' \neq x''$ then $\alpha(x') \neq \alpha(x'')$)

$c(x) = \{a \in \Lambda; a = \alpha(x')$ for some $x' \in s(x)\}$ is set of actions available at x

- Ξ is a collection of information sets and $H : \chi \rightarrow \Xi$ determines the information set of each node (if $H(x) = H(x')$ then $c(x) = c(x')$)

  $C(H) = \{a \in \Lambda; a \in c(x)$ for $x \in H\}$ is the set of actions available at H

- $\iota : H \rightarrow \{0, 1, ..., I\}$ determines the player at each info set (0 represents nature)

  $\Xi_i = \{H \in \Xi; \iota(H) = i\}$ is the set of info set at which player i is called to play

- $\rho : \Xi_0 \times \Lambda \rightarrow [0, 1]$ assigns probabilities to actions at the nature info sets ($\rho(H, a) = 0$ if $a \notin C(H)$ and $\sum_{a \in C(H)} \rho(H, a) = 1$ for all $H \in \Xi_0$)

- $u = (u_1(\cdot), ..., u_I(\cdot))$ where $u_i : T \rightarrow \mathbb{R}$ assigns (Bernoulli) utilities to each terminal node for each player

Extensions (Infinite Games)

- Continous set of actions $[a, b]$. Examples:
- Infinite set of decision nodes. Examples:
- Infinite number of players. Examples: