# Chapters 3 and 4: Firm Behaviour, Markets and Welfare

## Financial Microeconomics

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# 3.- Producer Theory

3.1.- Production functions: physical relation between inputs and outputs
3.2.- Cost minimisation (input choice)
3.3.- Profit maximisation (quantity choice) and individual supply function
3.4.- Market's supply function

# 3.1.- Production Functions

- Definition and isoquants
- Marginal rate of technical substitution
- Returns to scale
- Examples of production functions

# Input and Output

- Definition: A firm's *production function* for a given good $q$ is given by

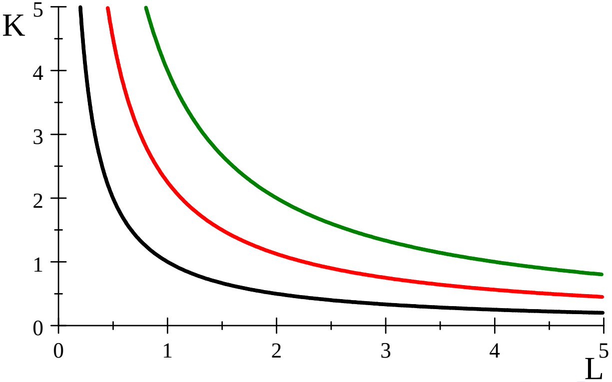
$$q = f(L, K, M...)$$

- Maximum amount of output that can be produced in a given period for each combination of labour ($L$), capital ($K$), raw material ($M$),...
- Example with two inputs $(L, K)$

$$q = 10L^{1/2}K^{1/2}$$

# Isoquant Maps

- Isoquant: combinations of inputs giving the same output $q_o$
- Ex: $q = f(L, K) = 10L^{1/2}K^{1/2}$, $q_o = 10, 15, 20$ (black, red, green)



# Marginal Rate of Technical Substitution (RTS)

- Rate at which labour can be substituted for capital holding output constant

$$RTS_{L,K}(L,K) \equiv -\left.\frac{\partial K}{\partial L}\right|_{q=q_o} = \frac{\frac{\partial f}{\partial L}(L,K)}{\frac{\partial f}{\partial K}(L,K)}$$

- Example: $q = f(L,K) = 10L^{1/2}K^{1/2}$

$$RTS(L,K) = RTS_{L,K}(L,K) = \frac{5L^{-1/2}K^{1/2}}{5L^{1/2}K^{-1/2}} = \frac{K}{L}$$

(a) $RTS(1,4) = 4$, (b) $RTS(1,1) = 1$ and (c) $RTS(4,1) = 1/4$

- Interpretation (a): for extra unit of labour, we can dispense of 4 of capital

# Returns to Scale

- How does output change to proportional increase in *all* inputs?
  (i) exactly proportionally or
  (ii) more than proportionally (division of labour and specialisation) or
  (iii) less than proportionally (managerial overseeing more difficult)
- For $m > 1$, the *returns to scale* of production are
  (i) constant if $f(mL, mK) = mf(L, K) = mq$
  (ii) increasing if $f(mL, mK) > mf(L, K) = mq$
  (iii) decreasing if $f(mL, mK) < mf(L, K) = mq$
- Real production functions often first increasing and then decreasing
- Why returns to scale are so important?

# Cobb-Douglas Example

- Suppose output could be produced by capital and labour according to

$$q = f(L, K) = AL^{\beta}K^{\alpha}$$

where $A > 0$ and $\alpha$,$\beta > 0$ are elasticities of output wrt capital and labour

- Given that

$$f(mL, mK) = A(mL)^{\beta}(mK)^{\alpha} = Am^{\alpha+\beta}L^{\beta}K^{\alpha} = m^{\alpha+\beta}f(L, K)$$

constant, increasing and decreasing returns to scale iff $\alpha + \beta = 1, > 1, < 1$

- In Cobb-Douglas, $A$, $\alpha$ and $\beta$ can be estimated econometrically:

$$\ln q = \ln A + \beta \ln L + \alpha \ln K + \varepsilon$$

# Examples of production functions (1)

- Linear production function

$$q = f(L, K) = \alpha K + \beta L$$

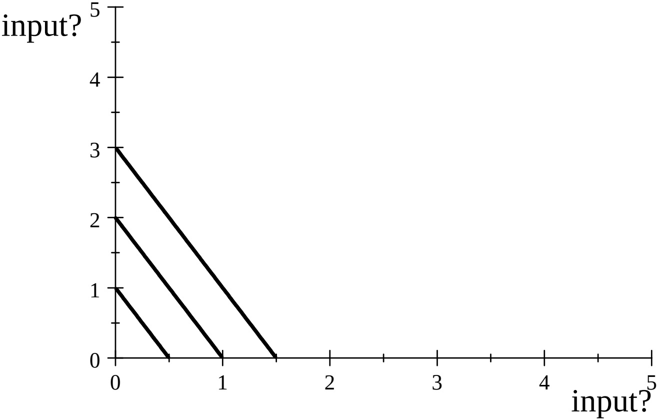
  for $\alpha,\beta > 0$

- Has constant returns to scale because for $m > 0$

$$f(mL, mK) = \beta mL + \alpha mK = mf(L, K)$$

- $RTS = ?$
- Reasonable? Examples?

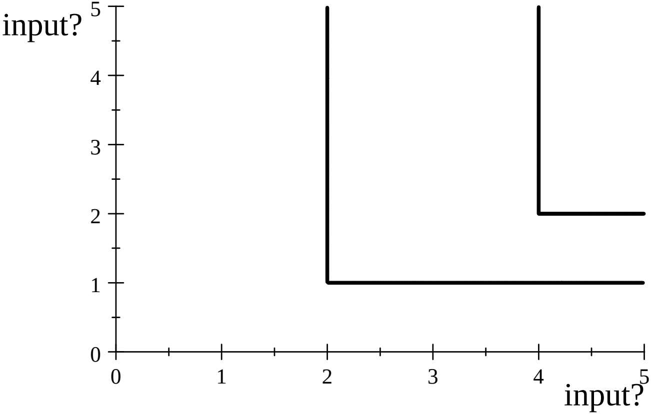
# Linear production function with alpha=2 and beta=1



# Examples of production functions (2)

- Fixed proportions
$$f(L,K) = \min\{\beta L, \alpha K\}$$
for $\alpha,\beta > 0$.
- A firm with this production will always operate at the corner
- Returns to scale?
- Examples?

# Fixed proportions with alpha=2 and beta=1



# Examples of production functions (3)

- Constant elasticity of substitution (CES) production function

$$q = f(L, K) = \left[K^{\rho} + L^{\rho}\right]^{\epsilon/\rho}$$

  where $\rho \leq 1$, $\rho \neq 0$, $\epsilon > 0$
- Returns to scale? Marginal rate technical substitution

$$RTS = \frac{(\epsilon/\rho)\, q^{1-\rho/\epsilon} \rho L^{\rho-1}}{(\epsilon/\rho)\, q^{1-\rho/\epsilon} \rho K^{\rho-1}} = \frac{L^{\rho-1}}{K^{\rho-1}} = \left(\frac{K}{L}\right)^{1-\rho},$$

- Then, $\ln RTS = (1-\rho)\ln(K/L)$ and elasticity of substitution

$$\sigma \equiv \frac{\partial \ln K/L}{\partial \ln RTS}\left(= \frac{\text{percent } \Delta(K/L)}{\text{percent } \Delta RTS}\right) = \frac{1}{1-\rho}$$

- Particular cases: linear, fixed proportions and Cobb-Douglas ($\rho = 1$, -$\infty$, 0 respectively) (proof requires limit arguments, see textbook)

# Production vs utility function?

- Output from inputs vs utility level from purchases
- Derived from technologies vs derived from preferences
- Cardinal (unique and specific amount) vs ordinal
- Marginal Product vs marginal utility
- Isoquant vs indifference curve
- Marginal rate of technical substitution vs marginal rate of substitution

# 3.2.- Cost Minimisation

- Definition of costs
- Cost minimisation choices
- Input demand and total cost function
- Average and marginal costs functions
- Economies of scale

# Input costs, but which costs?

- Accounting costs: out-of-pocket historical costs appropriately depreciated
- Economic (or opportunity) costs: payment needed to keep resources in place or, equivalently, the remuneration received in the next best alternative
- Ex: bought steel for £1m but price has gone up and is now worth £1.2m
  Economic and accounting costs?

- We'll always talk about economic costs
- Most of the time here, for simplicity, homogenous labour and capital costs
  - Denote $w$ labour cost (labour-hour)
  - Denote $r$ capital cost (price of machine-hour)

# Cost minimisation

- Firm selects $K$ and $L$ to minimise total costs of producing a given $q_o$

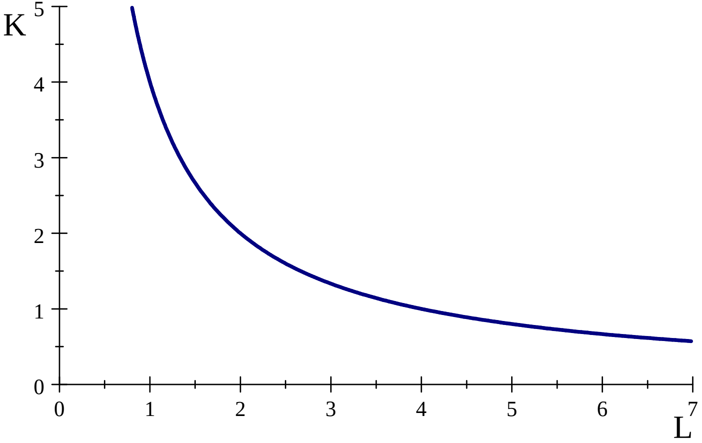
$$TC \equiv wL + rK$$

- First step towards maximisation of profits,

$$\pi \equiv TR - TC$$

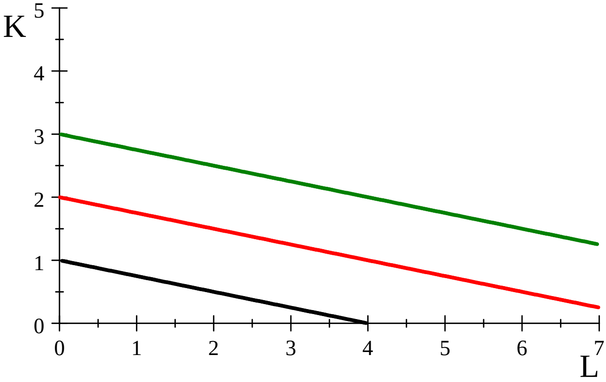
- But, it can accommodate more general objectives (e.g. regulated transmission firms are required to transport a certain quantity)

# Required production level



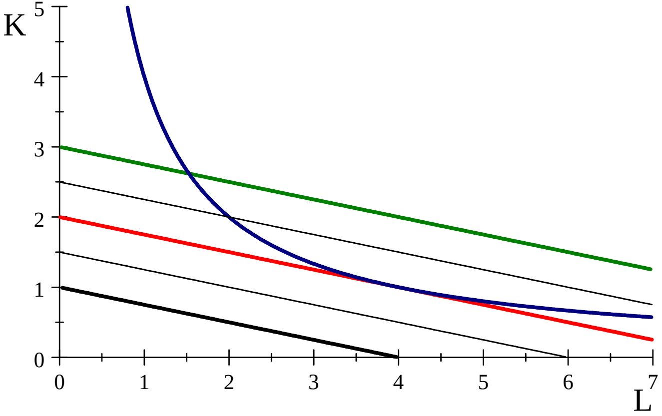

- Example: $q = f(L, K) = 10K^{1/2}L^{1/2}$ and $q_o = 20$ units are required

# Isocost lines




- Slope $-w/r$, vertical intercept $TC/r$ and horizontal $TC/w$
- Graph: $w = 0.25$ and $r = 1$, $TC = 3$ (green), 2 (red) and 1 (black)

# Which input combination?




- Combination s.t. its isoquant is tangent to its isocost line

# Mathematical Problem

- Find $K^*, L^*$ that solve

$$Min\ TC \equiv wL + rK$$
$$subject\ \text{to}\ q = f(L, K) = q_o$$

- This can be found by solving

$$Min_{L,K}\ \mathcal{L} = \ wL + rK + \lambda\left[q_o - f(L, K)\right]$$

or

$$\frac{\partial \mathcal{L}}{\partial L} = w - \lambda^* \frac{\partial f(L^*, K^*)}{\partial L} = 0 \text{ and } \frac{\partial \mathcal{L}}{\partial K} = r - \lambda^* \frac{\partial f(L^*, K^*)}{\partial K} = 0$$
$$\frac{\partial \mathcal{L}}{\partial \lambda} = q_o - f(L^*, K^*) = 0$$

# Input demand and total cost function

- Solution satisfies

$$RTS(L^*, K^*) = \frac{\frac{\partial f(L^*,K^*)}{\partial L}}{\frac{\partial f(L^*,K^*)}{\partial K}} = \frac{w}{r} \text{ and } q_o = f(K^*, L^*)$$

- One obtains the *input demands* and the *total cost function*:

$$L^*(w, r, q_o) \text{ and } K^*(w, r, q_o) \text{ and } TC^*(w, r, q_o)$$

# Hamburger production

- Production-hour ($q$) depends on grills-hour ($K$) and worker-hours ($L$)

$$q = 10K^{1/2}L^{1/2}$$

  and management wants to produce $q_o$ hamburgers per hour
- Optimal combination satisfies

$$RTS(L^*, K^*) = \frac{K^*}{L^*} = \frac{w}{r} \text{ and } q_o = 10\left(K^*\right)^{1/2}\left(L^*\right)^{1/2}$$

- Input demands:

$$L^*(w, r, q_o) = \frac{q_o}{10}\left(\frac{r}{w}\right)^{1/2} \text{ and } K^*(w, r, q_o) = \frac{q_o}{10}\left(\frac{w}{r}\right)^{1/2}$$

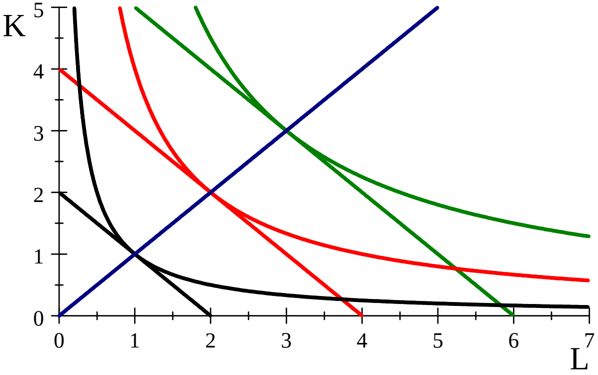
- If $q_o = 20$, $w = 0.25$, $r = 1$, then $L^* = 4$ and $K^* = 1$

# Price elasticity for input demand

| Input | Capital | Production | Non-production | Electricity |
|---|---|---|---|---|
| Industry | | labour | labour | |
| Textiles | -0.41 | -0.5 | -1.04 | -0.11 |
| Paper | -0.29 | -0.62 | -0.97 | -0.16 |
| Chemicals | -0.12 | -0.75 | -0.69 | -0.25 |
| Metals | -0.91 | -0.41 | -0.44 | -0.69 |

- Barnett, Reutter and Thompson, "Electricity Substitution: Some Local Industrial Evidence" Energy Economics 1998

# Firm's expansion path




- Larger quantity needed (10, 20, 30) implies higher cost (black, red, green)
- Normal (inferior) input if input demand increases (decreases) with output

# Average and marginal cost function

- Substituting $L^*$ and $K^*$ into total cost function

$$TC^*(w, r, q_o) \equiv TC(w, r, q)$$

(notation: we change $q$ for $q_o$ and take $^*$ out)

- In our hamburguer example

$$TC(w, r, q) = \frac{q}{5}(wr)^{1/2}$$

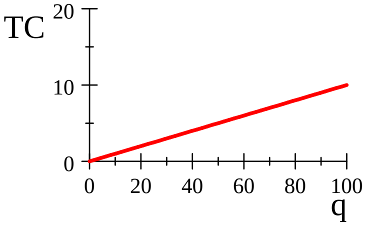
and therefore average cost function

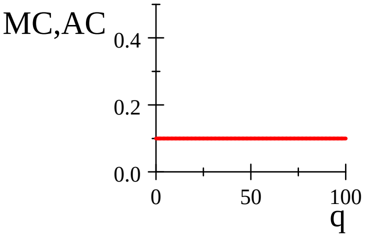
$$AC(w, r, q) = \frac{TC(w, r, q)}{q} = \frac{1}{5}(wr)^{1/2}$$

and the marginal cost function

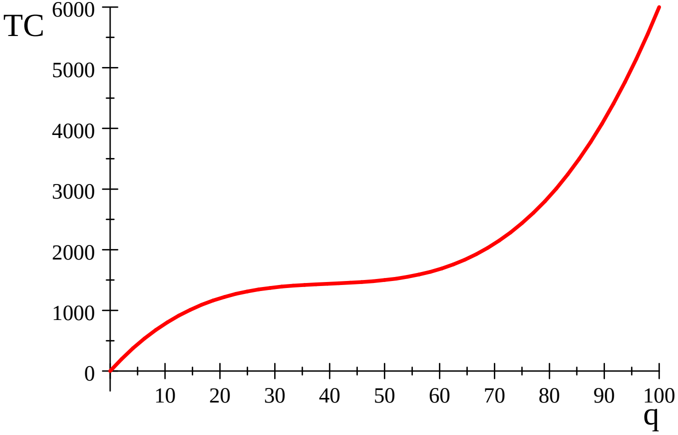
$$MC(w, r, q) = \frac{\partial TC(w, r, q)}{\partial q} = \frac{1}{5}(wr)^{1/2}$$

# Hamburgers example for w=0.25 and r=1



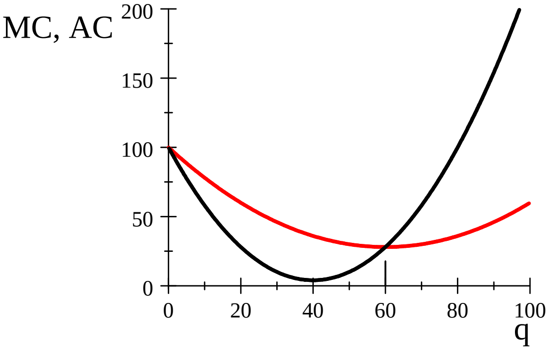



# Another total cost curve example (cubic)




- For given input prices: $TC(q) = 100q - 2.4q^2 + 0.02q^3$
- Can you hint the "returns to scale" here?

# Example: MC (black) and AC (red)



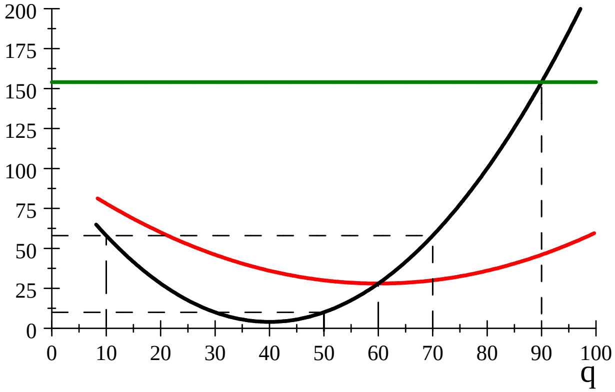

- $MC(q) = 100 - 4.8q + 0.06q^2$, $AC(q) = 100 - 2.4q + 0.02q^2$
- MC and AV coincide at the origin, why? AC is then higher than MC, why? If $AC > MC$ then $AC \downarrow$, why? Intersection at minimum of AC (60), why?

# Economies of scale

- Cost exhibits economies of scale if AC $\downarrow$: (=increasing returns to scale)
- Cost exhibits diseconomies of scale if AC $\uparrow$: (=decreasing returns to scale)
- Minimum of AC is called minimum efficiency scale (MES).
- MES for selected US food and beverage industries

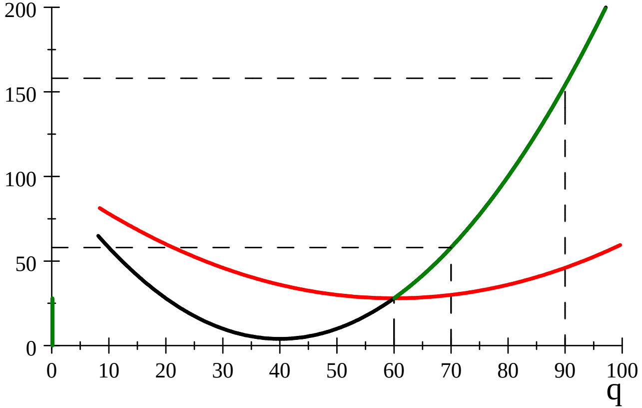
| Industry | MES as % of market output |
|---|---|
| Beet sugar (processed) | 1.87 |
| Cane sugar (processed) | 12.01 |
| Flour | 0.68 |
| Breakfast cereal | 9.47 |
| Baby food | 2.59 |

# 3.3.- Profit maximisation and supply




- $P = 154$ (green), $AC$ (red), $MC$ (black). Optimal quantity? Why?
- And if $P = 58$? $P = 10$?

# Supply function, graphically (in green)



# Another example, mathematically

- For the following total cost function

$$TC(q) = 2 + 0.25q^2/200$$

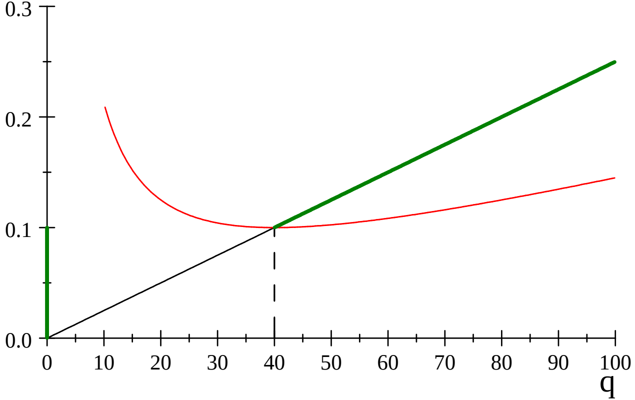
we have

$$MC(q) = 0.5q/200 \text{ and } AC(q) = 2/q + 0.25q/200$$

- Note (i) $MC'(q) > 0$ and (ii) $MC(q) > AC(q)$ if $q > 40$:

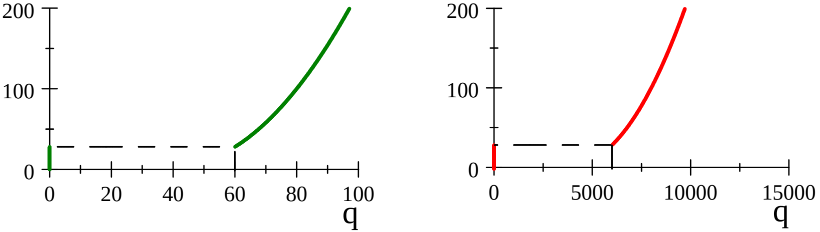
then $P = 0.5q/200$ if $q > 40$ and therefore $q = 400P$ if $P > 0.1$

# Graphically



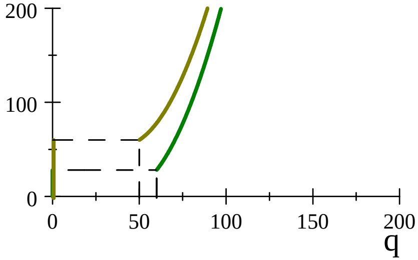

- *AC* (red), *MC* (black), Supply (green)

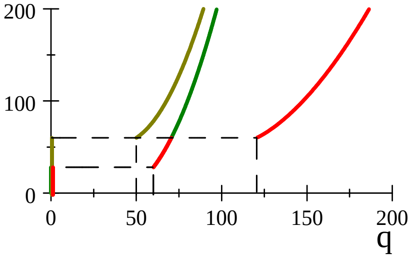
# 3.4. Market's supply function




- Market with 100 identical firms
- Individual (green, left) and market (red, right) supply
- Be careful about the scale difference!

# Another example




- Market with two different firms
- Individual (left, brown and green) and market (red, right) supply curves

# Mathematically

- Market supply function:

$$Q_s(w, r, P) = \sum_{i=1}^{n} q_i(w, r, P)$$

  where $q_i(w, r, P)$ $(= q_i^*)$ supply for each firm and $n$ number of firms
- Supply elasticity:

$$e_{Q_s,P} \equiv \frac{\partial Q_s(w, r, P)}{\partial P} \frac{P}{Q_s(w, r, P)}$$

- In our example, for $n = 100$ identical producers

$$Q_S = \sum_{i=1}^{100} 400P = 40000P \quad \text{and} \quad e_{Q_s,P} = 40000 \frac{P}{40000P} = 1$$

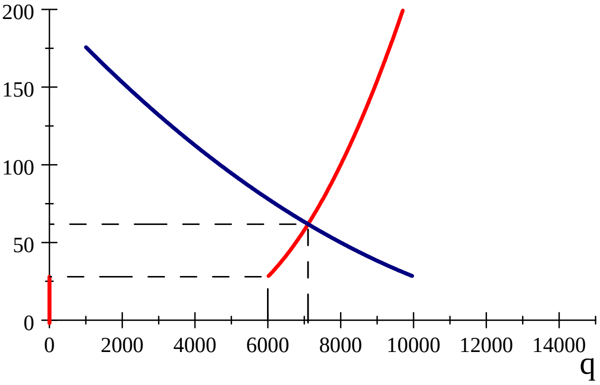
# 4.- Competitive Markets and Welfare

- Definition of competitive markets
- Price determination
- Shifts in demand and supply functions
- Profits and producer surplus
- Consumer and social welfare
- Pros and cons of markets

# Definition of "Competitive Markets"

- Large number of firms...
- ...producing a homogenous good (partial analysis)
- Firms maximise profits taking price as given
- All firms use same input and output prices
- Consumers and firms have perfect information
- Transactions are costless

# Price determination




- Market supply (red) and market demand (blue)
- Equilibrium price: $P^* = 61.822$ and quantity traded: $Q^* = 7104.5$

# Mathematically

- Equilibrium price ($P^*$) is defined as

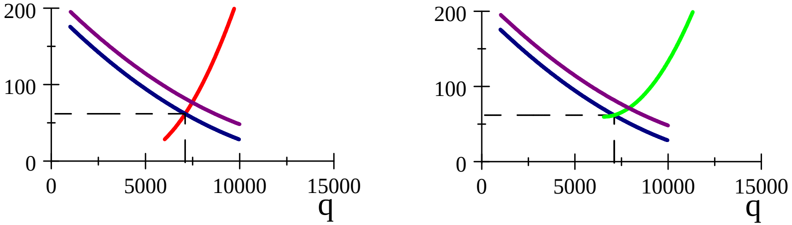
$$Q_D(P^*, P', I) = Q_S(P^*, r, w)(= Q^*)$$

  where $P'$ price of other goods, $I$ **vector** of incomes, $r, w$ input price
- $P^*$ serves to signal each (price-taker) producer how much to produce
  Each firm does $MR = MC$, given technology and input prices. Total: $Q^*$
- $P^*$ serves to signal each (price-taker) consumer how much to buy
  Each consumer maximises utility, given preferences and income. Total: $Q^*$
- Equilibrium since all can buy and sell what they demand and produce at $P^*$

# Supply and demand shifts

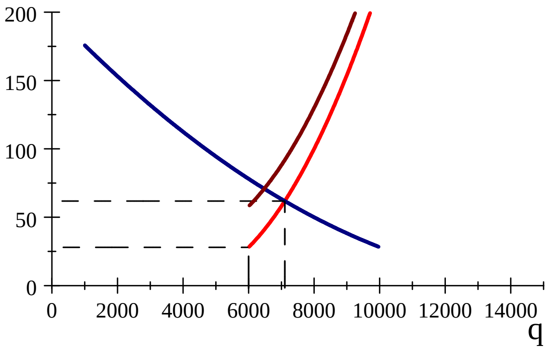
- Demand shifts because of a change in...
  Income(s), price(s) of other good(s), preferences
- Supply shifts because of a change in...
  Input price(s), number of producers, technology(ies)
- A change in either will change equilibrium price and quantity

# Demand shifts upwards



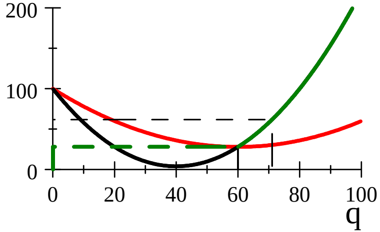

- What if demand shifts (from blue to purple)? Prices, quantities?
- How does the impact depend on the elasticity of supply?

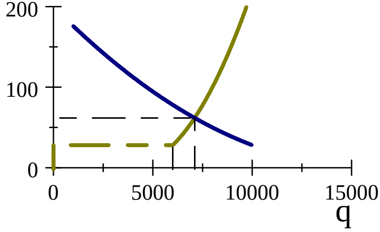
# Supply shifts upwards




- What if supply shifts (from red to magenta)? Prices, quantities?
- How does the impact depend on the elasticity of demand?

# Profits?




- Equilibrium: $P^* = 61.822$, $q_i^* = 71$, $Q^* = 7100$
- "Shut-down" price and quantity: $P^S = AC(q_i^S) = AC(Q^S)$
- In this example: $P^S = 28, q_i^S = 60, Q^S = 6000$
- *AC* (red) and *MC* (black), individual/market supply (green/brown)

# Profits and producer surplus (PS)

- Profits are equal to the area between individual supply and $P^*$:

$$\left(P^* - P^S\right) q_i^S + \int_{q_i^S}^{q_i^*} \left[P^* - MC_i(q)\right] dq =$$

$$\left(P^* - P^S\right) q_i^S + \left|P^* q - TC_i(q)\right|_{q=q_i^S}^{q=q_i^*}$$

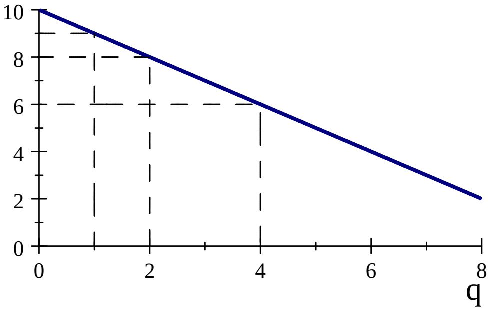
$$-P^S q_i^S + P^* q_i^* - TC_i(q_i^*) + TC_i(q_i^S) =$$

$$-AC_i(q_i^S) q_i^S + \pi_i + TC_i(q_i^S) = \pi_i$$

- Producer surplus $PS$: "societal" gains from production
- Here, individual $PS_i = \pi_i$. Total $PS$: sum of individual $PS_i$, i.e.

$$PS = \left(P^* - P^S\right) Q^S + \int_{Q^S}^{Q^*} \left[P^* - S(Q)\right] dQ$$

# Individual consumer welfare (CS)




- What about consumers? Need to provide monetary measure of "welfare"
- Individual demand can be viewed as willingness to pay per unit
- $CS_i$: difference between willingness to pay and price. $CS_i$ if $P = 6$?

# Consumer surplus

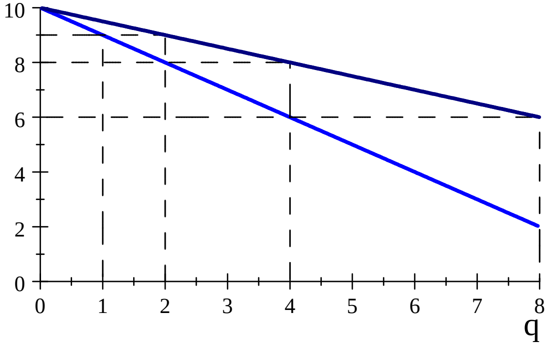
- Consumer surplus is the total sum of individual consumer welfare
- Formally

$$CS = \int_0^{Q^*} D(Q)dQ - P^*Q^* = \int_0^{Q^*} (D(Q) - P^*)dQ$$

where $D(Q)$ is "inverse demand" (sum of willingness to pay)

- $CS$ for market with two consumers with demand as in previous example?

# Consumer surplus




- Two consumers with identical individual demands
- Representation of consumer surplus if $P^* = 8$? If $P^* = 6$? Interpretation?

# Social surplus (or welfare) or total surplus (welfare)

- Social welfare is defined as the sum of consumer and producer welfare
- Suppose demand: $Q_D = 2(10 - P)$ and supply: $Q_S = 2P - 4$
- Equilibrium price and quantity: $P^* = 6$, $Q^* = 8$
- Consumer surplus:

$$CS = \int_0^8 (10 - Q/2)\, dQ - 6 * 8 = 16$$

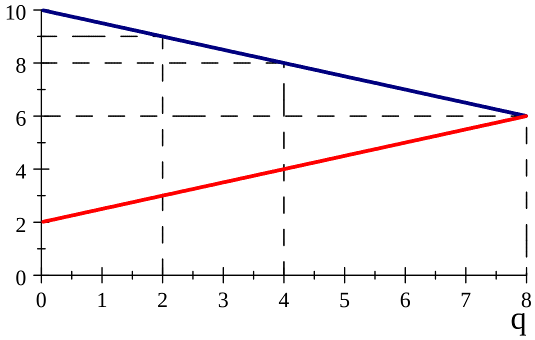
- Producer surplus:

$$PS = 6 * 8 - \int_0^8 (Q/2 + 2)\, dQ = 16$$

- Social (total) surplus:

$$TS = CS + PS = 32$$

# Graphically




- Representation of consumer, producer and social surplus?
- Could consumer and producer welfare be computed in a faster way here?
- If you were a (benevolent) social planner which price would you set?

# Efficiency of Perfectly Competitive Markets

- Perfectly competitive markets are "Pareto" efficient:
  Market outcome cannot be replaced by another that would increase the welfare of an individual without harming an other
  Buyers with highest willingness to pay obtain the goods
  Sellers with lowest costs produce the goods
- An efficient allocation maximizes the social surplus
  "It is not from the benevolence of the butcher, the brewer or the baker, that we expect our dinner, but from their regard to their own interest."
  Adam Smith, The Wealth of Nations (1776)

# Pros and cons of markets

- Responsiveness to changing economic conditions
- Mostly self-organising
- Benefits to the society through the pursuit of self-interest

- Losers as well as winners: resulting distribution may not be "desirable" (fair)
- "Market failures" may lead to inefficient outcomes