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Corporate Finance

# Chapter 1: Investment Decisions

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Albert Banal-Estanol

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# In this chapter...

- Part (a): Compute projects' cash flows :
  - Computing earnings, and free cash flows
  - Necessary inputs?
- Part (b): Evaluate risk-free projects:
  - Decide whether to invest in a project
  - Project selection
- Part (c): Adjusting for risk:
  - How to adjust the “discount rate”?
  - Portfolio theory, the CAPM and extensions

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## Part (a): Project's cash flows

- Computing earnings, and free cash flows
- Necessary inputs?
  
- Example:
  - Firm: Linksys (subsidiary of Cisco Systems, maker of consumer networking hardware)
  - Project: *Homenet*, wireless home network appliance, which would provide both hardware and software necessary to run an entire home from any Internet connection

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# Feasibility study

- ❑ Estimated life of the project: four years
- ❑ Revenue estimates:
  - Sales = 100,000 units/year
  - Per Unit Price = \$235
- ❑ Cost Estimates:
  - Up-Front R&D = \$15,000,000
  - Up-Front New Equipment = \$7,500,000
    - ❑ Expected life of the new equipment is 5 years (housed in existing lab)
  - Annual Overhead = \$3,000,000
  - Per Unit Cost = \$95
- ❑ Cost of the feasibility study \$300,000

# Incremental Earnings Forecast

	Year	0	1	2	3	4	5
<b>Incremental Earnings Forecast (\$000s)</b>							
1	Sales	—	23,500	23,500	23,500	23,500	—
2	Cost of Goods Sold	—	(9,500)	(9,500)	(9,500)	(9,500)	—
3	Gross Profit	—	14,000	14,000	14,000	14,000	—
4	Selling, General, and Administrative	—	(3,000)	(3,000)	(3,000)	(3,000)	—
5	Research and Development	(15,000)	—	—	—	—	—
6	Depreciation	—	(1,500)	(1,500)	(1,500)	(1,500)	(1,500)
7	EBIT	(15,000)	9,500	9,500	9,500	9,500	(1,500)
8	Income Tax at 40%	6,000	(3,800)	(3,800)	(3,800)	(3,800)	600
9	Unlevered Net Income	(9,000)	5,700	5,700	5,700	5,700	(900)

Are taxes relevant even if we make losses?

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# Capital Expenditures and Depreciation

- Investments in plant, property and equipment:
  - are a cash expense not directly listed as expense but
  - a fraction of cost deducted each year as depreciation
- Some methods:
  - Straight Line Depreciation: Asset's cost is divided equally over its life ( $\$7.5 \text{ million} \div 5 \text{ years} = \$1.5 \text{ million/year}$ )
  - Modified Accelerated Cost Recovery System (MACRS) depreciation (*see rates in next table, obtained from a complicated formula, which can be found in accounting textbooks*)

Tasa de depreciación (%) para cada periodo de recuperación SMARC en años						
Año	n = 3	n = 5	n = 7	n = 10	n = 15	n = 20
1	33.33	20.00	14.29	10.00	5.00	3.75
2	44.45	32.00	24.49	18.00	9.50	7.22
3	14.81	19.20	17.49	14.40	8.55	6.68
4	7.41	11.52	12.49	11.52	7.70	6.18
5		11.52	8.93	9.22	6.93	5.71
6		5.76	8.92	7.37	6.23	5.29
7			8.93	6.55	5.90	4.89
8			4.46	6.55	5.90	4.52
9				6.55	5.91	4.46
10				6.55	5.90	4.46
11				3.28	5.91	4.46
12					5.90	4.46
13					5.91	4.46
14					5.90	4.46
15					5.91	4.46
16					2.95	4.46
17 – 20						4.46
21						2.23

# From Earnings to Cash Flows

	Year	0	1	2	3	4	5
<b>Incremental Earnings Forecast (\$000s)</b>							
1	Sales	—	23,500	23,500	23,500	23,500	—
2	Cost of Goods Sold	—	(9,500)	(9,500)	(9,500)	(9,500)	—
3	<b>Gross Profit</b>	—	14,000	14,000	14,000	14,000	—
4	Selling, General, and Administrative	—	(3,000)	(3,000)	(3,000)	(3,000)	—
5	Research and Development	(15,000)	—	—	—	—	—
6	Depreciation	—	(1,500)	(1,500)	(1,500)	(1,500)	(1,500)
7	<b>EBIT</b>	(15,000)	9,500	9,500	9,500	9,500	(1,500)
8	Income Tax at 40%	6,000	(3,800)	(3,800)	(3,800)	(3,800)	600
9	<b>Unlevered Net Income</b>	(9,000)	5,700	5,700	5,700	5,700	(900)
<b>Free Cash Flow (\$000s)</b>							
10	Plus: Depreciation	—	1,500	1,500	1,500	1,500	1,500
11	Less: Capital Expenditures	(7,500)	—	—	—	—	—
12	Less: Increases in NWC	—	(2,100)	—	—	—	2,100
13	<b>Free Cash Flow</b>	(16,500)	5,100	7,200	7,200	7,200	2,700

Outflow Inflow



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# Net Working Capital (NWC)

- Definition

$$\begin{aligned}\text{Net Working Capital} &= \text{Current Assets} - \text{Current Liabilities} \\ &= \text{Cash} + \text{Inventory} + \text{Receivables} - \text{Payables}\end{aligned}$$

- Most projects require investment in NWC:

- Cash held at registers, safe box or checking account
- Inventories of raw materials or finished product
- Receivables: earned but not paid (credit offered to customers)
- Payables: spent but not paid (credit received by suppliers)

**Trade credit:** difference between receivables & payables

# Homenet NWC Requirements

	Year	0	1	2	3	4	5
Net Working Capital Forecast (\$000s)							
1	Cash Requirements	–	–	–	–	–	–
2	Inventory	–	–	–	–	–	–
3	Receivables (15% of Sales)	–	3,525	3,525	3,525	3,525	–
4	Payables (15% of COGS)	–	(1,425)	(1,425)	(1,425)	(1,425)	–
5	Net Working Capital	–	2,100	2,100	2,100	2,100	–

Investments in NWC reduce cash available to the firm:

$$\Delta NWC_t = NWC_t - NWC_{t-1}$$

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# Indirect effects and real-world complexities

(not considered here)

- Project Externalities
  - **Cannibalization** is when sales of a new product displaces sales of existing product
  - Would customers of HomeNet have purchased existing Linksys wireless routers?
- Opportunity costs
  - The value a resource could have provided in its best alternative use
  - Homenet's equipment will be housed in an existing lab, but what is the opportunity cost of not using the space in an alternative way (e.g., renting it out)?
- Further,
  - Sales, the average selling price, the average cost per unit will vary over time
- Where should we allocate the \$300,000 of the feasibility study?

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## Part (b): evaluating risk-free projects

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## Part (b): Evaluating risk-free projects

- Methods and rules to decide whether to invest:
  - Net present value rule
  - Internal rate of return rule
  - Payback period and payback rule
  - Profitability index
- Project selection:
  - Mutually exclusive projects
  - Scalable projects with limited resources

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# How to compare present and future?

- One euro today is worth more than one tomorrow!
  
- Why? Possible to earn interest! If interest is 10% a year...
  - Investing 10 million today gives 11 million in a year
  - The future value (in a year) of 10 million is 11 million
  - The present value of 11 million in a year is 10 million

# Future and Present Values

- Future Value: Amount to which an investment will grow after earning interest

$$FV = C_0 \times (1 + r)^t$$

- For example, 10 million after two years will be

$$FV = 10m \times (1 + 0.1)^2 = 12.1m$$

- Present Value: Value today of a future (expected) cash flow

$$PV = \frac{1}{(1 + r)^t} \times C_t$$

Discount factor

- For example, 12.1 million in two years is

$$PV = \frac{1}{(1 + 0.1)^2} \times 12.1m = 10m$$

Discount rate

# Net Present Value: an example

- Cash flows: immediate \$81.6 million “outflow” and an “inflow” of \$28 million per year for 4 years



- Therefore, if discount rate is  $r = 0.10$ , the NPV is:

$$NPV = -81.6 + \frac{28}{(1+0.1)^1} + \frac{28}{(1+0.1)^2} + \frac{28}{(1+0.1)^3} + \frac{28}{(1+0.1)^4}$$

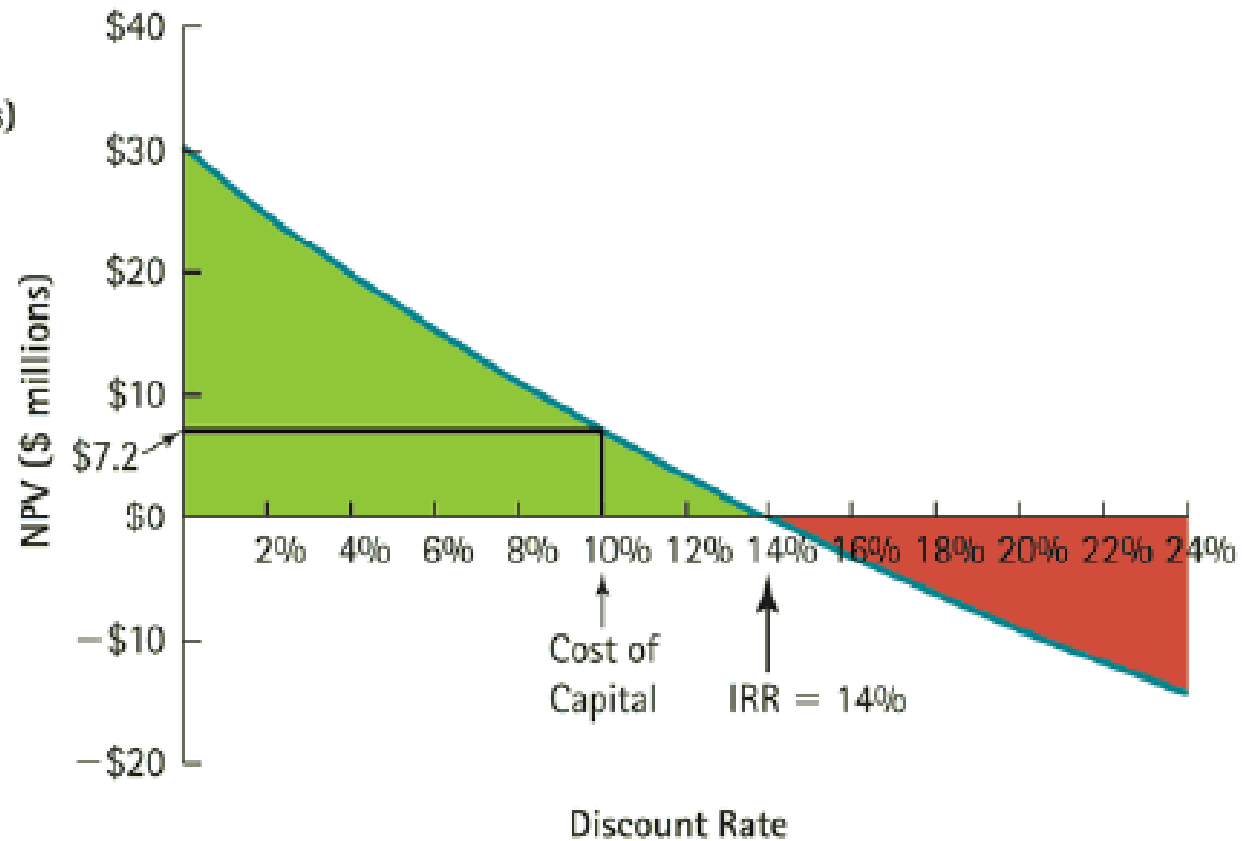
- Discount rate depends on the riskiness of the cash flows:
  - Equal to risk-free rate (government bond) if cash flows are certain
  - Higher risk implies greater discount and lower present value (more on that in part (c) of this chapter)



Panel (a)

Discount Rate	NPV (\$ millions)
0%	\$30.4
2%	\$25.0
4%	\$20.0
6%	\$15.4
8%	\$11.1
10%	\$7.2
12%	\$3.4
14%	\$0.0
16%	-\$3.3
18%	-\$6.3
20%	-\$9.1
22%	-\$11.8
24%	-\$14.3

Panel (b)



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## In general, the NPV rule:

Step 1: Forecast future cash flows

(see part (a) of this chapter)

Step 2: Estimate discount rate

(see part (c) of this chapter)

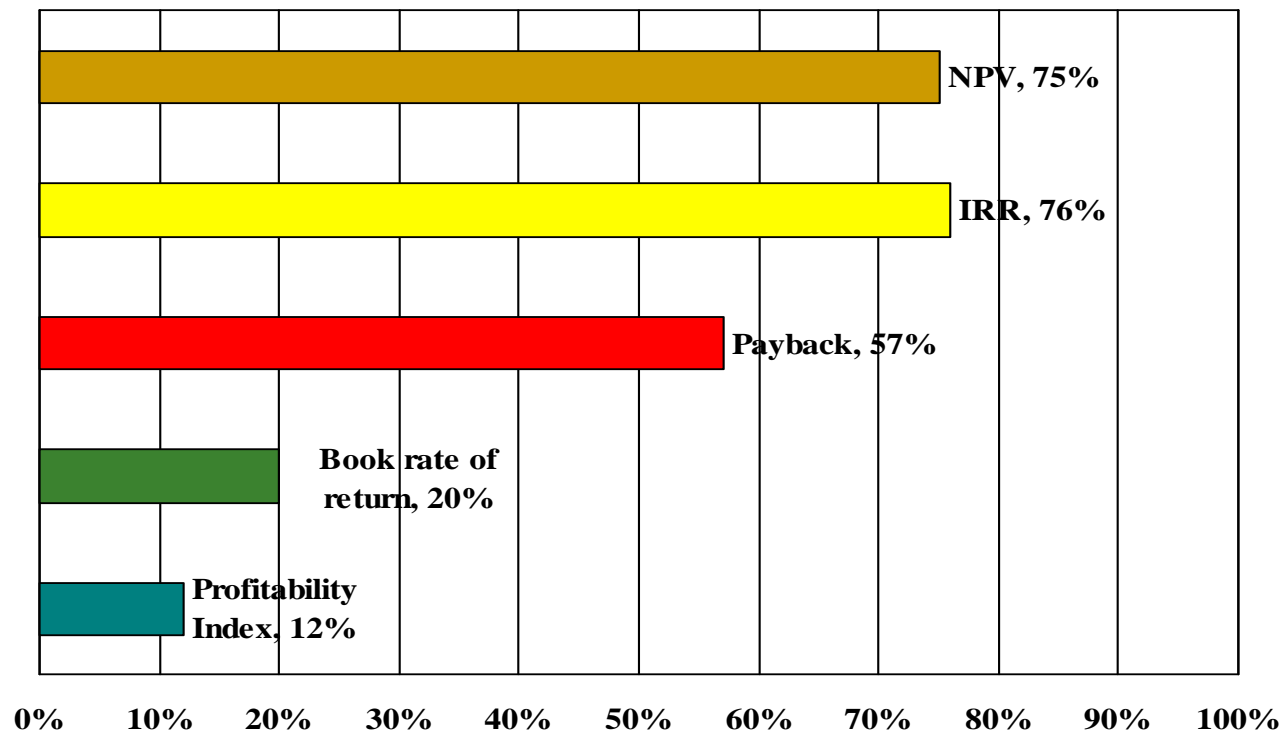
Step 3: Discount future cash flows

$$\text{NPV} = C_0 + \text{PV} = C_0 + \frac{C_1}{1+r_1} + \frac{C_2}{(1+r_2)^2} + \frac{C_3}{(1+r_3)^3} + \dots + \frac{C_T}{(1+r_T)^T}$$

Step 4: Go ahead if PV of payoff exceeds investment, i.e. if  $\text{NPV} > 0$

# But, are there other criteria?

## Survey Data on CFO Use of Investment Evaluation Techniques



SOURCE: Graham and Harvey, "The Theory and Practice of Finance: Evidence from the Field,"  
Journal of Financial Economics 61 (2001), pp. 187-243.

# Rate of return: an example

1. Buy and sell a firm (a project). Asset value in two subsequent periods:
  - ▣  $AV_0$ : 80m and  $AV_1$ : 96.8m
  - ▣ Return:  $r = (96.8 - 80)/80 = 0.21$  or 21%
2. Value in two non-subsequent periods:
  - ▣  $AV_0$ : 80m and  $AV_2$ : 96.8m, return: ?
  - ▣ Numerical method: find  $r$  such that Net Present Value (NPV) = 0

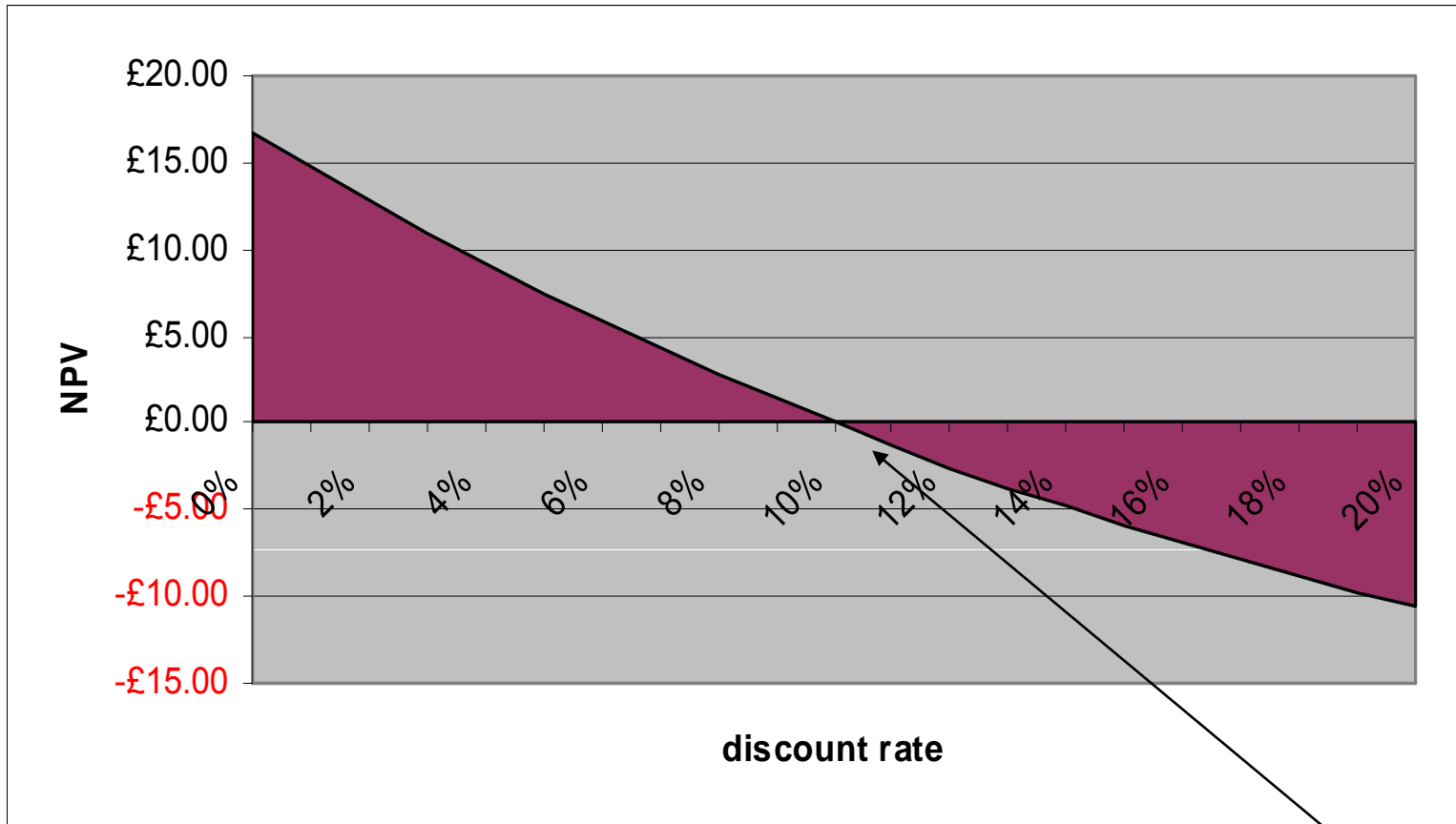
$$NPV = -80 + \frac{96.8}{(1+r)^2} = 0 \quad \text{or} \quad r = 0.10 = 10\%$$

- ▣ In other words,

$$80(1 + 0.1)(1 + 0.1) = 96.8$$

- ▣ What is the rate of return in the first example?

# Example 2



Rate of return: 10%

# Introducing revenues and costs

- No revenues:

- $AV_0$ : 80m and  $AV_2$ : 96.8m

$$NPV = -80 + \frac{96.8}{(1+r)^2} = 0 \quad \text{or } r = 10\%$$

- With revenues:

- $AV_0$ : 80m,  $AV_2$ : 96.8m,  $R_1$ : 2m,  $R_2$ : 2m

$$NPV = -80 + \frac{2}{(1+r)^1} + \frac{2}{(1+r)^2} + \frac{96.8}{(1+r)^2} = 0 \quad \text{or } r = 12.4\%$$

- With revenues and costs:

- $AV_0$ : 80m,  $AV_2$ : 96.8m,  $R_1$ : 2m,  $R_2$ : 2m,  $C_1$ : 1m,  $C_2$ : 1.2m

$$NPV = -80 + \frac{1}{(1+r)^1} + \frac{0.8}{(1+r)^2} + \frac{96.8}{(1+r)^2} = 0 \quad \text{or } r = 11.08\%$$

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## In general, the Rate of Return Rule:

- More generally, the internal rate of return of a cash flow stream is the interest rate  $y$  that makes the NPV of a project equal to 0:

$$0 = C_0 + \frac{C_1}{1+y} + \frac{C_2}{(1+y)^2} + \frac{C_3}{(1+y)^3} + \dots + \frac{C_T}{(1+y)^T}$$

- Accept investments offering rates of return in excess of the appropriate discount rate (“opportunity cost of capital”)

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# IRR and NPV

- Same criteria if NPV is decreasing wrt discount rate
- However, the IRR has some pitfalls:
  - If NPV increases (“lending” money instead of “borrowing”), we should ask for an IRR lower than opportunity cost of capital
  - There might be several IRRs or none
  - Ignores magnitude and cannot select among different projects
  - Even more problematic if we discount rates are not stable over time (with which one do we compare?)



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# Payback period and the payback rule

- The payback period is the number of periods (years) it takes before the cumulative forecasted cash flow equals the initial outlay
- The payback rule says only accept projects that “payback” in the desired time frame
- This method is flawed, primarily because it ignores later year cash flows and the present value of future cash flows

# Example

*Examine the three projects and note the mistake we would make if we insisted on only taking projects with a payback period of 2 years or less.*

Project	$C_0$	$C_1$	$C_2$	$C_3$	Payback Period	NPV@ 10%
A	-2000	500	500	5000	3	+ 2,624
B	-2000	500	1800	0	2	- 58
C	-2000	1800	500	0	2	+ 50

# Project Selection

- If only one from a set of positive NPV projects can be selected, we should select that with the largest NPV
- When resources are limited, the profitability index (PI) helps selecting among various project combinations and alternatives:
  - $PI = (NPV - C_0) / (-C_0) = PV / (-C_0)$
  - If resources are unlimited, we should select projects with  $PI > 1$ . Why?
  - Limited resources and projects can yield various combinations
  - Example: Eur100,000 for two scalable projects (numbers in Eur1,000)

Project	$C_0$	$C_1$	$C_2$	$PV @ 10\%$	$PI$
<i>A</i>	-1	+22	-12.1		
<i>B</i>	-4	+44	-24.2		

# Project Selection

- If only one from a set of positive NPV projects can be selected, we should select that with the largest NPV
- When resources are limited, the profitability index (PI) helps selecting among various project combinations and alternatives:
  - $PI = (NPV - C_0) / (-C_0) = PV / (-C_0)$
  - If resources are unlimited, we should select projects with  $PI > 1$ . Why?
  - Limited resources and projects can yield various combinations
  - Example: Eur100,000 for two scalable projects (numbers in Eur1,000)

Project	$C_0$	$C_1$	$C_2$	$PV @ 10\%$	$PI$
<i>A</i>	-1	+22	-12.1	10	10
<i>B</i>	-4	+44	-24.2	20	5

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Part (c): adjusting for risk!

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## Risky cash flows

Future cash flows should now be “expected cash flows”

Discount rate may need to be higher than risk-free rate

Example: a project expected to generate CF=\$100million per year for three years. PV for a discount rate  $r = 12\%$ ?

Project A		
Year	Cash Flow	PV @ 12%
1	100	89.3
2	100	79.7
3	100	71.2
PV Total		240.2

# Separating adjustments for risk and time?

If risk-free rate  $r_f = 6\%$ , which cash-flow reduction would you accept to get them with certainty (i.e. what is your certainty equivalent cash flow)?

$$\frac{CEQ_1}{1.06} = 89.3 \text{ or } CEQ_1 = 94.6$$

$$\frac{CEQ_t}{(1+r_f)^t} = \frac{C_t}{(1+r)^t} = PV$$

Project A		
Year	Cash Flow	PV @ 12%
1	100	89.3
2	100	79.7
3	100	71.2
PV Total		240.2

Project B		
Year	Cash Flow	PV @ 6%
1	94.6	89.3
2	89.6	79.7
3	84.8	71.2
PV Total		240.2

## Cash flow reductions

Year	Cash Flow	CEQ	Risk deduction
1	100	94.6	5.4
2	100	89.6	10.4
3	100	84.8	15.2

Larger risk deduction for later periods

Not necessary to discount at higher rates distant periods to generate a larger risk deduction



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## Risk premium

Risk premium makes risky and risk-free cash flows equally attractive

Formally, risk premium can be defined as the  $r_p$  such that...

$$\frac{C_t}{(1 + r_p)^t} = CEQ_t$$

In our case,

$$r_p = \frac{100}{94.6} - 1 = 0.57 \text{ or } 5.7\%$$

# Risk premium

Given that certainty-equivalent is defined as

$$\frac{CEQ_t}{(1+r_f)^t} = \frac{C_t}{(1+r)^t}$$

and risk-premium is given by

$$\frac{C_t}{(1+r_p)^t} = CEQ_t$$

We have that risk premium is given by

$$(1+r) = (1+r_f)(1+r_p)$$

and approximately...

$$r \approx r_f + r_p$$

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## Finding the discount rate or the “risk premium”

- Project’s cash flows should be appropriately discounted:
  - what is the return the firm can receive on similar but alternative investments (i.e. investments that bear the same risks?)
  - discount rate sometimes called “cost of capital” as it measures the opportunity cost of the funds
- How to compute the “cost of capital” of the project?
- Often start computing the whole firm’s cost of capital:
  - Loads of projects have similar risk as the firm as a whole
  - If not, good starting point that can be adjusted for:
    - If project has higher risk relative to the firm as a whole
    - For example, if project has high fixed costs, it will have more risk

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## Finding the discount rate and the “risk premium”

- Estimate the firm’s cost of capital:
  - First, suppose that firm is financed with equity only
  - Second, incorporate possibility that it has debt
- In this part (c), compute a firm’s expected equity return:
  - Basic tools for “portfolio” theory (risk-return trade-off)
  - Mean variance analysis and portfolio representation
  - The Capital Asset Pricing model (CAPM)
  - Factor models and the Arbitrage Pricing Theory (APT)
- In the next chapter: incorporate debt and compute the weighted average cost of capital (WACC)

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# Portfolio Tools and Diversification

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# Constructing “Portfolios”

- Investing in multiple stocks: constructing a “portfolio”

- Portfolio weight for stock  $j$ :  $x_j = \frac{\text{Dollars held in stock } j}{\text{Dollar value of the portfolio}}$

- Example of a portfolio: £100 in BT and £300 in BP

$$(x_{BT}, x_{BP}) = (1/4, 3/4)$$

- Properties:

- Weights should add up to 1
- Weights can be either positive (“long position”) or negative (“short”)

# Remember?

- Investment return (historical return) :  $r_{i,t} = \frac{p_t + d_t - p_{t-1}}{p_{t-1}}$   
 $r_1, r_2, r_3, \dots, r_N$  OR  $r_{BT}, r_{BP}$
- Expected return (forward looking):  
 $E(r_1), \dots, E(r_N)$  OR  $\overline{r_1}, \dots, \overline{r_N}$  OR  $\overline{r_{BT}}, \overline{r_{BP}}$
- Variance and standard deviation of an investment:  
 $\text{var}(r_i) = \sigma_i^2 = E[(r_i - \overline{r_i})^2]$        $\sigma_i = \sqrt{\text{var}(r_i)}$ 
  - Standard deviation has same units as returns
- Covariance of two investments 1 and 2:  
 $\text{cov}(r_i, r_j) = \sigma_{i,j} = E[(r_i - \overline{r_i})(r_j - \overline{r_j})]$ 
  - Interpretation: measure of relatedness. Move together?
  - Depends on units but...

# Remember?

- A useful measure of the co-movement of two returns is the correlation coefficient  $\rho$ .

- $$\rho_{i,j} = \frac{\text{COV}(r_i, r_j)}{\sigma_i \sigma_j} = \frac{\sigma_{i,j}}{\sigma_i \sigma_j} \quad \text{and} \quad \rho_{i,j} \in [-1, 1]$$

- When  $\rho_{i,j} = 1$  (or  $-1$ ), the assets' returns are perfectly positively (or negatively) correlated, i.e. always move together (or in opposite directions)
- When  $\rho_{i,j} = 0$ , the assets' returns are uncorrelated



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# Expected Return

- Portfolio return:
  - Portfolio-weighted average of returns of assets in the portfolio
  - Example: if BT's return has been 10% and BP's 5% and portfolio (0.25,0.75) then...
  - Portfolio return:  $0.25 \cdot 0.10 + 0.75 \cdot 0.05 = 0.0625$  or 6.25%
- Expected portfolio return:
  - Portfolio-weighted average of expected returns
  - If the portfolio is  $P = (x_1, \dots, x_N)$  then

$$E(r_P) = \sum_{i=1}^N x_i \bar{r}_i$$

# Variances and Covariances of a Portfolio

- For any two-stock portfolio...

$$\sigma_p^2 = \text{var}(x_1 r_1 + x_2 r_2) = x_1^2 \sigma_1^2 + x_2^2 \sigma_2^2 + 2x_1 x_2 \sigma_{1,2}$$

- Hence... larger covariance leads to higher portfolio variance

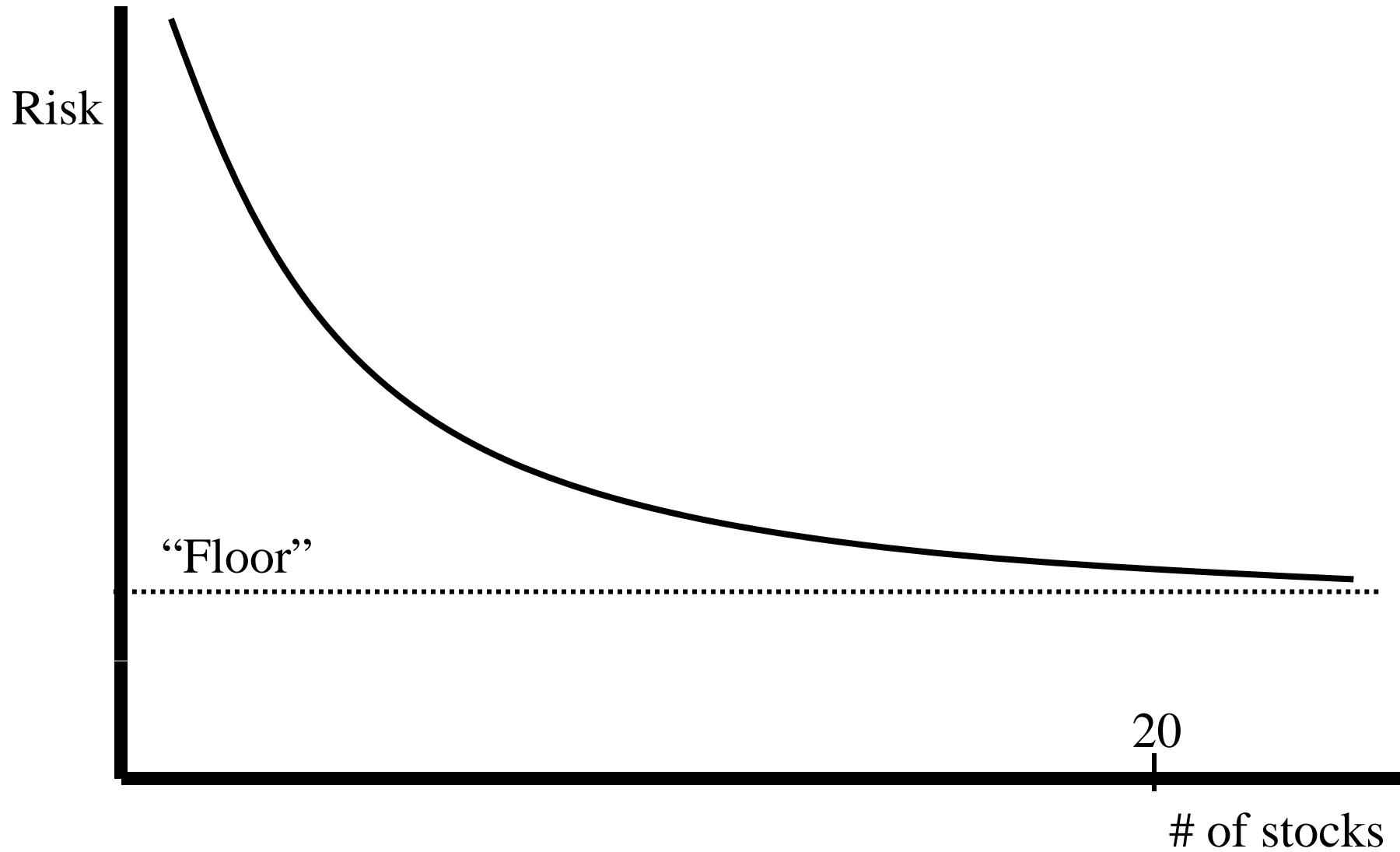
$$= x_1^2 \sigma_1^2 + x_2^2 \sigma_2^2 + 2x_1 x_2 \rho \sigma_1 \sigma_2$$

$$\leq x_1^2 \sigma_1^2 + x_2^2 \sigma_2^2 + 2x_1 x_2 \sigma_1 \sigma_2 = (x_1 \sigma_1 + x_2 \sigma_2)^2$$

- With strict inequality if  $\rho < 1$
- Thus..

$$\sigma_p \leq x_1 \sigma_1 + x_2 \sigma_2$$

# How Large Diversification Benefits are?



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# Mean Variance Analysis

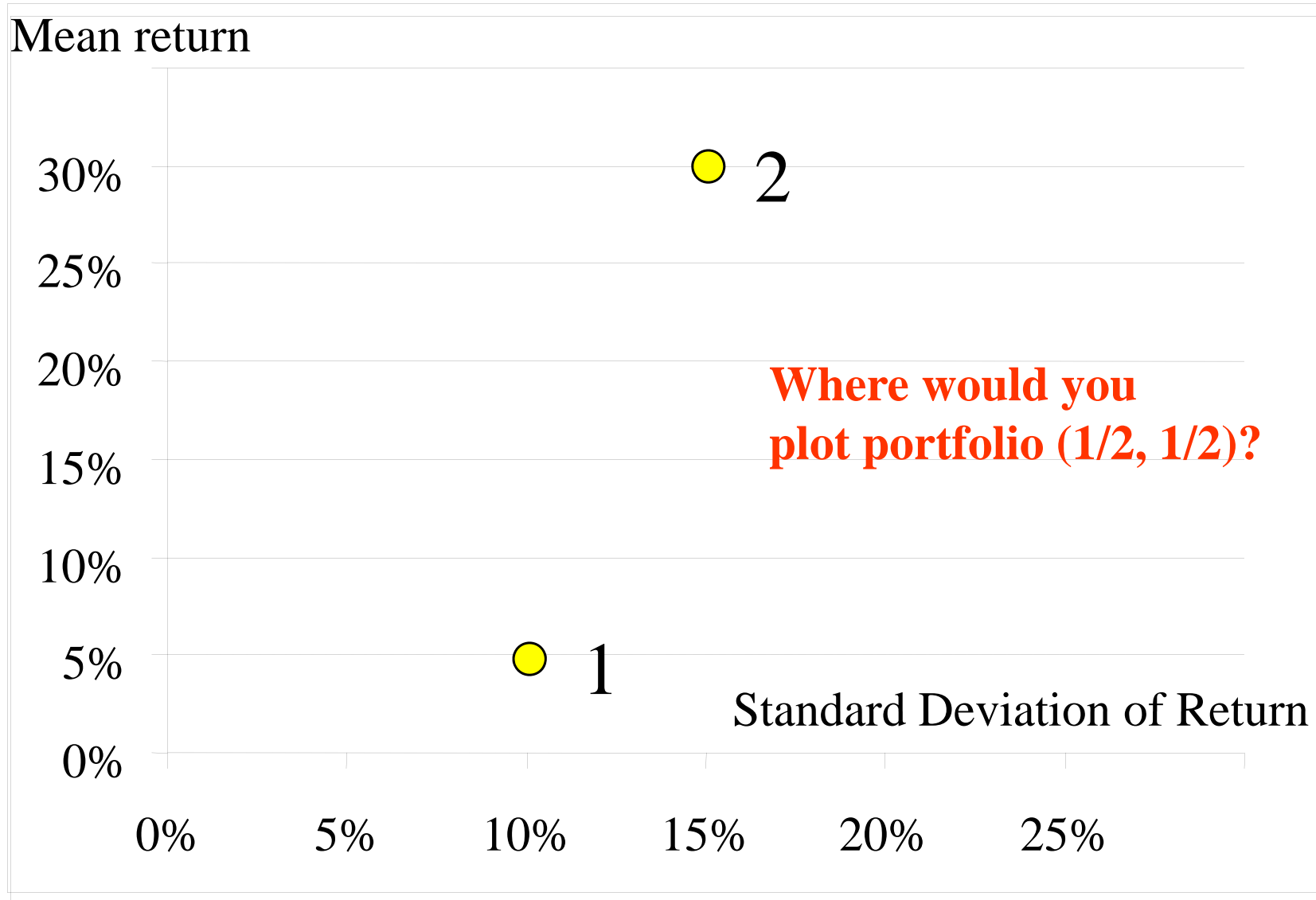
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# Portfolio problem

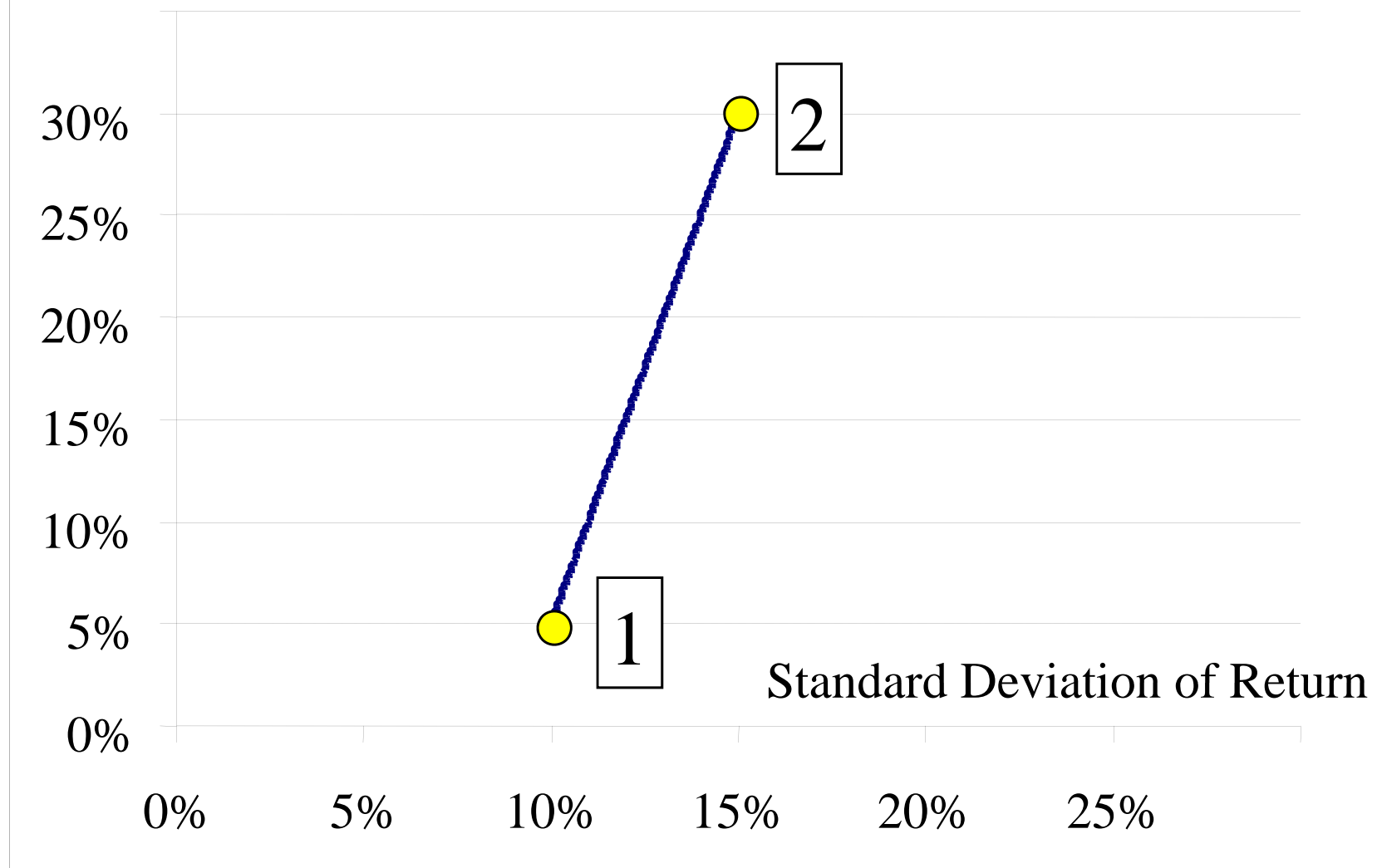
- How best to combine many assets in order to...
  - maximize expected return for a given variance (i.e. risk)
  - minimize variance (i.e. risk) for a given expected return.
- In other words, how to construct the set of “mean-variance efficient” portfolios?
- We assume frictionless markets:
  - all investments are tradable in any quantity (no restrictions on short-positions)
  - no transaction costs, regulations or tax consequences
- Assume first all assets are risky and then introduce risk-free asset

# Representation: Mean-Variance Diagrams

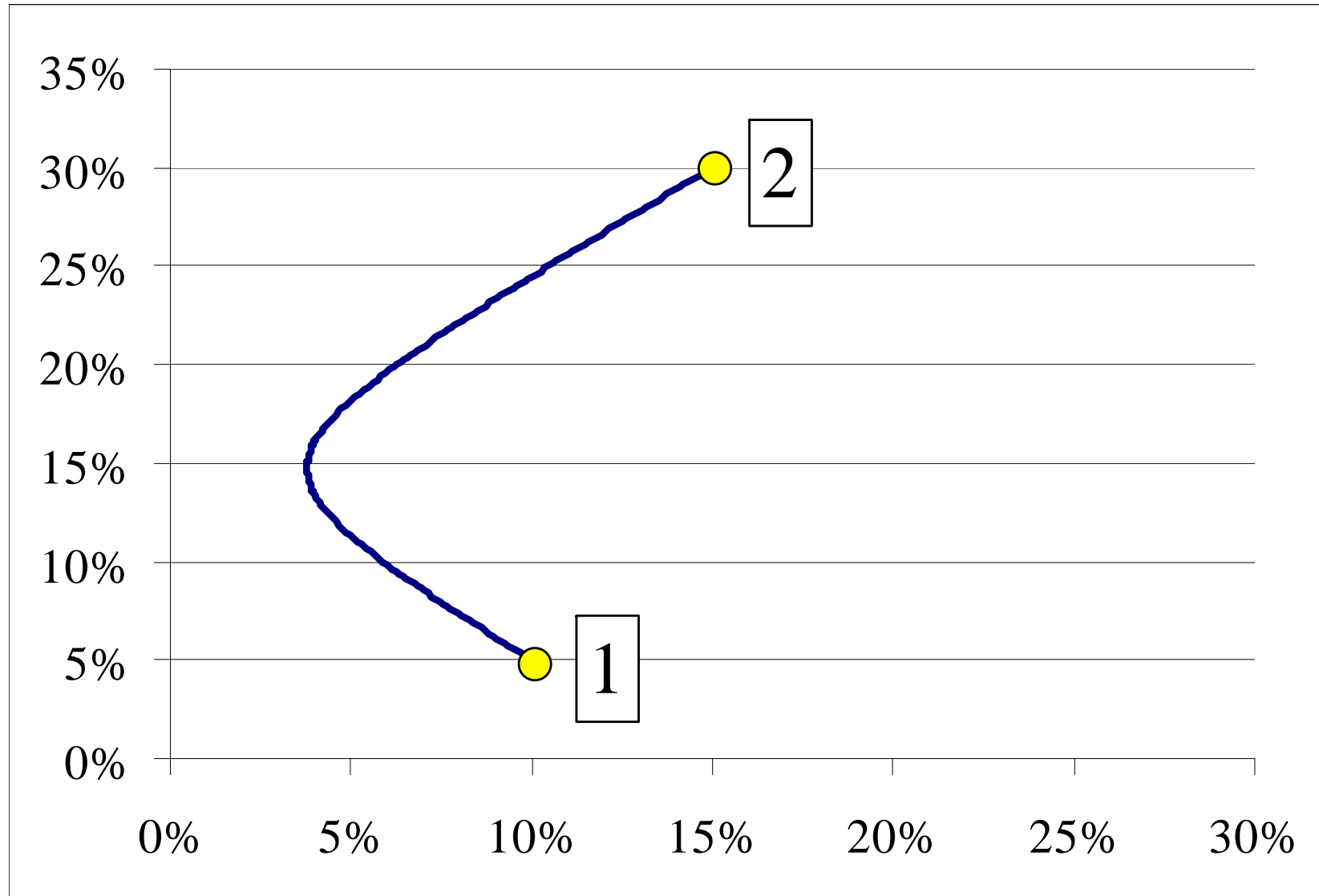


# Portfolio of two risky assets (Perfect Correlation)

Mean return

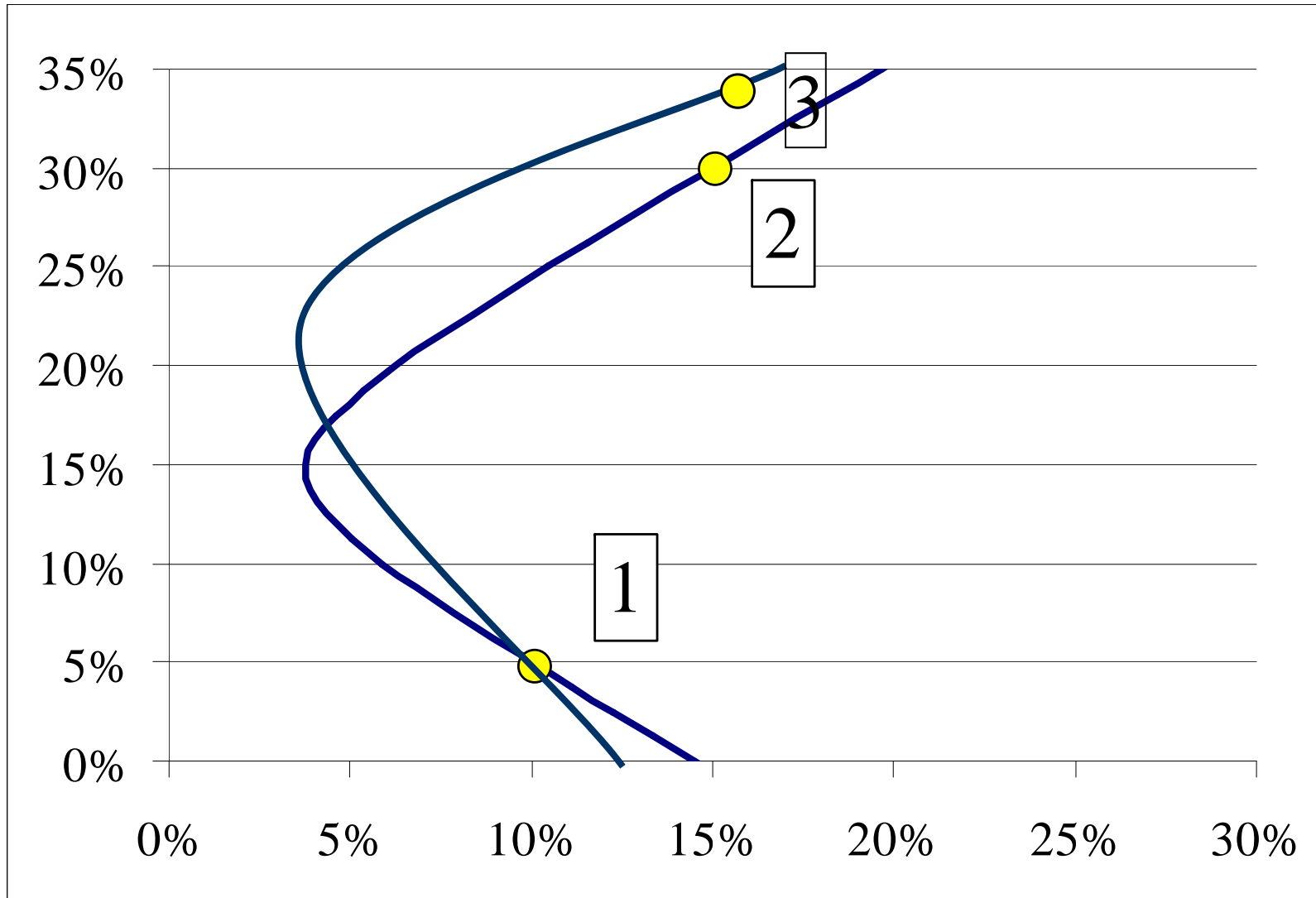


## Portfolio of two risky assets (Imperfect Correlation)

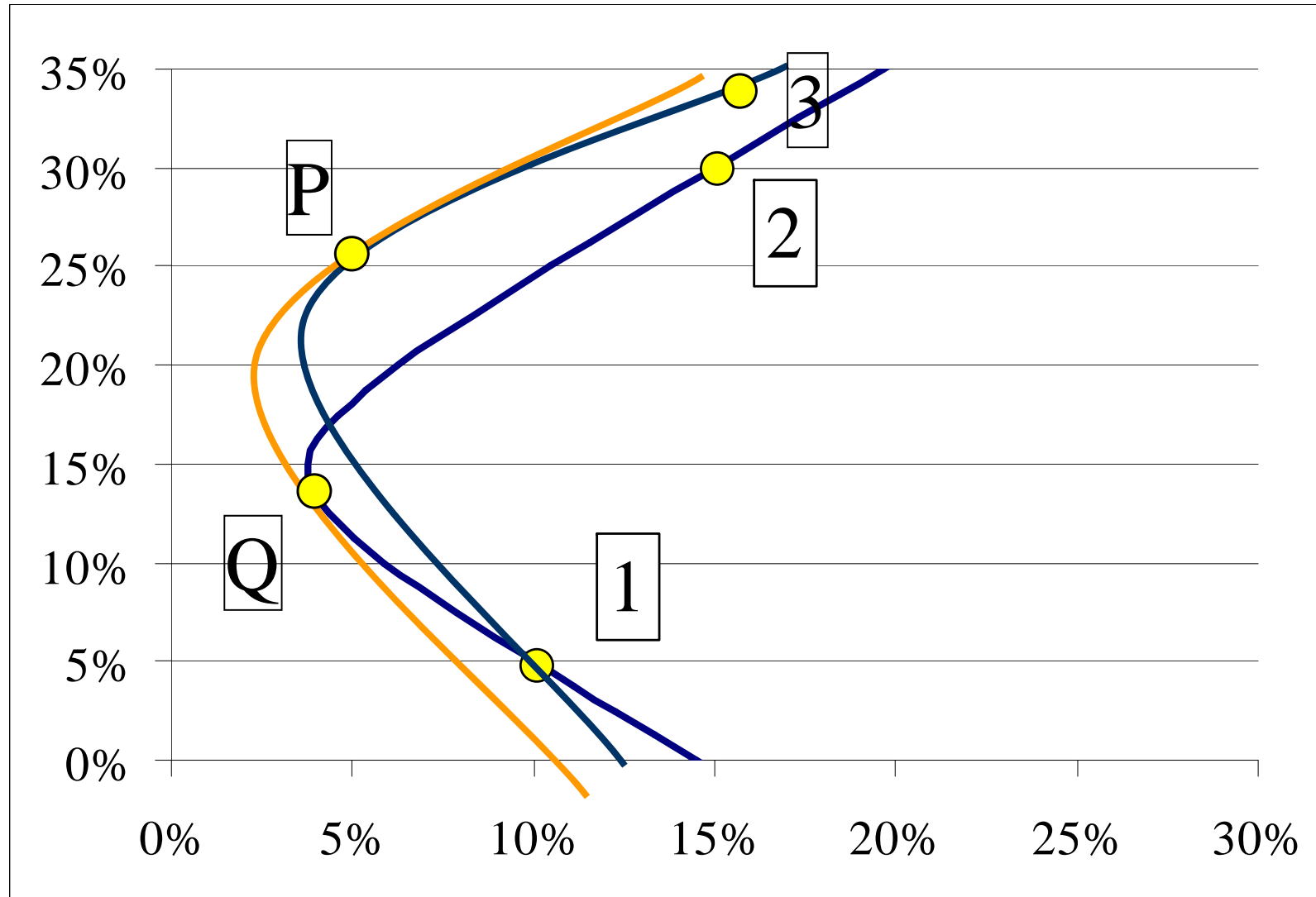


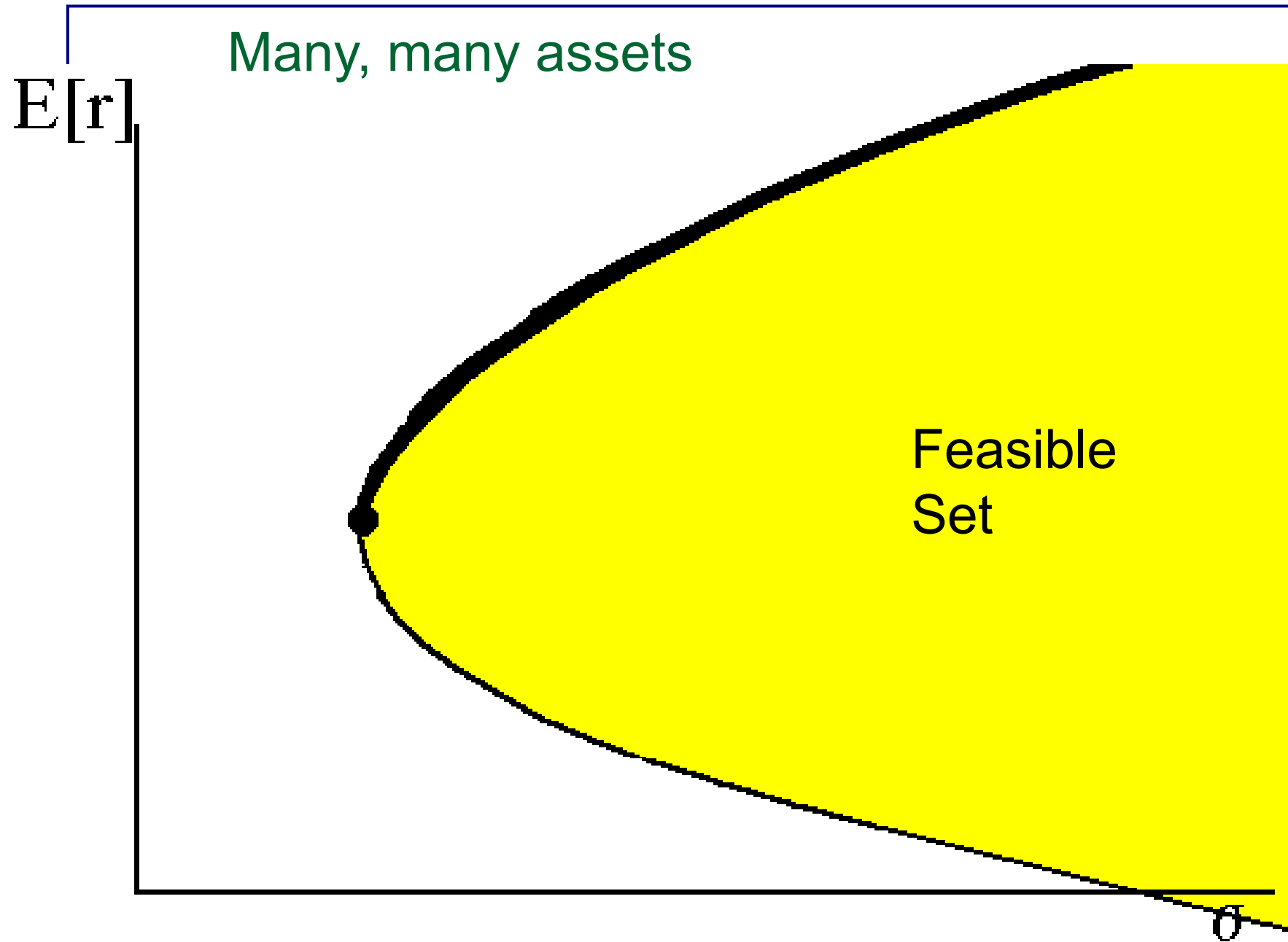


# New Asset



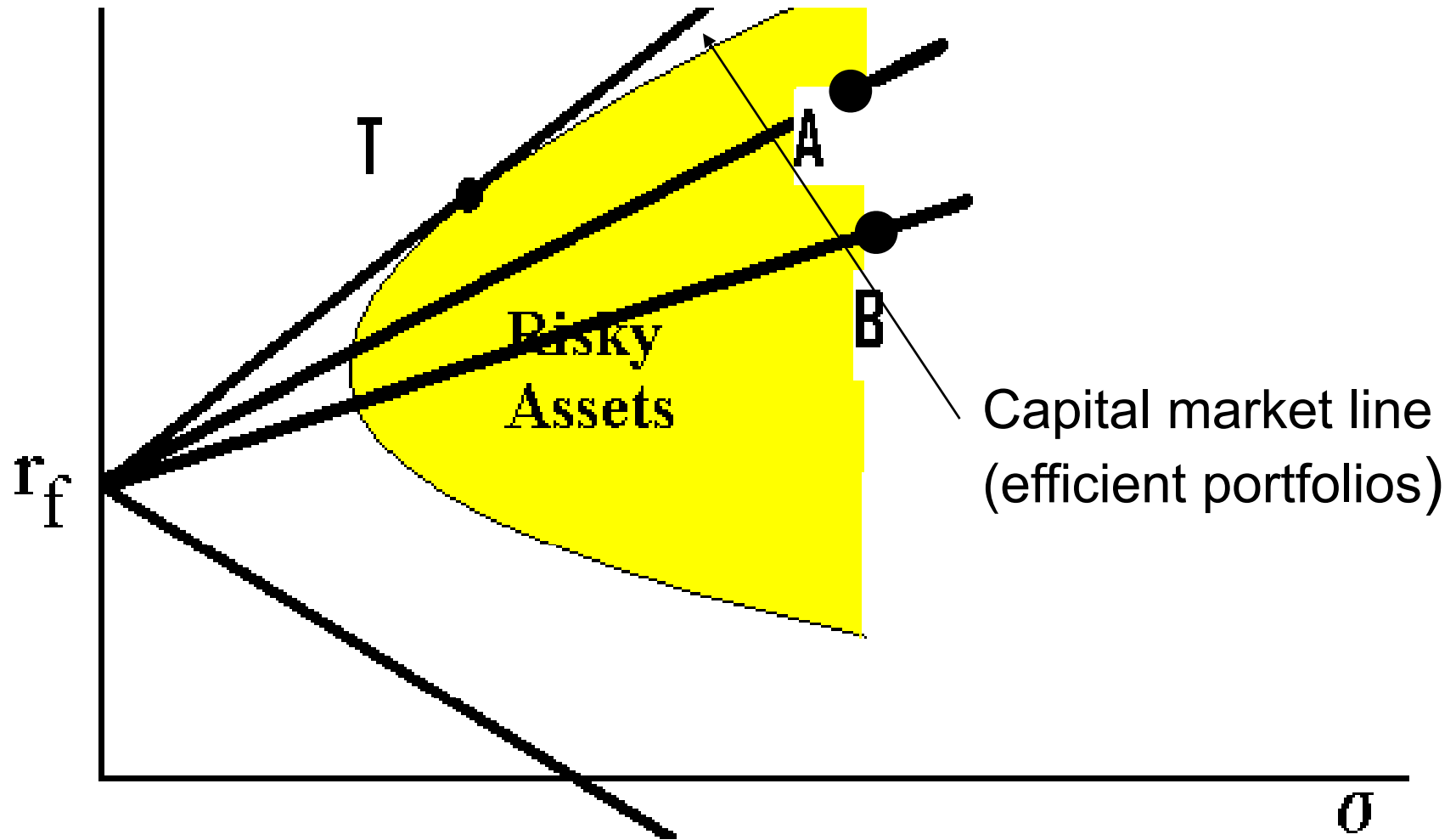
# Portfolio of Portfolios is another Portfolio!

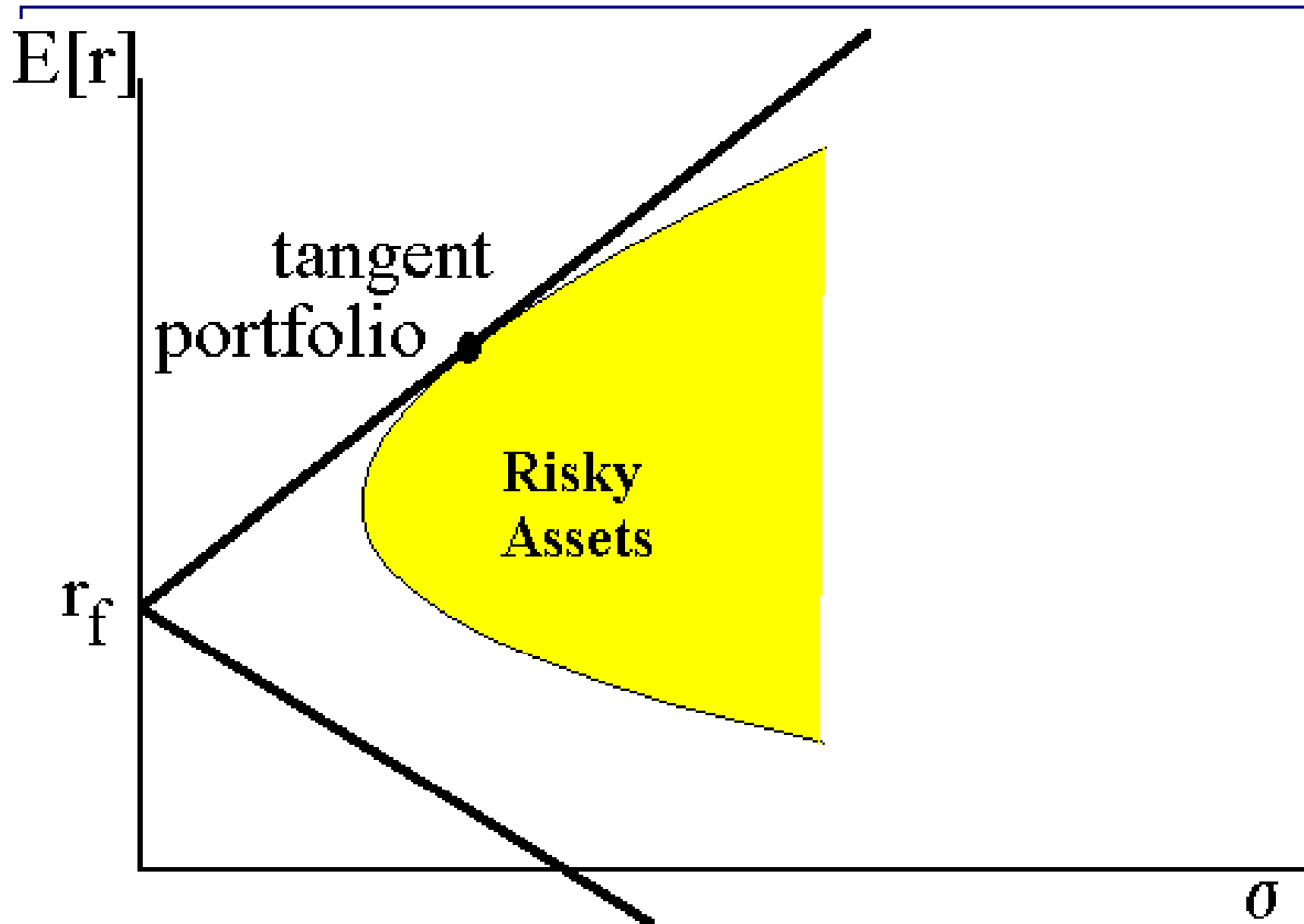




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# Adding a Risk-free Asset





## Nice Mathematical Result: Risk & Return

- For any investment  $i$ , it *can be shown* that

$$E[r_i] - r_f = \beta_{iT} (E[r_T] - r_f) \text{ where } \beta_{iT} = \frac{\sigma_{i,T}}{\sigma_T^2}$$

$$E[r_i] = r_f + \beta_{iT} (E[r_T] - r_f) \text{ where } \beta_{iT} = \frac{\sigma_{i,T}}{\sigma_T^2}$$

- Tangency portfolio key to relate expected return on any investment with a measure of its risk: the covariance.
- Allows us to estimate risk premium of any asset!

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# Finding the Tangency Portfolio

- Derive solving complex matrix algebra:
  - Relatively easy for investment across countries or asset types
  - Low number of investments and good parameter estimates
- But may be computationally demanding (or impossible):
  - Need to estimate all means and covariances
  - Difficult for individual investment selection: there are loads!
- CAPM tell us which should be the tangency portfolio

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# The CAPM

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# Assumptions and Conclusion

1. Markets are frictionless
  2. There is a risk-free asset that returns  $r_f$
  3. All investors want to hold efficient frontier portfolios;
  4. Supply equals demand in financial markets (we are in *equilibrium*)
  5. Investors have homogenous beliefs about means and st deviations
- These assumptions are sufficient to apply previous results
  - And, we also get: tangent portfolio is given by the market portfolio (m):
    - the portfolio of all risky assets, where the weight of each asset is the market value (market capitalisation) divided by the total market value
    - Examples of rough “approximations” of the market portfolio?

- Substituting in the previous formula

$$E[r_i] = r_f + b_i (E[r_m] - r_f) \quad \text{where } b_i = b_{mi} = \frac{\sigma_{mq}}{\sigma_m^2}$$

# Computing the market portfolio

Three-stock economy: HP, IBM, CPQ and US Treasuries.

	HP	IBM	CPQ
Price per share	\$33	\$95	\$20.25
Shares outstanding	2 bill	1.758 bill	1.7 bill

- Market portfolio: (UST, HP, IBM, CPQ) = (0, 0.25, 0.62, 0.13)  
Risk free asset: (UST, HP, IBM, CPQ) = (1, 0, 0, 0)
- All investors should hold a combination of the risk-free and the market portfolio (i.e. the same relative positions in risky assets)

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# Extensions: Factor Models and APT

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# Factor models

- The return of a risky investment is determined by:
  - Common factors (e.g. interest rates, inflation, productivity...)
  - A firm-specific component (new R&D results, fire in a plant,...)
- Return variances of large portfolios are determined by common factors, firm-specific ones can often be ignored
- Common factors do not affect all investments equally: each has its sensitivities to the factors (“factor betas”)
  - E.g stock of car company more sensitive to changes in interest rate than stock of a soft drink firm
  - Car companies are highly affected by interest rate (factor) risk
- Factor models can be used to estimate the expected rate of return of an investment, as an alternative to the CAPM:
  - Arbitrage pricing theory: relation of factor risk to expected return

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# A One-Factor Model: the Market Model

- Run the following regression:

$$r_{Dell} = \alpha_{Dell} + \beta_{Dell} r_{S\&P500} + \varepsilon_{Dell}$$

- If  $r_{S\&P}$  and  $\varepsilon_{Dell}$  are uncorrelated:

$$\sigma_{Dell}^2 = \beta_{Dell}^2 \sigma_{S\&P500}^2 + \sigma_{\varepsilon_{Dell}}^2$$

- Risk can be divided in two:
  - Systematic, market risk: part explained by market movements
  - Unsystematic risk: part not explained by market movements

# Unsystematic and Diversifiable Risk

- Unsystematic risk may be related to other factor risks:
  - Car company highly affected by interest rate risk
  - Part of this effect shows up in the residual of previous equation
  - As a result, not all unsystematic risk is diversifiable
- However, if for all the investments  $i$  we had

$$r_i = \alpha_i + \beta_i r_m + \varepsilon_i$$

such that all  $\varepsilon_i$  were uncorrelated then  $\varepsilon_i$  would be firm-specific and therefore the related risk would be diversifiable

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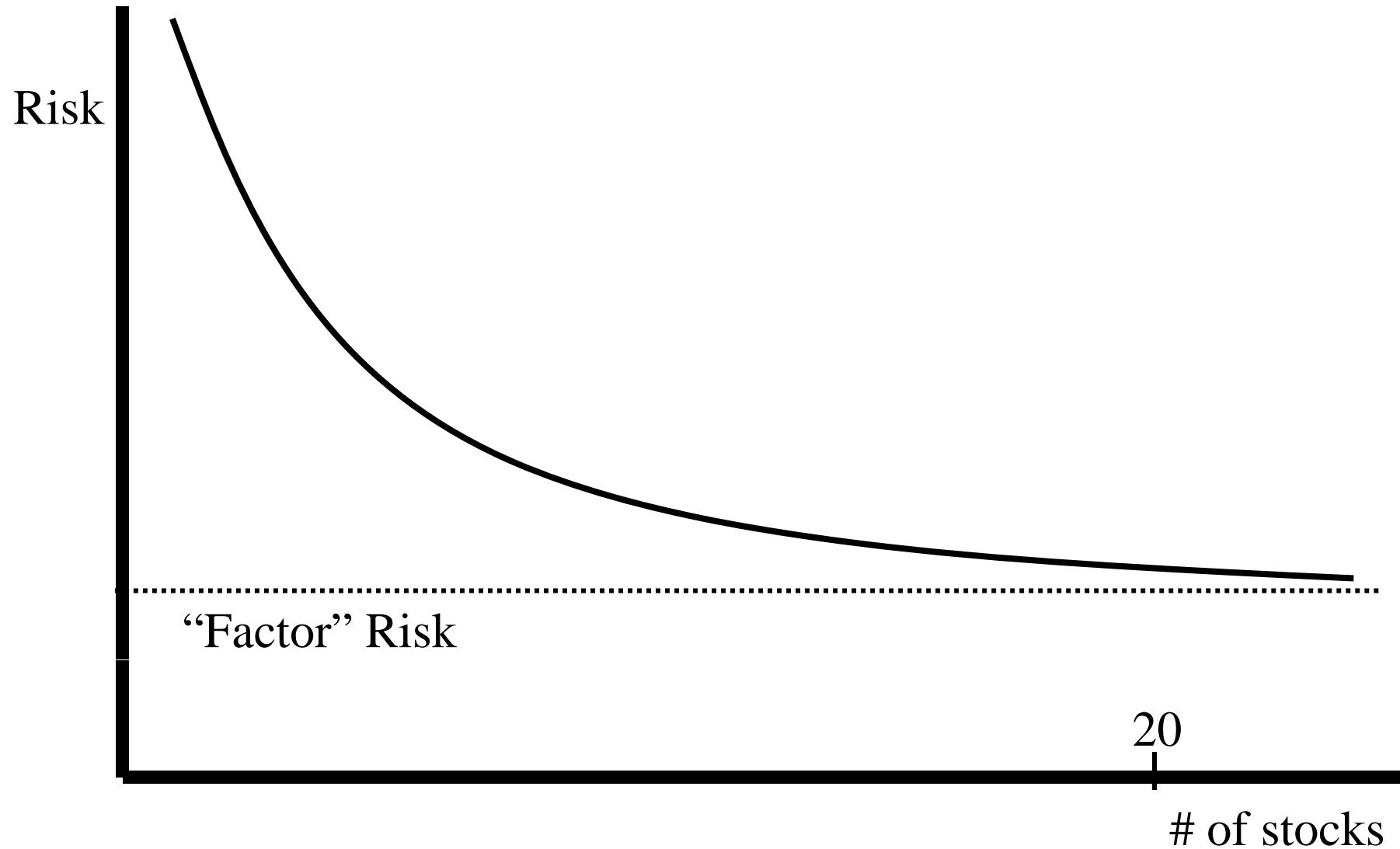
# More Factors

- The last assumption however is unrealistic
- Need more factors. More generally, one could specify:

$$r_i = \alpha_i + \beta_{i,1} r_{\text{Factor 1}} + \dots + \beta_{i,K} r_{\text{Factor K}} + \varepsilon_i$$

- Returns are assumed to be generated by relatively small number of factors
- Betas are the sensitivities to each factor
- $\varepsilon_i$  are uncorrelated firm-specific components
- Factors:
  - Other examples: industrial production, oil prices,..
  - Usually rescaled to have mean of zero
- Risk from...
  - Common factors cannot be eliminated by diversification
  - Unique factors can be eliminated and should be ignored

# How Large are Diversification Benefits?





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# Arbitrage Pricing Theory

- Under a set of assumptions:

$$E[r_i] - r_f = \beta_{i,1}(E[r_{Factor1}] - r_f) + \dots + \beta_{i,K}(E[r_{FactorK}] - r_f)$$

- A diversified portfolio with 0 sensitivity to each macro factor...
  - Is essentially risk-free and should offer no market premium
  - If the return is higher or lower than the risk-free rate then profits can be made by arbitrage
- A diversified portfolio with sensitivity to the factors...
  - Should offer a risk premium proportional to its sensitivity to the factor
  - Otherwise, profits from arbitrage can be made!

# Example: Arbitrage Pricing Theory

## Estimated risk premiums for taking on risk factors (1978-1990)

Factor	Estimated Risk Premium
	$E[r_{\text{factor}}] - r_f$
Yield spread	5.10%
Interest rate	-.61
Exchange rate	-.59
Real GNP	.49
Inflation	-.83
Market	6.36