Corporate Finance

Lecture 5: Factor Models and APT

Albert Banal-Estanol

Factor models

- The return of a risky investment is determined by:
  - Common factors (e.g. interest rates, inflation, productivity…)
  - A firm-specific component (new R&D results, fire in a plant,…)
- Return variances of large portfolios are determined by common factors, firm-specific ones can often be ignored
- Common factors do not affect all investments equally: each has its sensitivities to the factors ("factor betas")
  - E.g stock of car company more sensitive to changes in interest rate than stock of a soft drink firm
  - Car companies are highly affected by interest rate (factor) risk
- Factor models can be used to estimate the expected rate of return of an investment, as an alternative to the CAPM:
  - Arbitrage pricing theory: relation of factor risk to expected return

A One-Factor Model: the Market Model

- Run the following regression:

$$r_{Dell} = \alpha_{Dell} + \beta_{Dell} r_{S\&P500} + \varepsilon_{Dell}$$

- If $r_{S\&P}$ and ε_{Dell} are uncorrelated:

$$\sigma_{Dell}^2 = \beta_{Dell}^2 \sigma_{S\&P500}^2 + \sigma_{\varepsilon_{Dell}}^2$$

- Risk can be divided in two:
  - Systematic, market risk: part explained by market movements
  - Unsystematic risk: part not explained by market movements

Unsystematic and Diversifiable Risk

- Unsystematic risk may be related to other factor risks:
  - Car company highly affected by interest rate risk
  - Part of this effect shows up in the residual of previous equation
  - As a result, not all unsystematic risk is diversifiable
- However, if for all the investments i we had

$$r_i = \alpha_i + \beta_i r_{\mathrm{m}} + \varepsilon_i$$

such that all ε_i were uncorrelated then ε_i would be firm-specific and therefore the related risk would be diversifiable

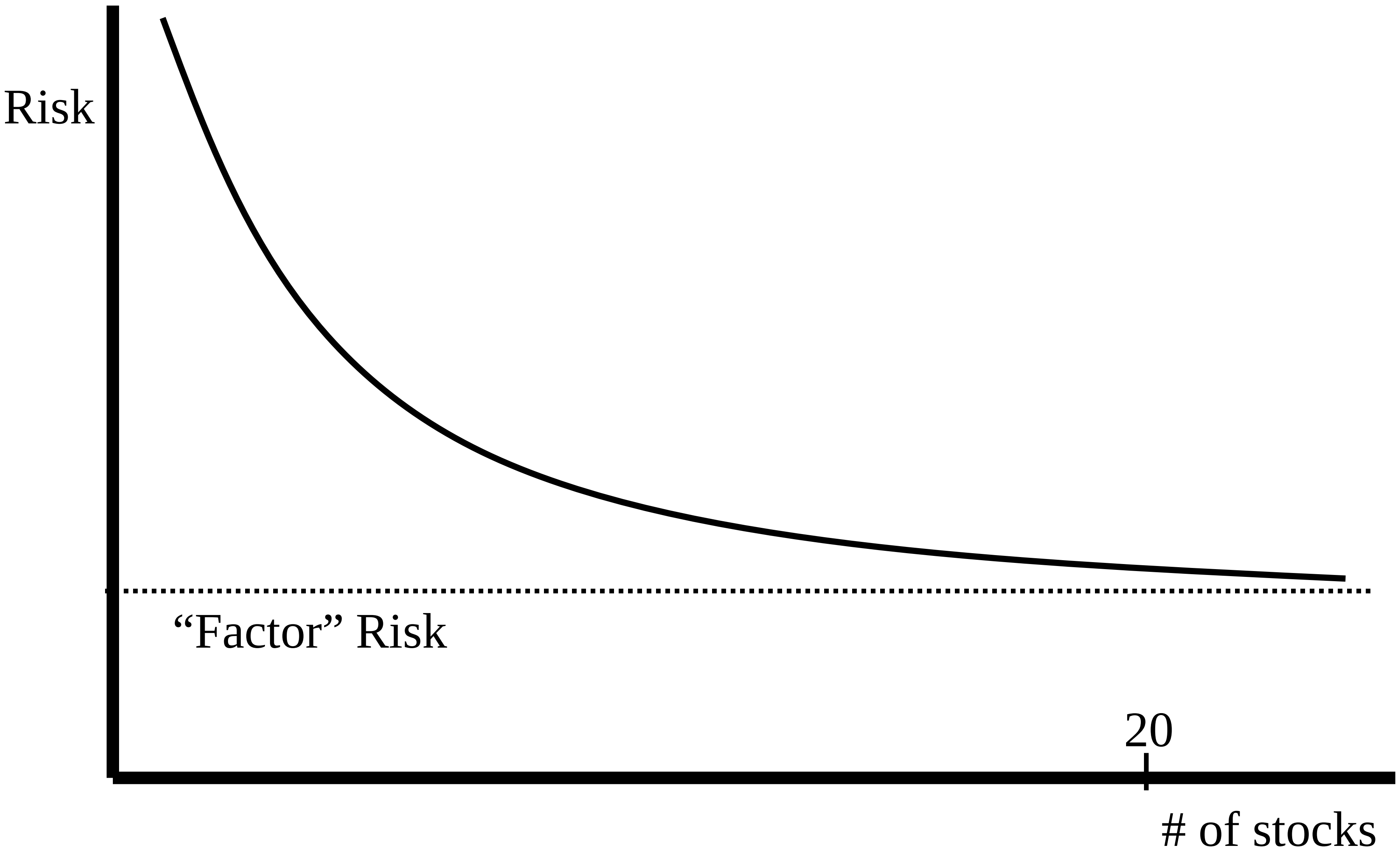
More Factors

- The last assumption however is unrealistic
- Need more factors. More generally, one could specify:

$$r_i = \alpha_i + \beta_{i,1} r_{\text{Factor 1}} + \ldots + \beta_{i,K} r_{\text{Factor K}} + \varepsilon_i$$

  - Returns are assumed to be generated by relatively small number of factors
  - Betas are the sensitivities to each factor
  - ε_i are uncorrelated firm-specific components
- Factors:
  - Other examples: industrial production, oil prices,..
  - Usually rescaled to have mean of zero
- Risk from…
  - Common factors cannot be eliminated by diversification
  - Unique factors can be eliminated and should be ignored

How Large are Diversification Benefits?



Arbitrage Pricing Theory

- Under a set of assumptions:

$$\bar{r}_i - r_f = \beta_{i,1}(\bar{r}_{\text{Factor 1}} - r_f) + \ldots + \beta_{i,K}(\bar{r}_{\text{Factor K}} - r_f)$$

- A diversified portfolio with 0 sensitivity to each macro factor…
  - Is essentially risk-free and should offer no market premium
  - If the return is higher or lower than the risk-free rate then profits can be made by arbitrage

- A diversified portfolio with sensitivity to the factors…
  - Should offer a risk premium proportional to its sensitivity to the factor
  - Otherwise, profits from arbitrage can be made!

Example: Arbitrage Pricing Theory

Estimated risk premiums for taking on risk factors (1978-1990)

Factor	Estimated Risk Premium $(r_{factor} - r_f)$
Yield spread	5.10%
Interest rate	-.61
Exchange rate	-.59
Real GNP	.49
Inflation	-.83
Market	6.36