

## Suggested Supervisions 4

1. (Voting by alternating veto, exercise 163.2 in Osborne (2004)) Two people select a policy that affects them both by alternately vetoing policies until only one remains. First person 1 vetoes a policy. If more than one policy remains, person 2 vetoes a policy. If more than one policy still remains, person 1 then vetoes another policy. The process continues until a single policy remains unvetoed. Suppose there are three possible policies, X, Y and Z, person 1 prefers X to Y to Z, and person 2 prefers Z to Y to X. Model this situation as an extensive game and find its Nash equilibria.
2. (Voting by alternating veto, exercise 173.3 in Osborne (2004)) Find the subgame perfect equilibria of the game in the previous exercise. Does the game have any Nash equilibrium that is not a subgame perfect equilibrium? Is any outcome generated by a Nash equilibrium not generated by any subgame perfect equilibrium? Consider variants of the game in which player 2's preferences may differ from those specified in the previous exercise. Are there any preferences for which the outcome in a subgame perfect equilibrium of the game in which player 1 moves first differs from the outcome in a subgame perfect equilibrium of the game in which player 2 moves first?
3. (Cournot's and Stackelberg's duopoly games with quadratic costs, exercises 59.1 and 189.1 in Osborne (2004)) Find the subgame perfect equilibrium of Stackelberg's duopoly game when  $C_i(q_i) = q_i^2$  for  $i = 1, 2$ , and  $P(q_1 + q_2) = \alpha - q_1 - q_2$  for all  $q_1 + q_2 \leq \alpha$  (with  $P(q_1 + q_2) = 0$  for all  $q_1 + q_2 > \alpha$ ). Compare the equilibrium outcome with the Nash equilibrium of the Cournot's game under the same assumptions.
4. (Sequential variant of Bertrand's duopoly game, exercise 192.1 in Osborne (2004)). Consider the variant of Bertrand's duopoly game in which first firm 1 chooses a price, then firm 2 chooses a price. Assume that each firm is restricted to choose a price that is an integral number of pence, that each firm's unit cost is constant and equal to  $c$  (an integral number of pence), and that the monopoly profit is positive.
  - a) Specify an extensive game with perfect information that models this situation.
  - b) Give an example of a strategy of firm 1 and an example of a strategy of firm 2.
  - c) Find the subgame perfect equilibria of the game.
5. (Firm-union bargaining, exercise 177.1 in Osborne (2004)). A firm's output is  $L(100 - L)$  when it uses  $L \leq 50$  units of labour, and 2500 when it

uses  $L > 50$  units of labour. The price of output is 1. A union that represents workers presents a wage demand (a nonnegative number  $w$ ), which the firm either accepts or rejects. If the firm accepts the demand, it chooses the number  $L$  of workers to employ (which you should take to be a continuous variable, not an integer); if it rejects the demand, no production takes place ( $L = 0$ ). The firm's payoffs are its profit; the union's payoffs are  $wL$ .

- a) Formulate this situation as an extensive game with perfect information.
- b) Find the subgame perfect equilibrium (equilibria?) of the game.
- c) Is there an outcome of the game that both parties prefer to any subgame perfect equilibrium outcome?
- d) Find a Nash equilibrium for which the outcome differs from any subgame perfect equilibrium outcome.