Lecture 5: Simultaneous Move Games
Application: Static Oligopoly

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Static Oligopoly

- Firms may compete in prices, advertising, quality, R&D,...
- Use game theory to model firms' competitive behaviour and predict outcomes
- Here, firms compete for only one period:

  Price competition (*Bertrand*) (homogenous goods)

  Quantity competition (*Cournot)* (homogenous goods)

  Price competition under capacity constraints (homogeneous goods)

  Price competition with product differentiation

- Concentrate in pure strategies

Price Competition: Elements

- Two firms produce and compete for the sale of a (homogenous) good:

  unit production costs are constant and equal to c, $C(q) = cq$

  consumers buy from the firm at the lowest price (half from each if it is the same) and

  total demand $D(p)$ is continuous, decreasing, and there exists $\overline{p}$ such that $D(\overline{p}) = 0$ for all $p \geq \overline{p}$

- Players: Firms 1 and 2. Strategies: $p_i \in (0, \infty)$ for $i = 1, 2$. Payoffs:

$$\Pi_i(p_i, p_j) = (p_i - c)D_i(p_i, p_j) \text{ where } D_i(p_i, p_j) = \begin{cases} D(p_i) \text{ if } p_i < p_j \\ \frac{D(p_i)}{2} \text{ if } p_i = p_j \\ 0 \text{ if } p_i > p_j \end{cases}$$

Price Competition: Behaviour

- $p_i \in (0, c)$ are weakly dominated strategies. Strictly?
- Proposition: $(p_i^*, p_j^*) = (c, c)$ is the unique NE. Proof:
  - a) $(p_i^*, p_j^*) = (c, c)$ is a NE because...
  - b) (p_i^*, p_j^*) such that $p_i^* > p_j^* > c$ is not a NE because...
  - c) (p_i^*, p_j^*) such that $p_i^* = p_j^* > c$ is not a NE because...
  - d) (p_i^*, p_j^*) such that $p_i^* > p_j^* = c$ is not a NE because...
  - e) (p_i^*, p_j^*) such that either $p_i^* < c$ or $p_j^* < c$ is not a NE because...
- Extension to more than two firms
- Puzzle: With more than one firm in the market, prices are again at competitive levels
- Extension to different unit costs, c_1 and c_2 and e.g. $c_1 < c_2$. NE:...

Quantity Competition: Linear Demand

- Two firms select the quantity that they are going to place in the market and... an auctioneer chooses the price according to $P()$ ($= D^{-1}()$)
  Assume unit costs c_i again but here linear demand

$$P(q_1 + q_2) = \begin{cases} 1 - q & \text{if } q_1 + q_2 \leq 1 \\ 0 & \text{if } \quad q_1 + q_2 > 1 \end{cases}$$

- Players: Firms 1 and 2. Strategies: $q_i \in [0, \infty)$. Payoffs:

$$\Pi_i(q_i, q_j) = \left[1 - (q_i + q_j)\right] q_i - c_i q_i$$

Quantity Competition: Linear Demand

- FOC (and best reply functions!):

$$B_i(q_j) = \frac{1 - q_j - c_i}{2}$$

- Solving (finding the intersection of the best-reply functions) we find the NE:

$$q_i^* = \frac{1 - 2c_i + c_j}{3}$$

- Total quantity and priceare given by:

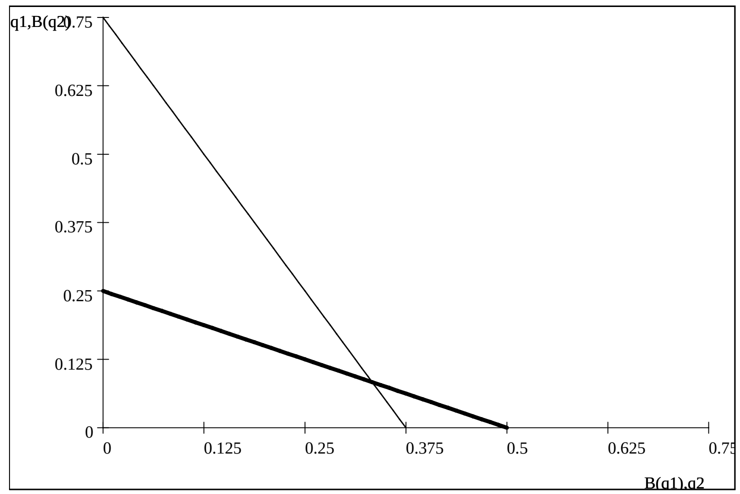
$$q_1^* + q_2^* = \frac{2 - c_i - c_j}{3} \text{ and } P\left(q_1^* + q_2^*\right) = \frac{1 + c_i + c_j}{3}$$

- Profits for each firm are given by:

$$\Pi_i\left(q_1^* + q_2^*\right) = \left(\frac{1 - 2c_i + c_j}{3}\right)^2$$

Quantity Competition: Linear Demand Example

- Example: $c_1 = 0.5$, $c_2 = 0.25$ and therefore NE: $(0.0833, 0.333)$



Best reply functions $B_1(q_2)$ (thick line, from horizontal to vertical axis) and $B_2(q_1)$ (thin line, from vertical to horizontal) and NE

Extensions

- More than two firms. Linear demand $1-(q_1+...+q_N)$ and symmetric firms c:

$$q_i^* = \frac{1-c}{n+1},\ nq_i^* = \frac{n(1-c)}{n+1} \text{ and profits } \Pi_i^* = \frac{(1-c)^2}{(n+1)^2}$$

  More firms, lower quantity per firm (higher in total) and lower profits

- Capacity constraints

- Product differentiation

- Competition for more than one period