S100 (Game Theory) Albert Banal-Estanol

Problem Set 3

1.- (More on NE and WPBE) For the game represented in Figure 1:

a) Find the Nash equilibria in pure strategies.

b) Find the weak perfect Bayesian equilibria in which each player's strategy is pure.

2.- (More on WPBE) Find the set of weak sequential equilibria of the game in Figure 2.

3.- (Making mistakes) Exercise 9.C.7 in Mas-Colell et al. [Hint: assume that you have a WPBE and consider, in turn, that in this WPBE player 1 plays B, T and a pure mixed strategy over this two strategies]. Printing mistake in the book: the WPBE is not unique.

4.- (War game) Consider the following strategic situation. Two opposed armies are poised to seize an island. Each army's general can choose either "attack" or "not attack." In addition, each army is either "strong" or "weak" with equal probability (the draws for each army are independent), and an army's type is known only to its general. Payoffs are as follows: The island is worth M if captured. An army can capture the island either by attacking when its opponent does not or by attacking when its rival does if it is strong and its rival is weak. If two armies of equal strength both attack, neither captures the island. An army also has a "cost" of fighting, which is s if it is strong and w if it is weak, where $s < w$. There is no cost of attacking if its rival does not. Identify all pure strategy Bayesian Nash equilibria of this game.

5.- (Another entry game) Two firms simultaneously decide whether to enter a market. Firm i's entry cost is $\theta_i \in (0, \infty)$. Firms' entry costs are private information and are independently drawn from the distribution P with strictly positive density p. Firm i's payoff is $\Pi^m - \theta_i$ if it enters but the other firm does not enter, $\Pi^d - \theta_i$ if both enter and 0 if it does not enter. Π^m and Π^d are the monopoly and duopoly profits gross of entry costs and are common knowledge. Assuming that $\Pi^m > \Pi^d$, compute a Bayesian Nash equilibrium.

6.- (Evolutionary Game by Maynard Smith (1991)) Some young animals expend energy begging for food from their parents- they squawk and bleat and scream, sometimes extravagantly. Can we expect these demands to signal their needs accurately? To answer this question, consider the following game.

A hungry parent has a piece of food that it may give to its offspring or keep for itself. It does not detect whether its offspring is hungry. In either case, the offspring may signal that it is hungry to its parent (by squawking, for example). An animal is stronger and thus produces more offspring (i.e. has a higher biological fitness) if it gets the food than if it does not. Normalize the parent's strength if it keeps the food to be 1, and denote its strength if it gives the food to its offspring by $S < 1$. If the offspring does not squawk, its strength is 1 if it gets the food, $V < 1$ if it is not hungry and does not get the food, and 0 if it is hungry and does not get the food. If the offspring squawks, its strength is multiplied by the factor $1 - t$, where $0 \leq t \leq 1$ (i.e. squawking may be costly). Denote the degree to which the parent and offspring are related by r, and take each player's payoff to be its strength plus r times the other player's strength. Evolutionary pressure will lead to behavior for each player that maximizes that player's payoff, given the other player's behavior.

a) Represent this game of incomplete information as a game of imperfect information (Bayesian game).

b) Find the conditions on r, in terms of S, V and t, under which the game has the following weak perfect Bayesian equilibrium ("separating equilibrium"): the offspring squawks if and only if it is hungry and the parent gives it the food if and only if it squawks.

c) Show that if the offspring's payoff from obtaining the food exceeds her payoff from not obtaining it, regardless of whether she is hungry (which means that $r < \frac{1-V}{1-S}$), then the game has such an equilibrium only if $t > 0$. That is, in this case an equilibrium exists in which the signal is accurate only if the signal is costly.

d) Show that if $r < \frac{1-S}{1-(1-\alpha)V}$, then the game has the following weak perfect Bayesian equilibrium ("pooling equilibrium"): the offspring is always quiet and the parent always keep the food. (For other parameter values, the game has a pooling equilibrium in which the offspring is always quiet and the parent always gives the food.)

7.- (Revisiting the War of Attrition, exercise 6 Problem set 1). First we are going to correct the solution of exercise 6 and show that the set of Nash equilibrium of the game is (t_1, t_2) such that either $0 = t_1 < t_2$ and $t_2 \geq v_1$ or $0 = t_2 < t_1$ and $t_1 \geq v_2$. Remarkably, the equilibrium outcomes do not depend on the valuations of the object. Second, using this example, we are going to show that in a Nash equilibrium players may use weakly dominated strategies.

Assume that a pair (t_1, t_2) is a Nash equilibrium.

(a) Show that $t_1 \neq t_2$ for any v_1, v_2.

(b) Show that we cannot have that $0 < t_1 < t_2$ or $0 < t_1 < t_2$ for any v_1, v_2.

(c) What conditions need to be satisfied to ensure that $0 = t_1 < t_2$ (or $0 = t_2 < t_1$) is a Nash equilibrium?

(d) What is the set of weakly dominated strategies for each player? Are some of them strictly dominated as well?

(e) Conclude that in a Nash equilibrium players may use weakly dominated strategies.

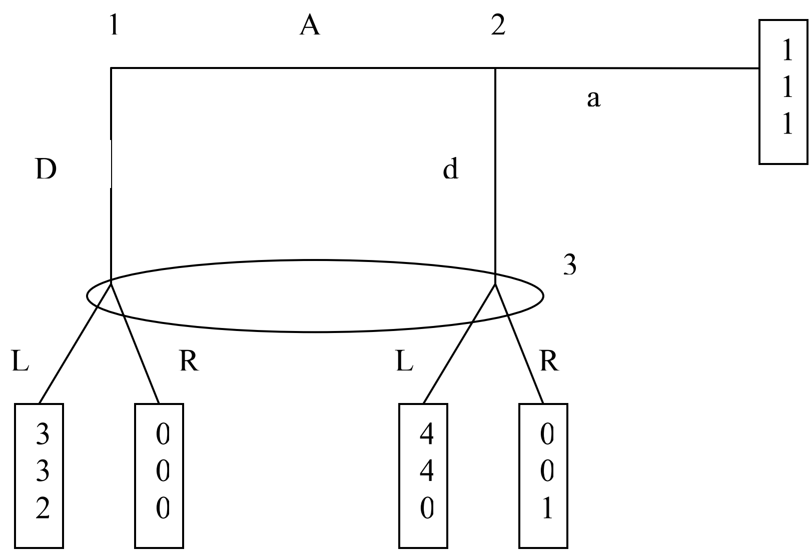

Figure 1

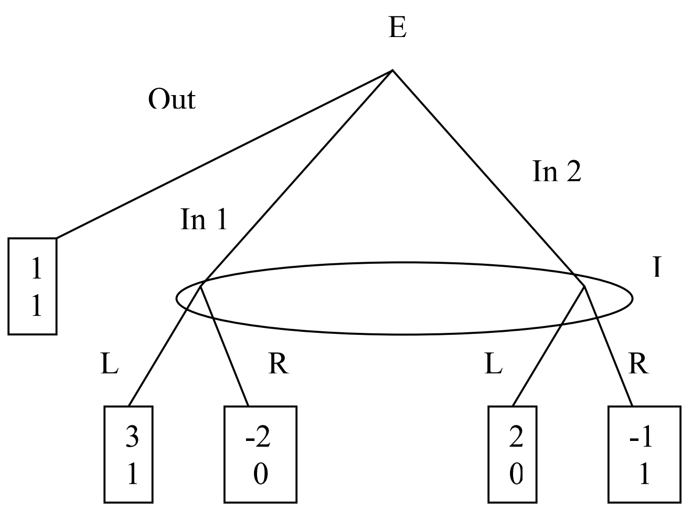

Figure 2