Problem Set 2

1. (NE, Backwards Induction and SPNE)

(a) Obtain the pure strategy Subgame Perfect Nash equilibria of the games described in Figures 1, 2 and 3 below. Can you think of a way of representing them in Normal form?

(b) Find the set of Nash equilibria obtained by backwards induction in Figure 4.

(c) Find all Nash equilibria (pure and mixed) and verify that the set of equilibria obtained in (b) is a subset of the set of equilibria.

(d) Show that (A,a) is not the play of a Nash equilibrium in the extensive form game of imperfect information depicted in Figure 5.

(e) Find a Nash equilibrium of the game of part (d). Is it a subgame perfect Nash equilibrium?

2. (Cournot and Stackelberg) There are two firms in a market producing quantities q_1 and q_2 respectively of a homogeneous good, with a market (inverse) demand function $p = f(q_1 + q_2)$ which is positive valued, strictly downward sloping, concave and twice differentiable (over the relevant range of outputs). Firm i has a cost function $c_i(q_i)$ which is strictly increasing, strictly convex, twice differentiable and satisfies $c_i'(0) = 0$.

First, suppose that firms move simultaneously.

a) Describe the elements of the normal form representation of this game.

b) Describe necessary first order conditions that must be satisfied by any Cournot-Nash equilibrium. Are these sufficient as well?

c) Show that the reaction function of firm 2 to firm 1's output, $\widehat{q_2}(q_1)$ must be downward sloping.

d) Among firm 2's strategies, which are strictly dominated?

Now suppose firm 1 is a leader, setting its output q_1 before firm 2 does. So firm 2 observes firm 1's output q_1 and then decides its own output q_2.

e) Draw an extensive form tree that represents this game.

f) Show that in any subgame perfect equilibrium of the resulting (Stackelberg) game, firm 1 produces more and firm 2 produces less compared to the Cournot-Nash equilibrium.

g) Provide an intuitive argument for why firm 1 must earn more profit in the SPNE Stackelberg equilibrium than in the Cournot-Nash equilibrium.

h) What is the highest profit firm 2 can achieve in a Nash Equilibrium of the Stackelberg game? Describe one of the NE in which firm 2 attains it.

3. (Hotelling) Suppose that we have two firms-stores located on a line of length 1 that produce the same good. The unit cost of the good for each store is c. Consumers are uniformly located in the line and they incur a

transportation cost tx^2 for travelling a length of x. Consumers have unit demands; each consumer derives a utility of $\overline{v}$ for consumption.

Suppose that firm 1 is located at point $a \geq 0$ and firm 2 at point $1 - b$, where $b \geq 0$ and without loss of generality, $1 - a - b \geq 0$ (firm 1 is to the left of firm 2; $a = b = 0$ can be interpreted as maximal differentiation and $a + b = 1$ can be interpreted as minimal differentiation). Assume that the market is covered and firms sell positive quantities. Firms compete in prices.

a) For any pair of prices, (p_1, p_2). Which consumer is indifferent between buying from firm 1 or firm 2? Which consumers will buy from firm 1? Find the demand for each firm as a function of the prices.

b) Show that in a NE, given a and b, firm 1 will charge

$$p_1 = c + t(1 - a - b)(1 + \frac{a - b}{3})$$

whereas firm 2 will charge

$$p_2 = c + t(1 - a - b)(1 + \frac{b - a}{3}).$$

c) Find firms' market shares of the market.

d) Now consider a game where prior to price competition and knowing the prices that will be chosen given their locations, the two firms choose their locations simultaneously. Where do they locate?

e) What would be the socially optimal location of the two firms? Compare with the market outcome.

4. (Tacit collusion in Bertrand) Consider an infinitely repeated Bertrand oligopoly model. In each stage game, N firms compete in prices. Customers purchase from the firm with the lowest announced price, dividing equally in the event of ties. Quantity purchased is given by a continuous, strictly decreasing function $D(p)$. Firms produce with constant marginal cost c. Letting $\pi(p) \equiv (p - c)D(p)$, assume that $\pi(p)$ is increasing in p on $[c, p^m$ (where p^m is the monopoly price).

(a) Find the Nash equilibrium of the stage game.

(b) Assuming that players discount future payoffs, when can we sustain all firms pricing at the monopoly level in each period as the outcome path of a NE of the supergame? Construct the NE strategy profile.

(c) Cooperation becomes more or less difficult when there are more firms in the market?

(d) Can other prices be sustained in a NE?

(e) Are the NE of (a) and (d) SPNE as well?

(f) Can cooperation be sustained in a SPNE if the stage game was only repeated a finite number of times?

5. (Tacit collusion in Cournot) Consider an infinitely repeated Cournot duopoly with discount factor $\delta < 1$, unit costs $c > 0$, and inverse demand function $p(q) = a - bq$, with $a > c$ and $b > 0$.

a) Under what conditions can the symmetric joint monopoly outputs $(q_1, q_2) = (q^m/2, q^m/2)$ be sustained with strategies that call for $(q^m/2, q^m/2)$ to be played if no one has deviated yet and for the single period Cournot (Nash) equilibrium to be played otherwise? Is it easier or more difficult to sustain cooperation than in (Bertrand) price competition? Why?

b) Derived the minimal level of δ such that output levels $(q_1, q_2) = (q, q)$ with $q \in [\frac{a-c}{4b}, \frac{a-c}{3b}]$ are sustainable through Nash reversion strategies. Show that this level of δ, $\delta(q)$, is a decreasing, differentiable function of q.

6. (More on Repeated games) Consider the following game in normal form:

	L	C	R
U	5, 3	5, 5	3, 4
M	4, 10	9, 9	4, 11
D	3, 3	11, 4	5, 5

a) Suppose that this game is repeated a finite number of times, and that the discount factor (δ) is unity. Characterize the set of subgame perfect equilibria.

b) Suppose that the game is repeated infinitely, and that δ is less than unity. For what values of δ can the players sustain the cooperative choice (M, c) in every period through Nash reversion?

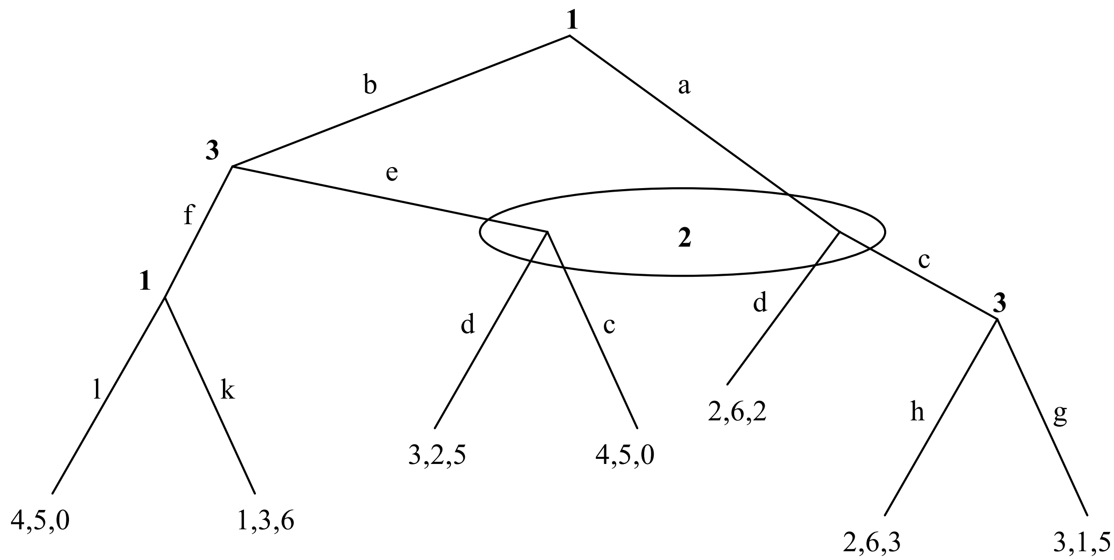


Figure 1

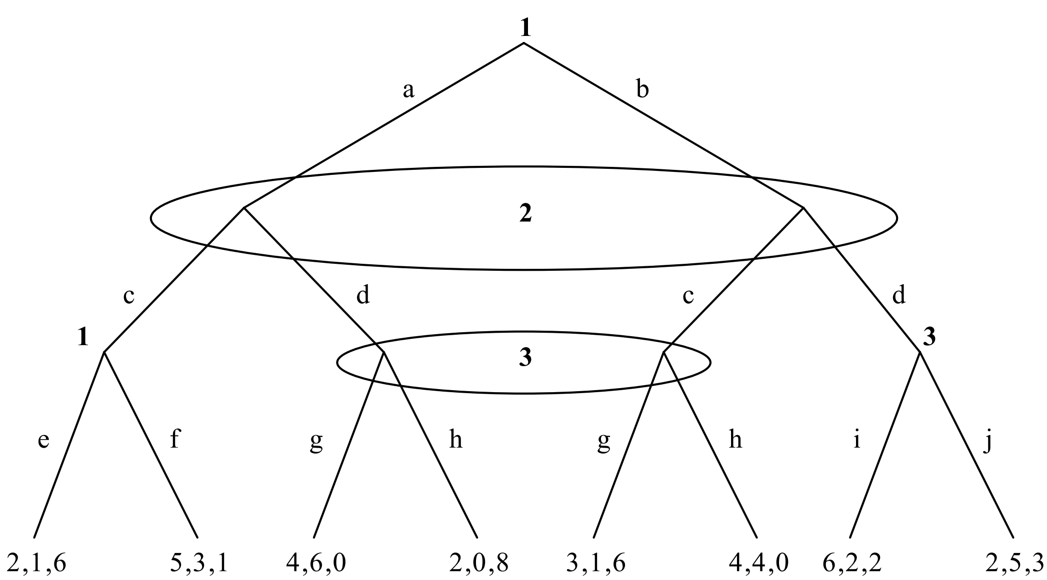


Figure 2

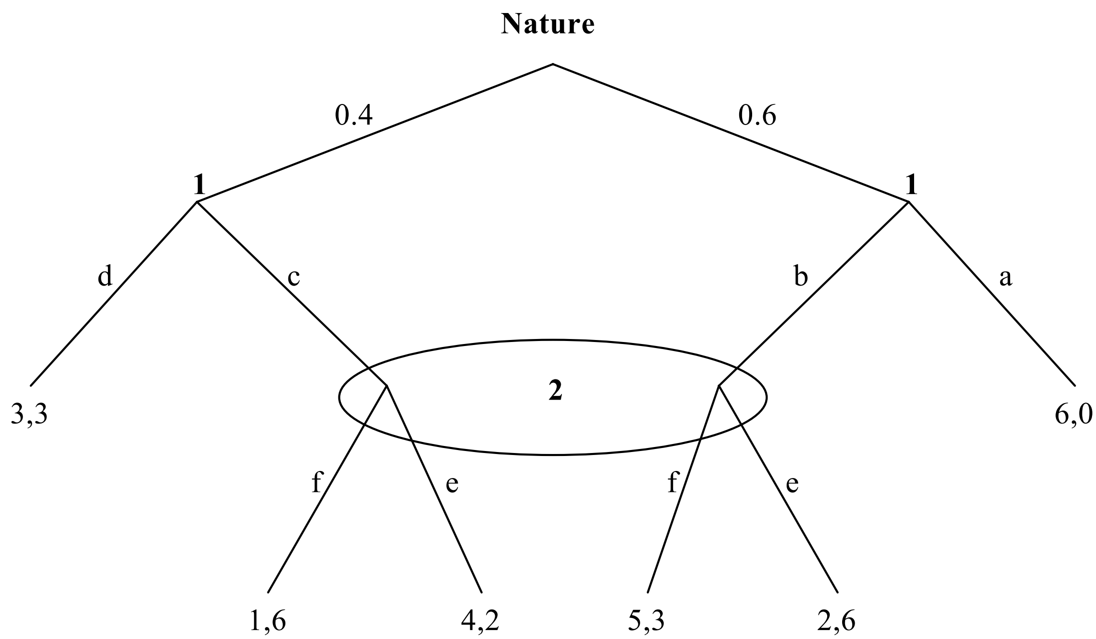


Figure 3

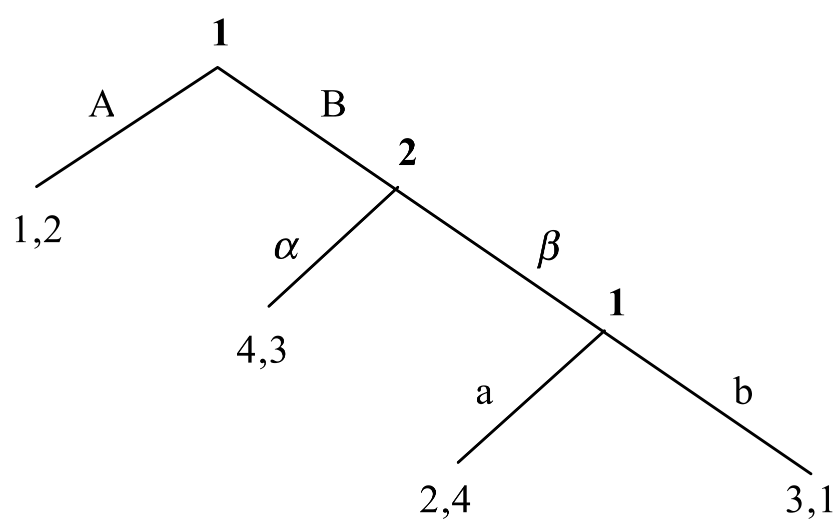
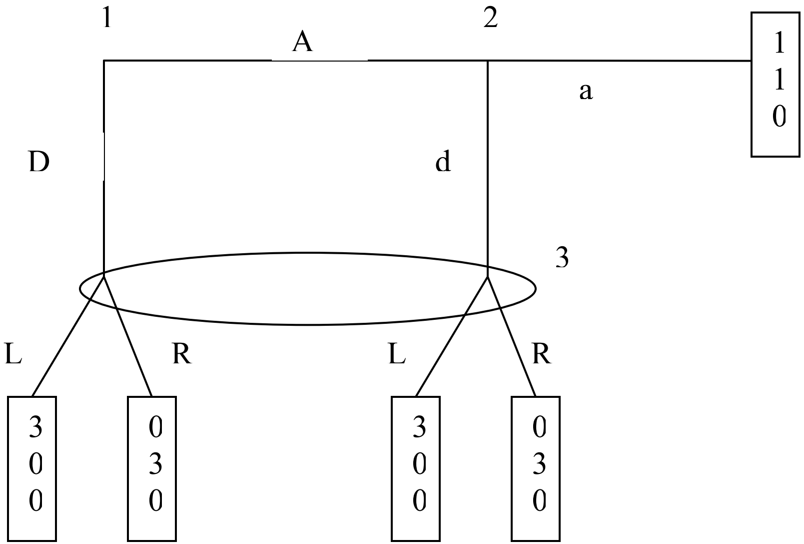


Figure 4



1
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