# Lecture 3: Application: Static Oligopoly

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# Static Oligopoly

- Firms may compete in prices, advertising, quality, R&D,...
- Use game theory to model firms' competitive behaviour and predict outcomes
- Here, firms compete for only one period:

  Price competition (*Bertrand*) (homogenous goods)

  Quantity competition (*Cournot)* (homogenous goods)

  Price competition under capacity constraints (homogeneous goods)

  Price competition with product differentiation

- Concentrate in pure strategies

# Price competition: Representation

- Two firms produce and compete for the sale of a (homogenous) good:

  unit production costs are constant and equal to $c$

  consumers buy from the firm at the lowest price (half from each if it is the same) and

  total demand $D(p)$ is continuous, decreasing, and there exists $\overline{p}$ such that $D(\overline{p}) = 0$ for all $p \geq \overline{p}$

- Players: Firms 1 and 2. Strategies: $p_i \in (0, \infty)$ for $i = 1, 2$. Payoffs:

$$\Pi_i(p_i, p_j) = (p_i - c)D_i(p_i, p_j) \text{ where } D_i(p_i, p_j) = \begin{cases} D(p_i) \text{ if } p_i < p_j \\ \frac{D(p_i)}{2} \text{ if } p_i = p_j \\ 0 \text{ if } p_i > p_j \end{cases}$$

# Price competition: Behaviour

- $p_i \in (0, c)$ are weakly dominated strategies. Strictly?
- Proposition: $(p_i^*, p_j^*) = (c, c)$ is the unique NE. Proof:

  a) $(p_i^*, p_j^*) = (c, c)$ is a NE because...

  b) $(p_i^*, p_j^*)$ such that $p_i^* > p_j^* > c$ is not a NE because...

  c) $(p_i^*, p_j^*)$ such that $p_i^* = p_j^* > c$ is not a NE because...

  d) $(p_i^*, p_j^*)$ such that $p_i^* > p_j^* = c$ is not a NE because...

  e) $(p_i^*, p_j^*)$ such that either $p_i^* < c$ or $p_j^* < c$ is not a NE because...
- Extension to more than two firms
- Puzzle: With more than one firm in the market, prices are again at competitive levels
- Extension to different unit costs, $c_1$ and $c_2$ and e.g. $c_1 < c_2$. NE:...

# Quantity competition: Linear Demand

- Two firms select the quantity that they are going to place in the market and... an auctioneer chooses the price according to $P()$ ( $= D^{-1}()$ )
  Assume here $P(q_1 + q_2) = 1 - (q_1 + q_2)$ and again unit costs $c_i$
- Players: Firms 1 and 2. Strategies: $q_i \in [0, \infty)$. Payoffs:

$$\Pi_i(q_i, q_j) = \left[1 - (q_i + q_j)\right] q_i - c_i q_i$$

- Assuming $q_i \in [0, M]$ for large $M$, existence conditions satisfied (verify them!)

# Quantity competition: Linear Demand

- FOC (and best reply functions!):

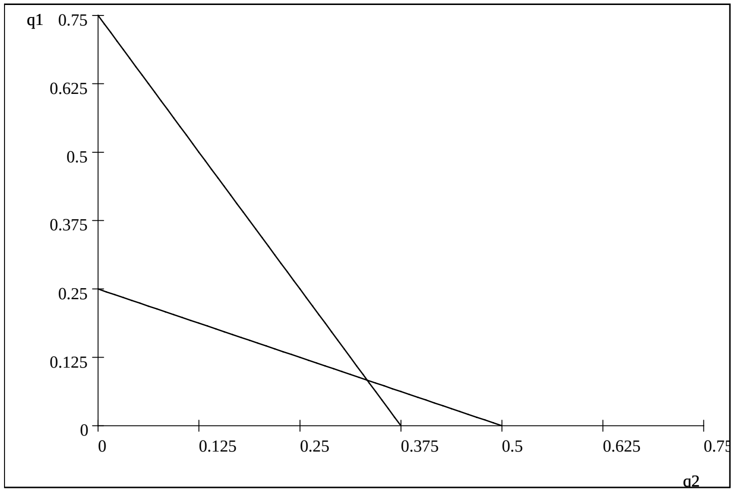
$$b_i(q_j) = \frac{1 - q_j - c_i}{2}$$

- Solving (finding the intersection of the best-reply functions) we find the NE:

$$q_i^* = \frac{1 - 2c_i + c_j}{3}$$

# Quantity competition: Linear Demand Example

- Example: $c_1 = 0.5$, $c_2 = 0.25$ and therefore NE: $(0.0833, 0.333)$




Best reply functions $b_1(q_2)$ (from horizontal to vertical axis) and $b_2(q_1)$ (from vertical to horizontal) and NE

# Iterated Deletion in Cournot

- Denote $q_1^m = b_1(0)$ (0.25 in the example) and $q_2^m = b_2(0)$ (0.375)
- $S_i^1 = (q_i^m, \infty)$ are never best-response
- Assuming that Player $j$ never plays $S_j^1$ then $[0, q_i^*)$, where $q_i^*$ is such that $b_i(q_i^*) = q_j^m$, are never best response
- Iterating successively we can eliminate all strategies except the Nash Equilibrium
- A similar process takes place if one deletes strictly dominated strategies (2-player game):

  Strategies $[q_i^m, \infty)$ are dominated by $q_1^m$ (can you see why?)

# Quantity competition: General Model

- Assume that...
  the demand function $P()$ is twice-differentiable, $P' < 0$, $P'' \leq 0$
  total costs $C_i(q_i)$ are twice-differentiable with $C_i'' \geq 0$ and $P(0) > C_i'(0)$
- Players: Firms 1 and 2. Strategies: $q_i \in [0, \infty)$. Payoffs:

$$\Pi_i(q_i, q_j) = P(q_i + q_j)q_i - C_i(q_i)$$

- Maximise over $q_i \geq 0$. FOC (since $q_i > 0$ and SOC satisfied):

$$P(q_i + q_j) - C_i'(q_i) + P'(q_i + q_j)q_i = 0$$

- Competitive had no third term and the monopoly had $P'(q_i + q_j)[q_i + q_j]$
- Not industry-efficient: firms do not take into account the effect on the other firms and marginal costs are not equalised

# Extensions

- More than two firms. Linear demand $1-(q_1+...+q_N)$ and symmetric firms $c$:

$$q_i^* = \frac{1-c}{n+1},\ nq_i^* = \frac{n(1-c)}{n+1} \text{ and profits } \Pi_i^* = \frac{(1-c)^2}{(n+1)^2}$$

More firms, lower quantity per firm (higher in total) and lower profits

- Capacity constraints (see IO section)

- Product differentiation (see IO section)

- Competition for more than one period (next week)