Lecture 2b: Trembling-Hand Perfection for Extensive Games

Albert Banal-Estanol

May 2006
Trembling-Hand Again

- Motivation: No need to think about off-equilibrium path beliefs if players make mistakes at all information sets

- Problem: (normal form) trembling-hand perfect equilibria (NFTHP) may not be SPNE

- Refinement: extensive form trembling-hand perfection (EFTHP)
Example

- Entry game again:
Normal form:

<table>
<thead>
<tr>
<th></th>
<th>A if In</th>
<th>F if In</th>
</tr>
</thead>
<tbody>
<tr>
<td>Out, A if In</td>
<td>2,2</td>
<td>2,2</td>
</tr>
<tr>
<td>Out, F if In</td>
<td>2,2</td>
<td>2,2</td>
</tr>
<tr>
<td>In, A if In</td>
<td>4,1</td>
<td>1,0</td>
</tr>
<tr>
<td>In, F if In</td>
<td>0,0</td>
<td>0,1</td>
</tr>
</tbody>
</table>

NE: (In, A if In, A if In), (Out, A if In, F if In), (Out, F if In, F if In)

SPNE: only the first

Recall that in 2-player games NFTHP is equivalent to NE with non-weakly dominated strategies

In this example: no player has a weakly dominant strategy. NFTHP: all three NE

Non-SPNE can survive NFTHP because trembles may place more weight on (In, F if In) than on (In, A if In)

Does this make sense? Given In, A should be better than F for E
Extensive form trembling-hand perfection

- Idea: players tremble independently among all actions at each info set
- Definition: the *agent normal form* is obtained by selecting a different agent to make decisions at each info set, and these agents have the same payoffs as the original player
- Example: (1a chooses rows, 1b chooses columns, 2 chooses boxes)

\[
\begin{array}{ccc}
1a/1b & A \text{ if } \text{In} & F \text{ if } \text{In} \\
\text{In} & 4,4,1 & 0,0,0 \\
\text{Out} & 2,2,2 & 2,2,2 \\
\end{array}
\]

- Definition: a NE is *extensive-form trembling hand perfect* (EFTHP) if it the agent-normal form is trembling hand perfect
- EFTHP in example: \((\text{In}, A \text{ if } \text{In}, A \text{ if } \text{In})\)
  - If 1a plays \(\text{In}\) with \(\text{prob} > 0\) then 1b will prefer \(A \text{ if } \text{In}\) to \(F \text{ if } \text{In}\)
Relation between EFTHP and SE

- Both use trembles in the extensive form to derive beliefs out of equilibrium path
- SE easier to compute because it only requires to think about best responses to the limiting strategies
- SE entails a specific treatment of beliefs
- \( \{EFTHP\} \subset \{SE\} \)
- But in generic games, \( \{EFTHP\} = \{SE\} \) (see Fudenberg Tirole)