Suggested Supervisions Chapter 4 (and Chapter 3)

1. (Tacit collusion in Bertrand) Consider an infinitely repeated Bertrand oligopoly model. In each stage game, N firms compete in prices. Customers purchase from the firm with the lowest announced price, dividing equally in the event of ties. Quantity purchased is given by a continuous, strictly decreasing function $D(p)$. Firms produce with constant marginal cost c. Letting $\pi(p) \equiv (p-c)D(p)$, assume that $\pi(p)$ is increasing in p on $[c, p^m$ (where p^m is the monopoly price).

(a) Find the Nash equilibrium of the stage game.

(b) Assuming that players discount future payoffs, when can we sustain all firms pricing at the monopoly level in each period as the outcome path of a NE of the supergame? Construct the NE strategy profile.

(c) Cooperation becomes more or less difficult when there are more firms in the market?

(d) Can other prices be sustained in a NE?

(e) Are the NE of (a) and (d) SPNE as well?

(f) Can cooperation be sustained in a SPNE if the stage game was only repeated a finite number of times?

2. (Tacit collusion in Cournot) Consider an infinitely repeated Cournot duopoly with discount factor $\delta < 1$, unit costs $c > 0$, and inverse demand function $p(q) = a - bq$, with $a > c$ and $b > 0$.

a) Under what conditions can the symmetric joint monopoly outputs $(q_1, q_2) = (q^m/2, q^m/2)$ be sustained with strategies that call for $(q^m/2, q^m/2)$ to be played if no one has deviated yet and for the single period Cournot (Nash) equilibrium to be played otherwise? Is it easier or more difficult to sustain cooperation than in (Bertrand) price competition? Why?

b) Derived the minimal level of δ such that output levels $(q_1, q_2) = (q, q)$ with $q \in [\frac{a-c}{4b}, \frac{a-c}{3b}]$ are sustainable through Nash reversion strategies. Show that this level of δ, $\delta(q)$, is a decreasing, differentiable function of q.

3. (More on Repeated games) Consider the following game in normal form:

	L	C	R
U	$5,3$	$5,5$	$3,4$
M	$4,10$	$9,9$	$4,11$
D	$3,3$	$11,4$	$5,5$

a) Suppose that this game is repeated a finite number of times, and that the discount factor (δ) is unity. Characterize the set of subgame perfect equilibria.

b) Suppose that the game is repeated infinitely, and that δ is less than unity. For what values of δ can the players sustain the cooperative choice (M, c) in every period through Nash reversion?

4. (War game) Consider the following strategic situation. Two opposed armies are poised to seize an island. Each army's general can choose either "attack" or "not attack." In addition, each army is either "strong" or "weak" with equal probability (the draws for each army are independent), and an army's type is known only to its general. Payoffs are as follows: The island is worth M if captured. An army can capture the island either by attacking when its opponent does not or by attacking when its rival does if it is strong and its rival is weak. If two armies of equal strength both attack, neither captures the island. An army also has a "cost" of fighting, which is s if it is strong and w if it is weak, where $s < w$. There is no cost of attacking if its rival does not. Identify all pure strategy Bayesian Nash equilibria of this game.

5. (Another entry game) Two firms simultaneously decide whether to enter a market. Firm i's entry cost is $\theta_i \in (0, \infty)$. Firms' entry costs are private information and are independently drawn from the distribution P with strictly positive density p. Firm i's payoff is $\Pi^m - \theta_i$ if it enters but the other firm does not enter, $\Pi^d - \theta_i$ if both enter and 0 if it does not enter. Π^m and Π^d are the monopoly and duopoly profits gross of entry costs and are common knowledge. Assuming that $\Pi^m > \Pi^d$, compute a Bayesian Nash equilibrium.