

Suggested Supervisions Chapter 1

1. (A parlor game) Represent the following parlor as an extensive-form game with imperfect information. First player 1 receives a card that is either H or L with equal probabilities. Player 2 does not see the card. Player 1 may announce that her card is L and pay 1 dollar to player 2, or may claim that her card is H. In the latter, player 2 may choose to concede or to insist on seeing player 1's card. If player 2 concedes then he must pay 1 dollar to player 1. If he insists on seeing player 1's card then player 1 must pay him 4 dollars if her card is L and he must pay her 4 dollars if her card is H.
2. (Voting game) A committee with three members, $\{1, 2, 3\}$, has to choose a new member of a club among a set of four candidates, $\{a, b, c, d\}$. Each member of the committee has veto power which is used in a successive way, starting by member 1, and finishing with member 3. Each member of the committee has to veto one and only one of the candidates that have not been vetoed yet.
 - (a) Draw the extensive form of the game, writing in the terminal nodes the name of the elected candidate.
 - (b) How many strategies does each player have? Do not try to write them (player 3 should have a lot).
3. (Labour market) There are two firms ($i=1,2$) in an industry. Each of them has a job opening. Firm i offers a wage w_i . There are two workers that decide simultaneously to which firm they apply (they can only apply to one). If only one worker applies to a given firm that worker gets the job; if the two workers apply to the same firm, the firm hires one of them randomly and the other remains unemployed (that has payoff zero).
 - (a) Represent this game in extensive form.
 - (b) Represent this game in normal form.
4. (Political economy) Each of I people chooses whether to become or not a political candidate, and if so which position to take. There is a continuum of citizens, each of whom has a favorite position; the distribution of favorite positions is given by a density function f on $[0,1]$ with $f(x) > 0 (\forall x \in [0,1])$. A candidate attracts the votes of those citizens whose favorite positions are closer to his position than to the position of any other candidate; if k candidates choose the same position then each receives the fraction $\frac{1}{k}$ of the votes that the position attracts. The winner of the competition is the candidate who receives the most votes. Each person prefers to be the unique winning candidate than tie for the first place, prefers to tie for the first place than to stay out of the competition, and prefers to stay out of

the competition than to enter and lose. Represent (i.e. find the elements) in normal form.

5. (War of attrition) Two players are involved in a dispute over an object. The value of the object to player i is $v_i > 0$. Time is modeled as a continuous variable that starts at 0 and runs indefinitely. Each player chooses when to concede the object to the other player; if the player to concede does so at time t , the other player obtains the object at that time. If both players concede simultaneously, the object is split equally between them, player i receiving a payoff of $\frac{v_i}{2}$. Time is valuable: until the first concession each player loses one unit of payoff per unit of time. Represent (i.e. find the elements) in normal form.