

# Lecture 5: Dynamic Games of Complete Information

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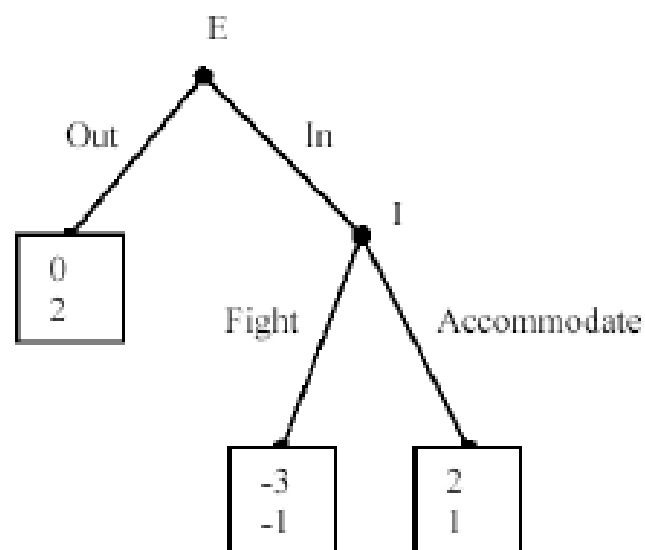
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## Today's Lecture

- Motivation:  
e.g. firms usually compete for several periods  
in a bargaining process there are offers and counteroffers,...
  - Start by representing the normal form and using the Nash Equilibrium concept
  - Problem of credibility and the principle of sequential rationality
  - Solving by backwards induction in perfect information games
  - Subgame Perfect Nash Equilibrium (extension to imperfect information games)
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## A Simple Entry Game

- Story: A potential entrant  $E$  decides whether to enter and if so... the incumbent decides whether to fight or accommodate. Payoffs are as follows:



## Normal Form and Nash Equilibrium

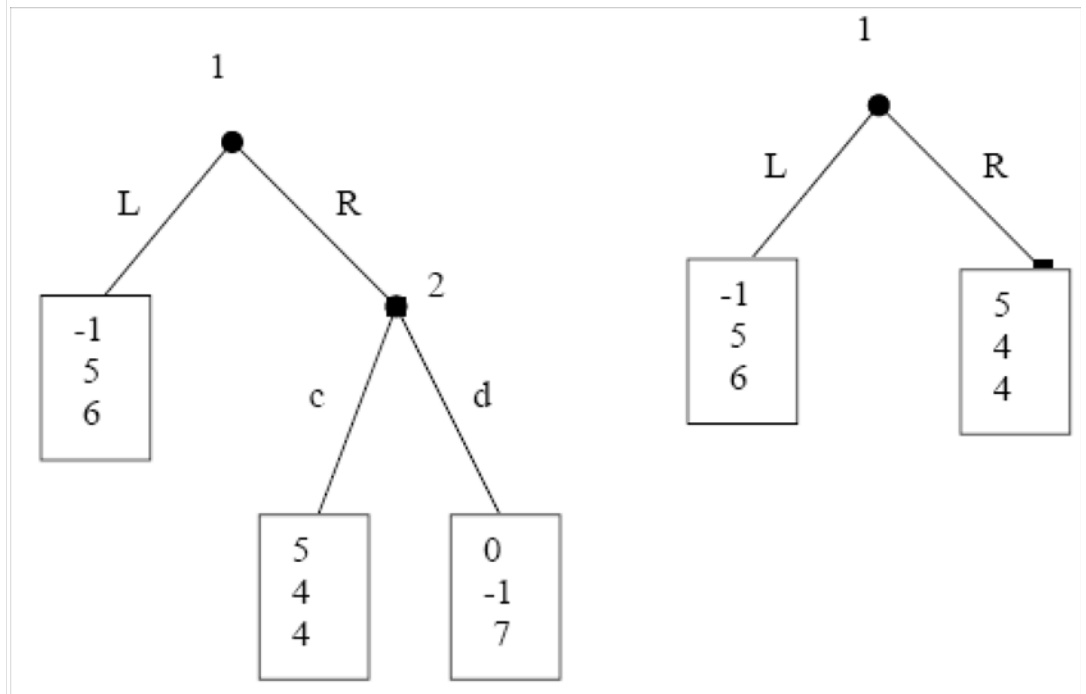
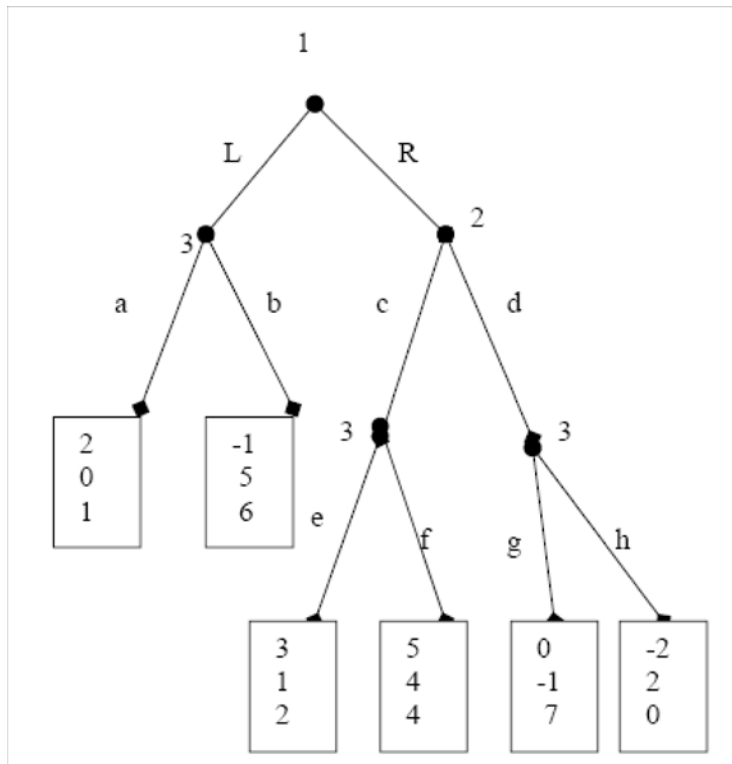
- Represented in normal form:

$E \setminus I$	$F$ if $In$	$A$ if $In$
$Out$	0,2	0,2
$In$	-3,-1	2,1

- Nash equilibria:
  - Is the first reasonable? Problem of credibility. Other examples
  - Shouldn't prediction satisfy sequential rationality (i.e. "rational behaviour at each point in time")?
  - Yes! Need to refine Nash Equilibrium concept
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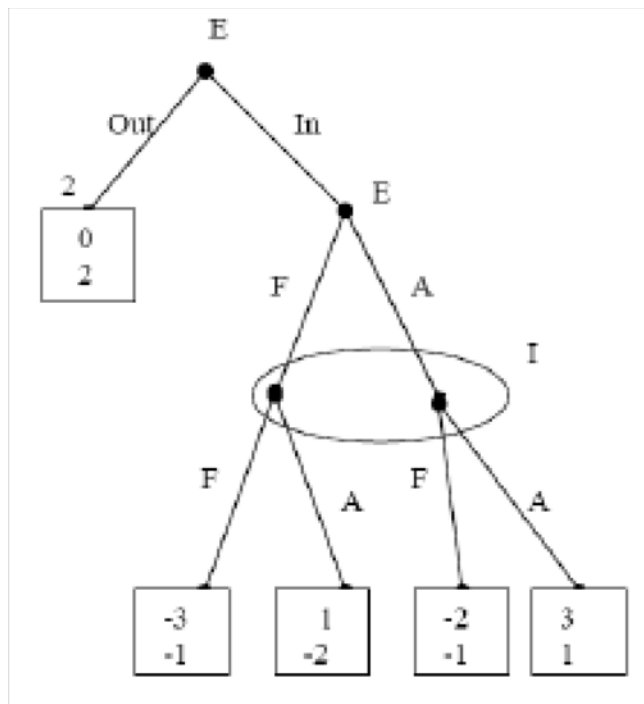
## Backward Induction Method

- Find sequential rational actions in perfect information extensive games:
    - a) Find the optimal action at each of the predecessors of the terminal nodes
    - b) Associate these nodes with the payoffs of the anticipated terminal node
    - c) Start again the process with this reduced game
  - Previous example: solve for I's optimal decision in the post-entry game (*Accomodate*) and then, anticipating I's decision, solve for E's optimal decision (*In*).  $[(In, A)]$
  - Example in next slide: one obtains  $[R, c, (b, f, g)]$ . This is one of the NE.
  - Proposition (Zermelo's theorem): Every finite game of perfect information  $\Gamma_E$  has a pure strategy NE that can be derived through backwards induction. Moreover, if no player has the same payoffs at any two terminal nodes, then there is a unique NE that can be derived in this way.
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## Subgame Perfect Nash Equilibrium: Example

- Extension of backwards induction to imperfect information games



## Subgame Perfect Nash Equilibrium

- Definition: A subgame of  $\Gamma_E$  is a subset of  $\Gamma_E$  that satisfies:
    - a) Begins at an info set that has only one node and contains all its successors
    - b) If a node of an info set is in it, then the other nodes of the info set also are
  - Previous example: whole subgame and subgame starting at E decision node
  - Definition: A strategy profile  $(\sigma_1, \dots, \sigma_I)$  in  $\Gamma_E$  is a subgame perfect Nash equilibrium if it induces a NE in every subgame
  - By definition every SPNE is a NE (method 1 to find SPNE: select among NE)
  - In games of perfect information, SPNE is equal to the set of NE derived by backwards induction
  - More generally, SPNE can be found by finding NE in every subgame and substituting backwards (method 2 to find SPNE)
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## Example (continued)

- Normal form:

$E \setminus I$	$A \text{ if } In$	$F \text{ if } In$
$Out, A \text{ if } In$	0,2	0,2
$Out, F \text{ if } In$	0,2	0,2
$In, A \text{ if } In$	3,1	-2,-1
$In, F \text{ if } In$	1,-2	-3,-1

- NE:
  - Do all of them induce a NE in every subgame? SPNE:
  - Other examples in Industrial Organisation: choosing degree of differentiation before competing in prices
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