Direcció Financera II

Chapter 1: Investment Decisions

Albert Banal-Estanol

In this chapter…

- Part (a): Compute projects' cash flows :
  - Computing earnings, and free cash flows
  - Necessary inputs?
- Part (b): Evaluate risk-free projects:
  - Decide whether to invest in a project
  - Project selection
- Part (c): Adjusting for risk:
  - How to adjust the "discount rate"?
  - Portfolio theory, the CAPM and extensions

Part (a): Project's cash flows

- Computing earnings, and free cash flows
- Necessary inputs?

- Example:
  - Firm: Linksys (subsidiary of Cisco Systems, maker of consumer networking hardware)
  - Project: *Homenet*, wireless home network appliance, which would provide both hardware and software necessary to run an entire home from any Internet connection

Feasibility study

- Estimated life of the project: four years
- Revenue estimates:
  - Sales = 100,000 units/year
  - Per Unit Price = $235
- Cost Estimates:
  - Up-Front R&D = $15,000,000
  - Up-Front New Equipment = $7,500,000
    - Expected life of the new equipment is 5 years (housed in existing lab)
  - Annual Overhead = $3,000,000
  - Per Unit Cost = $95
- Cost of the feasibility study $300,000

Incremental Earnings Forecast

		Year	0	1	2	3	4	5
Incremental Earnings Forecast ($000s)								
1	Sales		—	23,500	23,500	23,500	23,500	—
2	Cost of Goods Sold		—	(9,500)	(9,500)	(9,500)	(9,500)	—
3	**Gross Profit**		—	14,000	14,000	14,000	14,000	—
4	Selling, General, and Administrative		—	(3,000)	(3,000)	(3,000)	(3,000)	—
5	Research and Development		(15,000)	—	—	—	—	—
6	Depreciation		—	(1,500)	(1,500)	(1,500)	(1,500)	(1,500)
7	**EBIT**		(15,000)	9,500	9,500	9,500	9,500	(1,500)
8	Income Tax at 40%		6,000	(3,800)	(3,800)	(3,800)	(3,800)	600
9	**Unlevered Net Income**		**(9,000)**	**5,700**	**5,700**	**5,700**	**5,700**	**(900)**

Are taxes relevant even if we make losses?

Capital Expenditures and Depreciation

- Investments in plant, property and equipment:
  - are a cash expense not directly listed as expense but
  - a fraction of cost deducted each year as depreciation
- Some methods:
  - Straight Line Depreciation: Asset's cost is divided equally over its life ($7.5 million ÷ 5 years = $1.5 million/year)
  - Modified Accelerated Cost Recovery System (MACRS) depreciation *(see rates in next table, obtained from a complicated* formula, which can be found in accounting textbooks)

Tasa de depreciación (%) para cada período de recuperación SMARC en años						
Año	n = 3	n = 5	n = 7	n = 10	n = 15	n = 20
1	33.33	20.00	14.29	10.00	5.00	3.75
2	44.45	32.00	24.49	18.00	9.50	7.22
3	14.81	19.20	17.49	14.40	8.55	6.68
4	7.41	11.52	12.49	11.52	7.70	6.18
5		11.52	8.93	9.22	6.93	5.71
6		5.76	8.92	7.37	6.23	5.29
7			8.93	6.55	5.90	4.89
8			4.46	6.55	5.90	4.52
9				6.55	5.91	4.46
10				6.55	5.90	4.46
11				3.28	5.91	4.46
12					5.90	4.46
13					5.91	4.46
14					5.90	4.46
15					5.91	4.46
16					2.95	4.46
17 – 20						4.46
21						2.23

From Earnings to Cash Flows

	Year	0	1	2	3	4	5
Incremental Earnings Forecast ($000s)							
1	Sales	—	23,500	23,500	23,500	23,500	—
2	Cost of Goods Sold	—	(9,500)	(9,500)	(9,500)	(9,500)	—
3	**Gross Profit**	—	14,000	14,000	14,000	14,000	—
4	Selling, General, and Administrative	—	(3,000)	(3,000)	(3,000)	(3,000)	—
5	Research and Development	(15,000)	—	—	—	—	—
6	Depreciation	—	(1,500)	(1,500)	(1,500)	(1,500)	(1,500)
7	**EBIT**	(15,000)	9,500	9,500	9,500	9,500	(1,500)
8	Income Tax at 40%	6,000	(3,800)	(3,800)	(3,800)	(3,800)	600
9	**Unlevered Net Income**	**(9,000)**	**5,700**	**5,700**	**5,700**	**5,700**	**(900)**
Free Cash Flow ($000s)							
10	Plus: Depreciation	—	1,500	1,500	1,500	1,500	1,500
11	Less: Capital Expenditures	(7,500)	—	—	—	—	—
12	Less: Increases in NWC	—	(2,100)	—	—	—	2,100
13	**Free Cash Flow**	**(16,500)**	**5,100**	**7,200**	**7,200**	**7,200**	**2,700**

Outflow Inflow

Net Working Capital (NWC)

- Definition

$$\text{Net Working Capital} = \text{Current Assets} - \text{Current Liabilities}$$
$$= \text{Cash} + \text{Inventory} + \text{Receivables} - \text{Payables}$$

- Most projects require investment in NWC:
  - Cash held at registers, safe box or checking account
  - Inventories of raw materials or finished product
  - Receivables: earned but not paid (credit offered to customers)
  - Payables: spent but not paid (credit received by suppliers)

Trade credit: difference between receivables & payables

Homenet NWC Requirements

	Year	0	1	2	3	4	5
Net Working Capital Forecast ($000s)							
1	Cash Requirements	—	—	—	—	—	—
2	Inventory	—	—	—	—	—	—
3	Receivables (15% of Sales)	—	3,525	3,525	3,525	3,525	—
4	Payables (15% of COGS)	—	(1,425)	(1,425)	(1,425)	(1,425)	—
5	**Net Working Capital**	**—**	**2,100**	**2,100**	**2,100**	**2,100**	**—**

Investments in NWC reduce cash available to the firm:

$$\Delta NWC_t = NWC_t - NWC_{t-1}$$

Indirect effects and real-world complexities

(not considered here)

- Project Externalities
  - **Cannibalization** is when sales of a new product displaces sales of existing product
  - Would customers of HomeNet have purchased existing Linksys wireless routers?
- Opportunity costs
  - The value a resource could have provided in its best alternative use
  - Homenet's equipment will be housed in an existing lab, but what is the opportunity cost of not using the space in an alternative way (e.g., renting it out)?
- Further,
  - Sales, the average selling price, the average cost per unit will vary over time
- Where should we allocate the $300,000 of the feasibility study?

Part (b): evaluating risk-free projects

Part (b): Evaluating risk-free projects

- Methods and rules to decide whether to invest:
  - Net present value rule
  - Internal rate of return rule
  - Payback period and payback rule
  - Profitability index
- Project selection:
  - Mutually exclusive projects
  - Scalable projects with limited resources

How to compare present and future?

- One euro today is worth more than one tomorrow!

- Why? Possible to earn interest! If interest is 10% a year…
  - Investing 10 million today gives 11 million in a year
  - The future value (in a year) of 10 million is 11 million
  - The present value of 11 million in a year is 10 million

Future and Present Values

- <u>Future Value</u>: Amount to which an investment will grow after earning interest

$$FV = C_0 \times (1+r)^t$$

  - For example, 10 million after two years will be

$$FV = 10m \times (1+0.1)^2 = 12.1m$$

- <u>Present Value</u>: Value today of a future (expected) cash flow

$$PV = \frac{1}{(1+r)^t} \times C_t$$

Discount factor

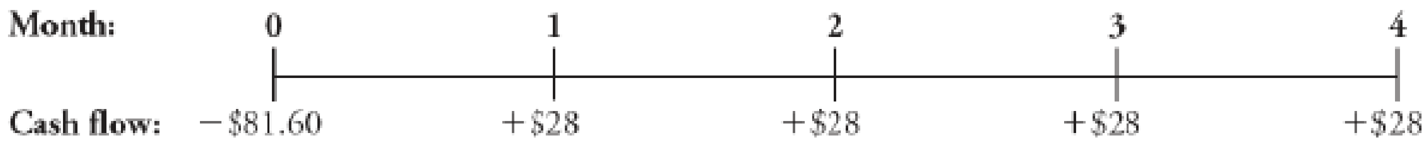
Discount rate

  - For example, 12.1 million in two years is

$$PV = \frac{1}{(1+0.1)^2} \times 12.1m = 10m$$

Net Present Value: an example

- Cash flows: immediate \$81.6 million "outflow" and an "inflow" of \$28 million per year for 4 years



- Therefore, if discount rate is *r = 0.10*, the NPV is:

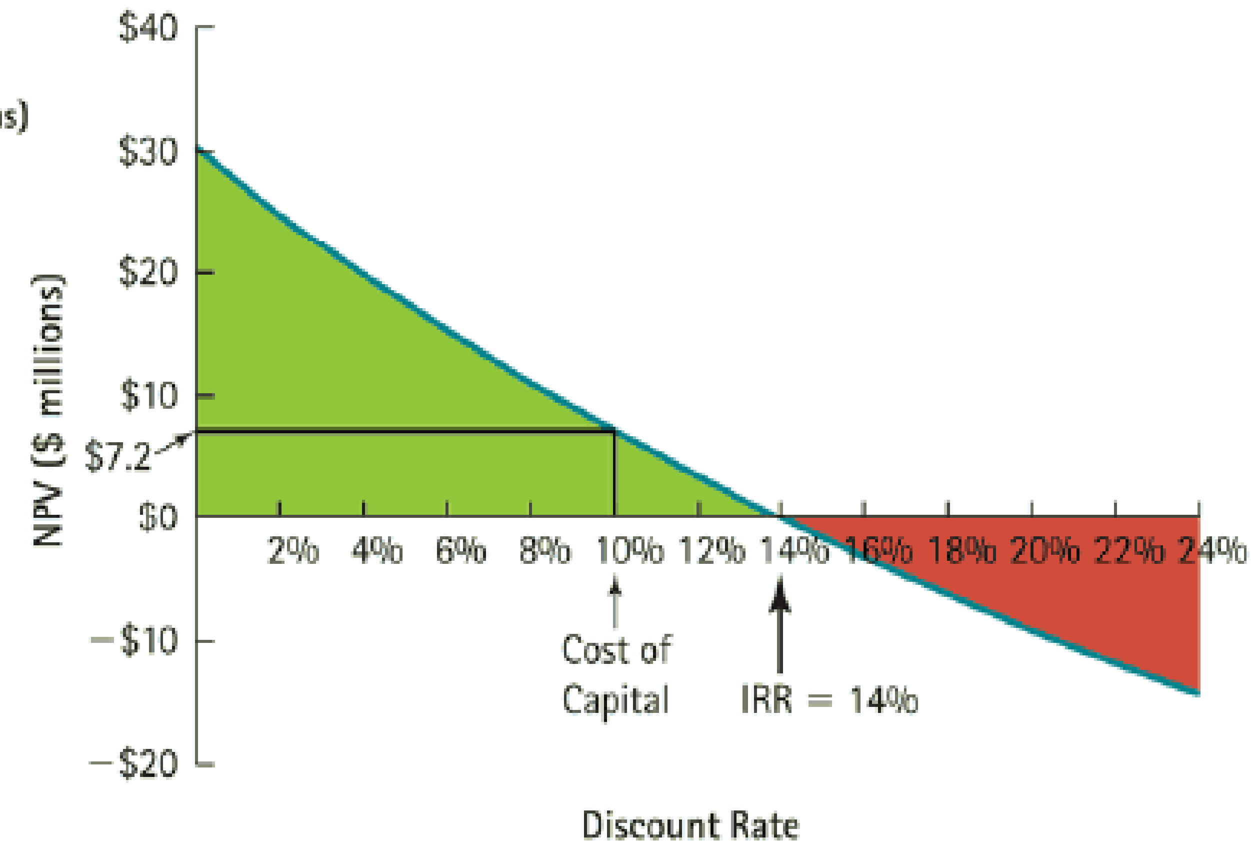
$$NPV = -81.6 + \frac{28}{(1+0.1)^1} + \frac{28}{(1+0.1)^2} + \frac{28}{(1+0.1)^3} + \frac{28}{(1+0.1)^4}$$

- Discount rate depends on the riskiness of the cash flows:
  - Equal to risk-free rate (government bond) if cash flows are certain
  - Higher risk implies greater discount and lower present value (more on that in part (c) of this chapter)

Panel (a)

Discount Rate	NPV ($ millions)
0%	$30.4
2%	$25.0
4%	$20.0
6%	$15.4
8%	$11.1
10%	$7.2
12%	$3.4
14%	$0.0
16%	−$3.3
18%	−$6.3
20%	−$9.1
22%	−$11.8
24%	−$14.3

Panel (b)



In general, the NPV rule:

Step 1: Forecast future cash flows

(see part (a) of this chapter)

Step 2: Estimate discount rate

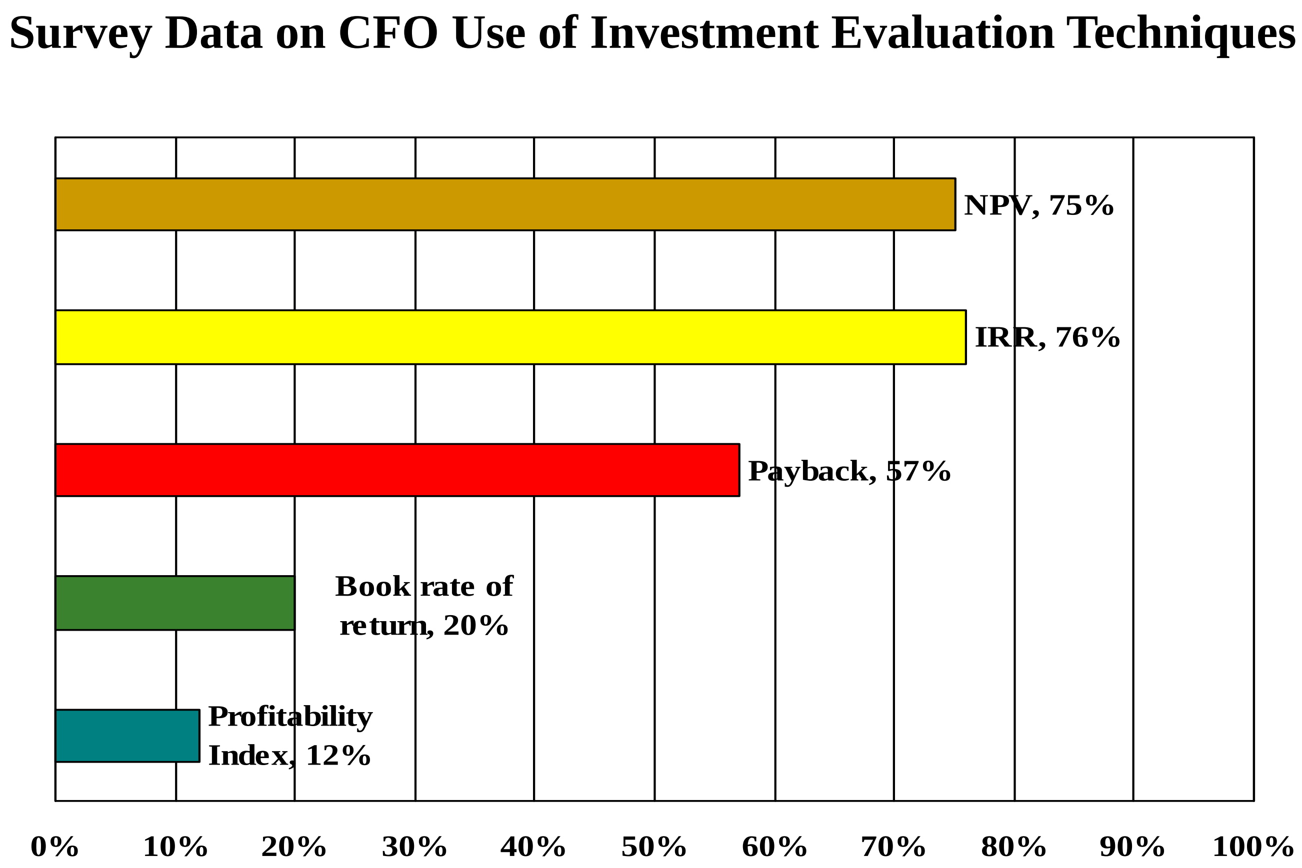
(see part (c) of this chapter)

Step 3: Discount future cash flows

$$\text{NPV} = \text{C}_0 + \text{PV} = \text{C}_0 + \frac{C_1}{1+r_1} + \frac{C_2}{(1+r_2)^2} + \frac{C_3}{(1+r_3)^3} + \ldots + + \frac{C_T}{(1+r_T)^T}$$

Step 4: Go ahead if PV of payoff exceeds investment, i.e. if NPV > 0

But, are there other criteria?



SOURCE: Graham and Harvey, "The Theory and Practice of Finance: Evidence from the Field," Journal of Financial Economics 61 (2001), pp. 187-243.

Rate of return: an example

1. Buy and sell a firm (a project). Asset value in two subsequent periods:
   - AV_0: 80m and AV_1: 96.8m
   - Return: r = (96.8 – 80)/80 = 0.21 or 21%
2. Value in two non-subsequent periods:
   - AV_0: 80m and AV_2: 96.8m, return: ?
   - Numerical method: find *r* such that Net Present Value (NPV) = 0

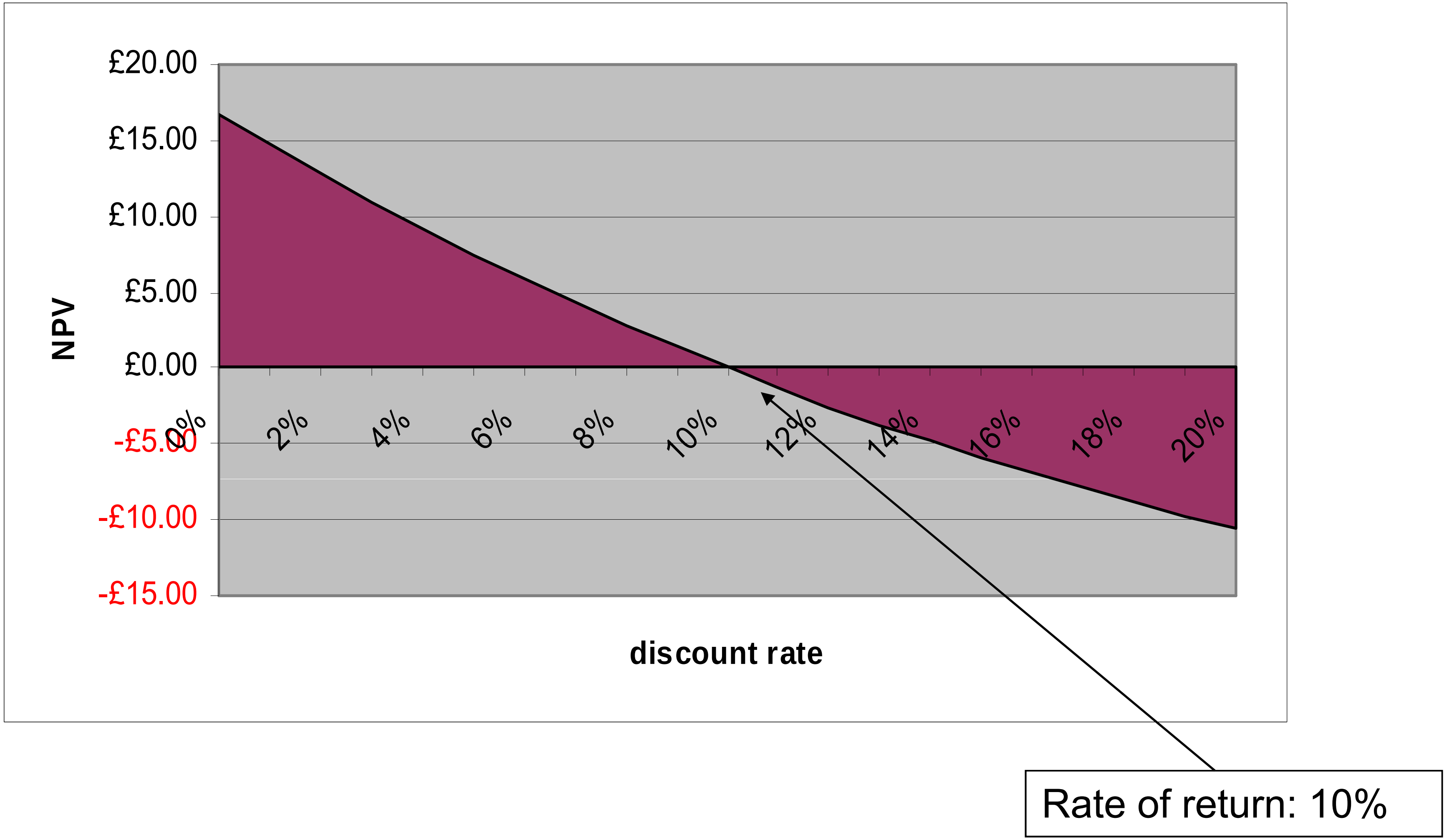
$$NPV = -80 + \frac{96.8}{(1+r)^2} = 0 \quad \text{or} \quad r = 0.10 = 10\%$$

   - In other words,

$$80(1+0.1)(1+0.1) = 96.8$$

   - What is the rate of return in the first example?

Example 2


£20.00
£15.00
£10.00
£5.00
£0.00
-£5.00
-£10.00
-£15.00
NPV
0%
2%
4%
6%
8%
10%
12%
14%
16%
18%
20%
discount rate
Rate of return: 10%

Introducing revenues and costs

- No revenues:
  - AV_0: 80m and AV_2: 96.8m

$$NPV = -80 + \frac{96.8}{(1+r)^2} = 0 \quad \text{or} \quad r = 10\%$$

- With revenues:
  - AV_0: 80m, AV_2: 96.8m, R_1: 2m, R_2: 2m

$$NPV = -80 + \frac{2}{(1+r)^1} + \frac{2}{(1+r)^2} + \frac{96.8}{(1+r)^2} = 0 \quad \text{or} \quad r = 12.4\%$$

- With revenues and costs:

AV_0: 80m, AV_2: 96.8m, R_1: 2m, R_2: 2m, C_1: 1m, C_2: 1.2m

$$NPV = -80 + \frac{1}{(1+r)^1} + \frac{0.8}{(1+r)^2} + \frac{96.8}{(1+r)^2} = 0 \quad \text{or} \quad r = 11.08\%$$

In general, the Rate of Return Rule:

- More generally, the internal rate of return of a cash flow stream is the interest rate y that makes the NPV of a project equal to 0:

$$0 = C_0 + \frac{C_1}{1+y} + \frac{C_2}{(1+y)^2} + \frac{C_3}{(1+y)^3} + \ldots + + \frac{C_T}{(1+y)^T}$$

- Accept investments offering rates of return in excess of the appropriate discount rate (“opportunity cost of capital”)

IRR and NPV

- Same criteria if NPV is decreasing wrt discount rate
- However, the IRR has some pitfalls:
  - If NPV increases ("lending" money instead of "borrowing"), we should ask for an IRR lower than opportunity cost of capital
  - There might be several IRRs or none
  - Ignores magnitude and cannot select among different projects
  - Even more problematic if we discount rates are not stable over time (with which one do we compare?)

Payback period and the payback rule

- The payback period is the number of periods (years) it takes before the cumulative forecasted cash flow equals the initial outlay

- The payback rule says only accept projects that "payback" in the desired time frame

- This method is flawed, primarily because it ignores later year cash flows and the present value of future cash flows

Example

Examine the three projects and note the mistake we would make if we insisted on only taking projects with a payback period of 2 years or less.

Project	C_0	C_1	C_2	C_3	Payback Period	NPV@ 10%
A	- 2000	500	500	5000	3	+ 2,624
B	- 2000	500	1800	0	2	- 58
C	- 2000	1800	500	0	2	+ 50

Project Selection

- If only one from a set of positive NPV projects can be selected, we should select that with the largest NPV
- When resources are limited, the profitability index (PI) helps selecting among various project combinations and alternatives:
  - $PI = (NPV - C_0) / (-C_0) = PV / (-C_0)$
  - If resources are unlimited, we should select projects with PI>1. Why?
  - Limited resources and projects can yield various combinations
  - Example: Eur100,000 for two scalable projects (numbers in Eur1,000)

Project	C_0	C_1	C_2	PV @10%	PI
A	-1	$+22$	-12.1		
B	-4	$+44$	-24.2		

Project Selection

- If only one from a set of positive NPV projects can be selected, we should select that with the largest NPV
- When resources are limited, the profitability index (PI) helps selecting among various project combinations and alternatives:
  - PI = (NPV - C_0) / (-C_0) = PV / (-C_0)
  - If resources are unlimited, we should select projects with PI>1. Why?
  - Limited resources and projects can yield various combinations
  - Example: Eur100,000 for two scalable projects (numbers in Eur1,000)

Project	C_0	C_1	C_2	PV @10%	PI
A	−1	+22	−12.1	9	9
B	−4	+44	−24.2	16	4

Part (c): adjusting for risk!

Risky cash flows

Future cash flows should now be "expected cash flows"

Discount rate may need to be higher than risk-free rate

Example: a project expected to generate CF=$100milllion per year for three years. PV for a discount rate *r = 12%*?

Project A

Year	Cash Flow	PV @ 12%
1	100	89.3
2	100	79.7
3	100	71.2
	PV Total	240.2

Separating adjustments for risk and time?

If risk-free rate $r_f = 6\%$, which cash-flow reduction would you accept to get them with certainty (i.e. what is your certainty equivalent cash flow)?

$$\frac{\mathrm{CEQ}_1}{1.06} = 89.3 \text{ or } \mathrm{CEQ}_1 = 94.6$$

$$\frac{CEQ_t}{(1+r_f)^t} = \frac{C_t}{(1+r)^t} = PV$$

Project A

Year	Cash Flow	PV @ 12%
1	100	89.3
2	100	79.7
3	100	71.2
	PV Total	240.2

Project B

Year	Cash Flow	PV @ 6%
1	94.6	89.3
2	89.6	79.7
3	84.8	71.2
	PV Total	240.2

Cash flow reductions

Year	Cash Flow	CEQ	Risk deduction
1	100	94.6	5.4
2	100	89.6	10.4
3	100	84.8	15.2

Larger risk deduction for later periods

Not necessary to discount at higher rates distant periods to generate a larger risk deduction

Risk premium

Risk premium makes risky and risk-free cash flows equally attractive

Formally, risk premium can be defined as the r_i such that…

$$\frac{C_t}{(1+r_i)^t} = CEQ_t$$

In our case,

$$r_i = \frac{100}{94.6} - 1 = 0.57 \text{ or } 5.7\%$$

Risk premium

Given that certainty-equivalent is defined as

$$\frac{CEQ_t}{(1+r_f)^t} = \frac{C_t}{(1+r)^t}$$

and risk-premium is given by

$$\frac{\mathrm{C_t}}{(1+r_\mathrm{i})^t} = \mathrm{CEQ_t}$$

We have that risk premium is given by

$$(1+r) = (1+r_\mathrm{f})(1+r_\mathrm{i})$$

and approximately…

$$r \approx r_\mathrm{f} + r_\mathrm{i}$$

Finding the discount rate or the "risk premium"

- Project's cash flows should be appropriately discounted:
  - what is the return the firm can receive on similar but alternative investments (i.e. investments that bear the same risks?)
  - discount rate sometimes called "cost of capital" as it measures the opportunity cost of the funds

- How to compute the "cost of capital" of the project?

- Often start computing the whole firm's cost of capital:
  - Loads of projects have similar risk as the firm as a whole
  - If not, good starting point that can be adjusted for:
    - If project has higher risk relative to the firm as a whole
    - For example, if project has high fixed costs, it will have more risk

Finding the discount rate and the “risk premium”

- Estimate the firm’s cost of capital:
  - First, suppose that firm is financed with equity only
  - Second, incorporate possibility that it has debt
- In this part (c), compute a firm’s expected equity return:
  - Basic tools for “portfolio” theory (risk-return trade-off)
  - Mean variance analysis and portfolio representation
  - The Capital Asset Pricing model (CAPM)
  - Factor models and the Arbitrage Pricing Theory (APT)
- In the next chapter: incorporate debt and compute the weighted average cost of capital (WACC)

Portfolio Tools and Diversification

Constructing "Portfolios"

- Investing in multiple stocks: constructing a "portfolio"

- Portfolio weight for stock j: $x_j = \dfrac{\text{Dollars held in stock j}}{\text{Dollar value of the portfolio}}$

  - Example of a portfolio: £100 in BT and £300 in BP

$$(x_{BT}, x_{BP}) = (1/4, 3/4)$$

- Properties:
  - Weights should add up to 1
  - Weights can be either positive ("long position") or negative ("short")

Remember?

- Investment return (historical return) : $r_{i,t} = \dfrac{p_t + d_t - p_{t-1}}{p_{t-1}}$

  $$r_1, r_2, r_3, \ldots r_N \text{ or } r_{BT}, r_{BP}$$

- Expected return (forward looking):

  $$\overline{r_1}, \ldots \overline{r_N} \text{ or } \overline{r_{BT}}, \overline{r_{BP}}$$

- Variance and standard deviation of an investment:

  $$\text{var}(r_i) = \sigma_i^2 = E[(r_i - \overline{r_i})^2] \qquad \sigma_i = \sqrt{\text{var}(r_i)}$$

  - Standard deviation has same units as returns

- Covariance of two investments 1 and 2:

  $$\text{cov}(r_i, r_j) = \sigma_{i,j} = E[(r_i - \overline{r_i})(r_j - \overline{r_j})]$$

  - Interpretation: measure of relatedness. Move together?
  - Depends on units but…

Remember?

- A useful measure of the co-movement of two returns is the correlation coefficient ρ.

- $\rho_{i,j} = \dfrac{\text{cov}(r_i, r_j)}{\sigma_i \sigma_j} = \dfrac{\sigma_{i,j}}{\sigma_i \sigma_j}$ and $\rho_{i,j}$ ε [-1,1]

  - When $\rho_{i,j}$ = 1 (or -1), the assets' returns are perfectly positively (or negatively) correlated, i.e. always move together (or in opposite directions)

  - When $\rho_{i,j}$ = 0, the assets' returns are uncorrelated

Expected Return

- Portfolio return:
  - Portfolio-weighted average of returns of assets in the portfolio
  - Example: if BT's return has been 10% and BP's 5% and portfolio (0.25,0.75) then…
  - Portfolio return: 0.25*0.10+0.75*0.05=0.0625 or 6.25%

- Expected portfolio return:
  - Portfolio-weighted average of expected returns
  - If the portfolio is $P=(x_1,\ldots, x_N)$ then

$$E(r_P) = \sum_{t=1}^{N} x_i \overline{r_i}$$

Variances and Covariances of a Portfolio

- For any two-stock portfolio…

$$\sigma_p^2 = \mathrm{var}(x_1 r_1 + x_2 r_2) = x_1^2 \sigma_1^2 + x_2^2 \sigma_2^2 + 2 x_1 x_2 \sigma_{1,2}$$

- Hence… larger covariance leads to higher portfolio variance

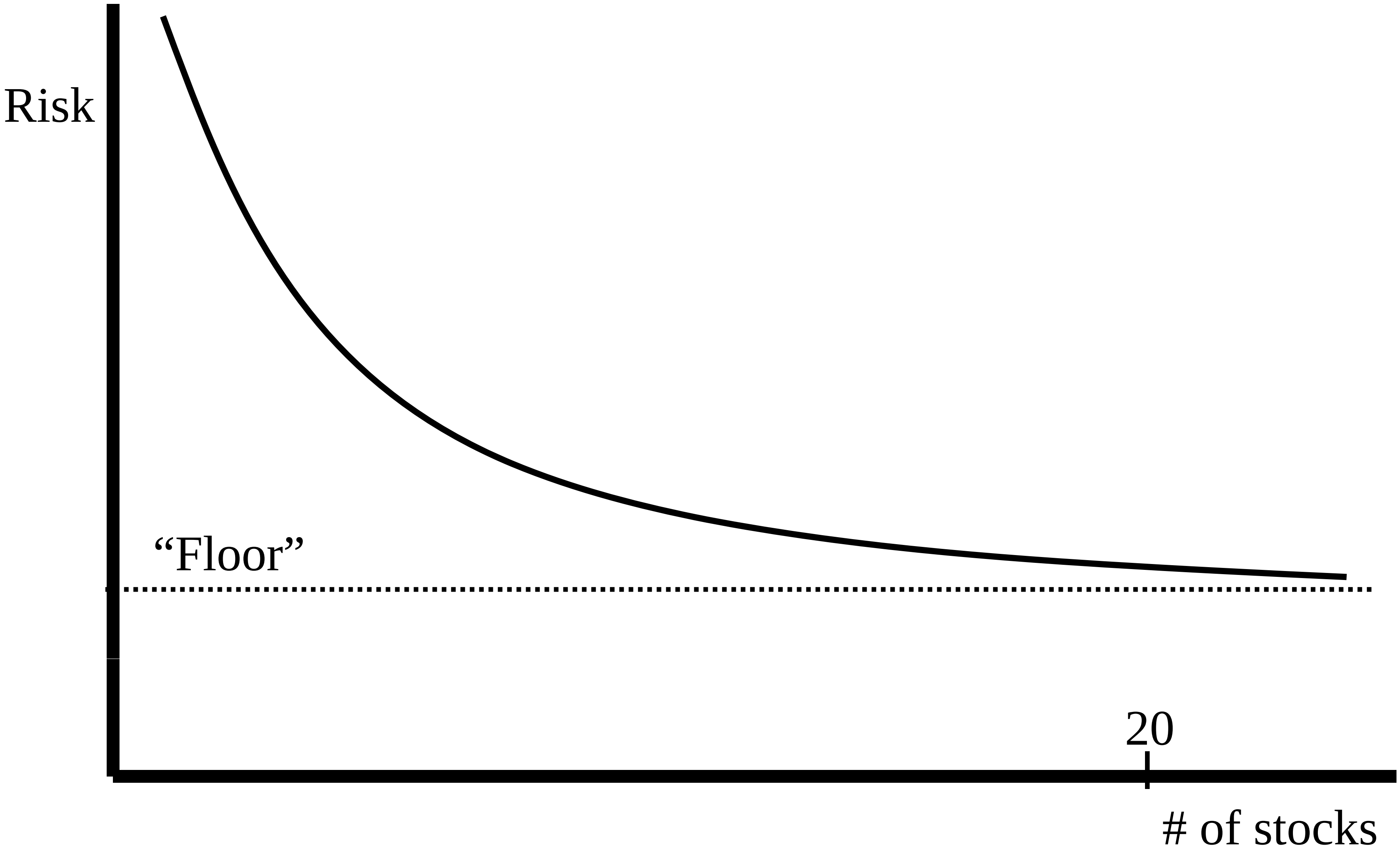
$$= x_1^2 \sigma_1^2 + x_2^2 \sigma_2^2 + 2 x_1 x_2 \rho \sigma_1 \sigma_2$$

$$\leq x_1^2 \sigma_1^2 + x_2^2 \sigma_2^2 + 2 x_1 x_2 \sigma_1 \sigma_2 = (x_1 \sigma_1 + x_2 \sigma_2)^2$$

- With strict inequality if ρ<1
- Thus..

$$\sigma_p \leq x_1 \sigma_1 + x_2 \sigma_2$$

How Large Diversification Benefits are?

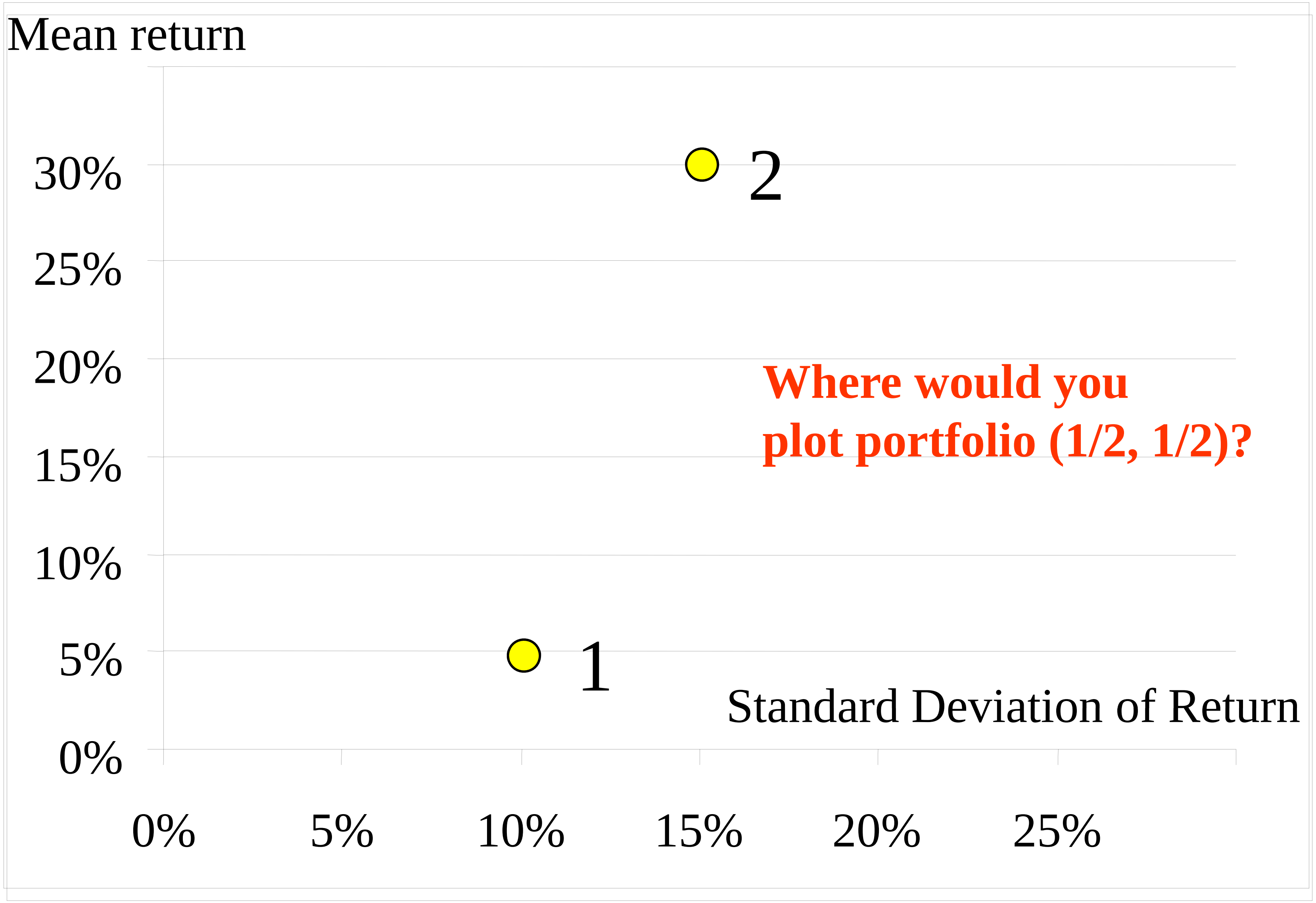


Mean Variance Analysis

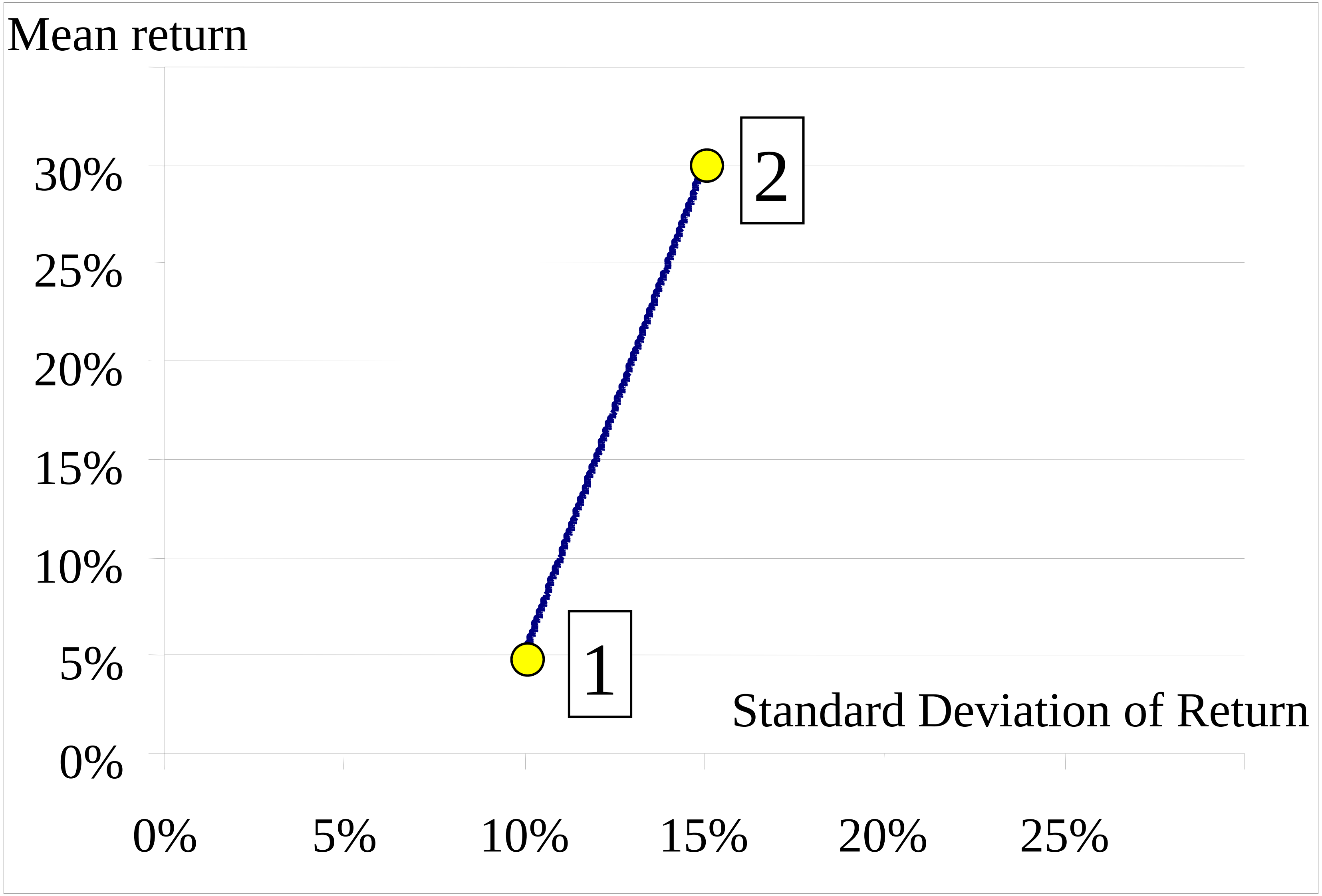
Portfolio problem

- How best to combine many assets in order to…
  - maximize expected return for a given variance (i.e. risk)
  - minimize variance (i.e. risk) for a given expected return.
- In other words, how to construct the set of "mean-variance efficient" portfolios?
- We assume frictionless markets:
  - all investments are tradable in any quantity (no restrictions on short-positions)
  - no transaction costs, regulations or tax consequences
- Assume first all assets are risky and then introduce risk-free asset

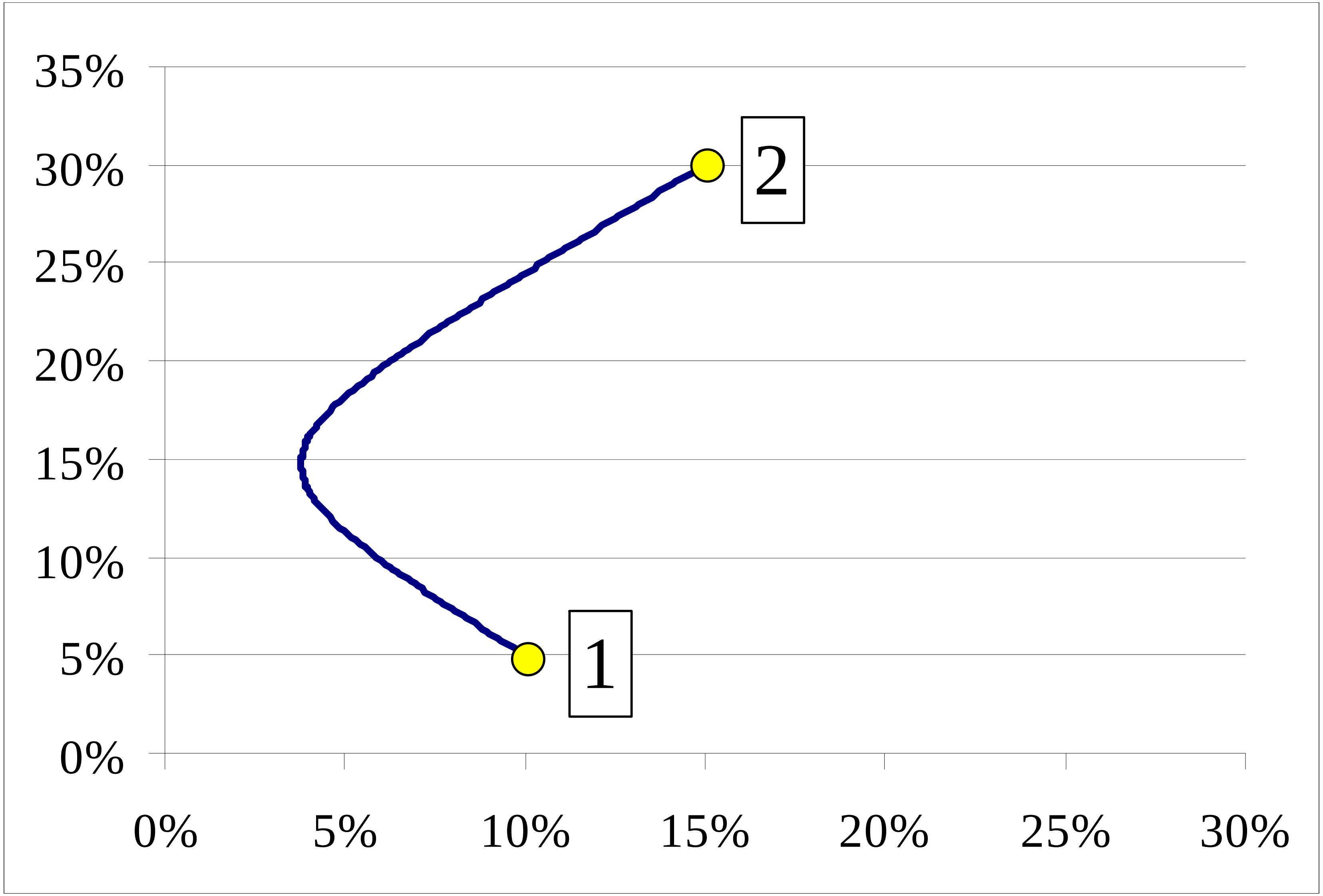
Representation: Mean-Variance Diagrams



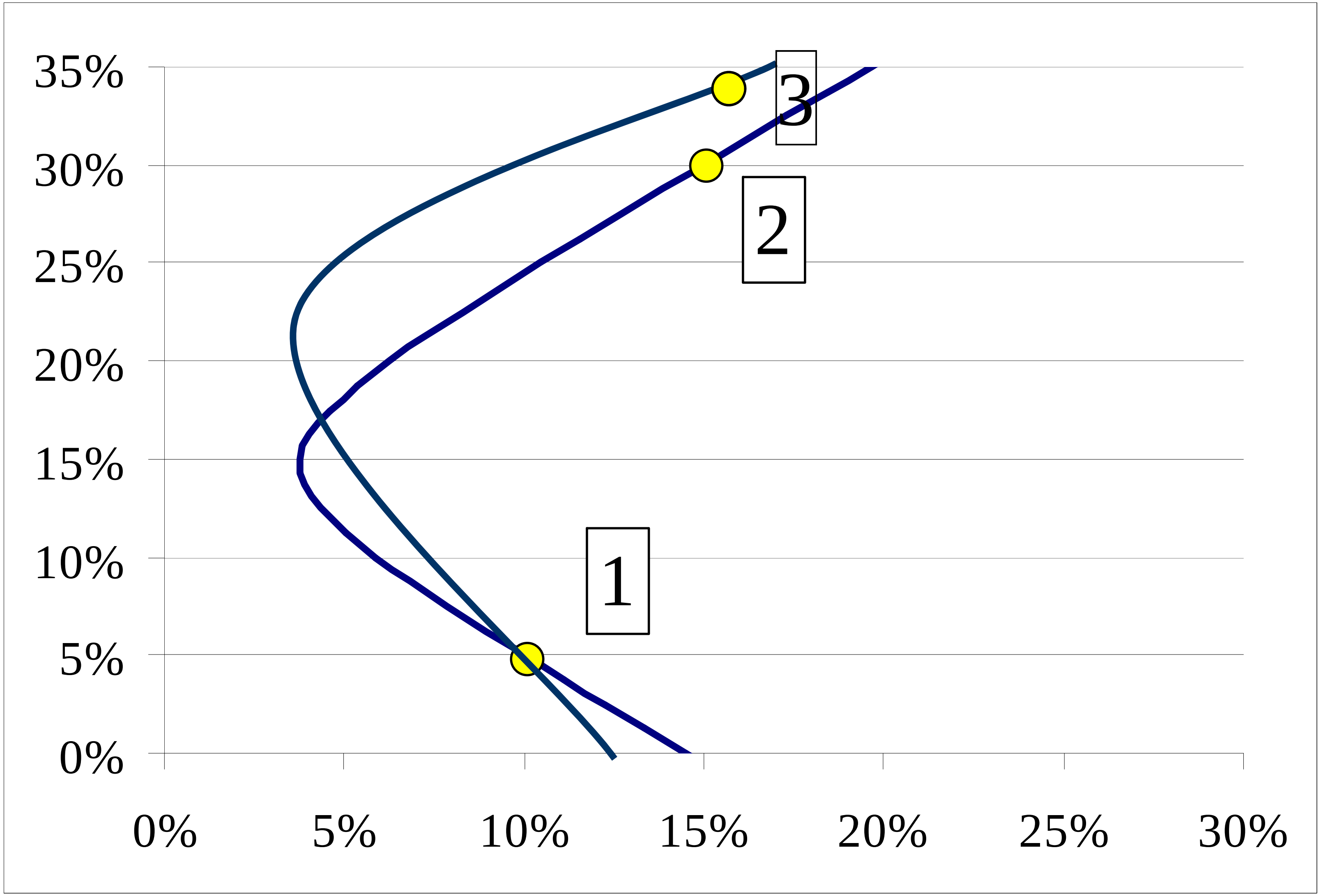
Portfolio of two risky assets (Perfect Correlation)



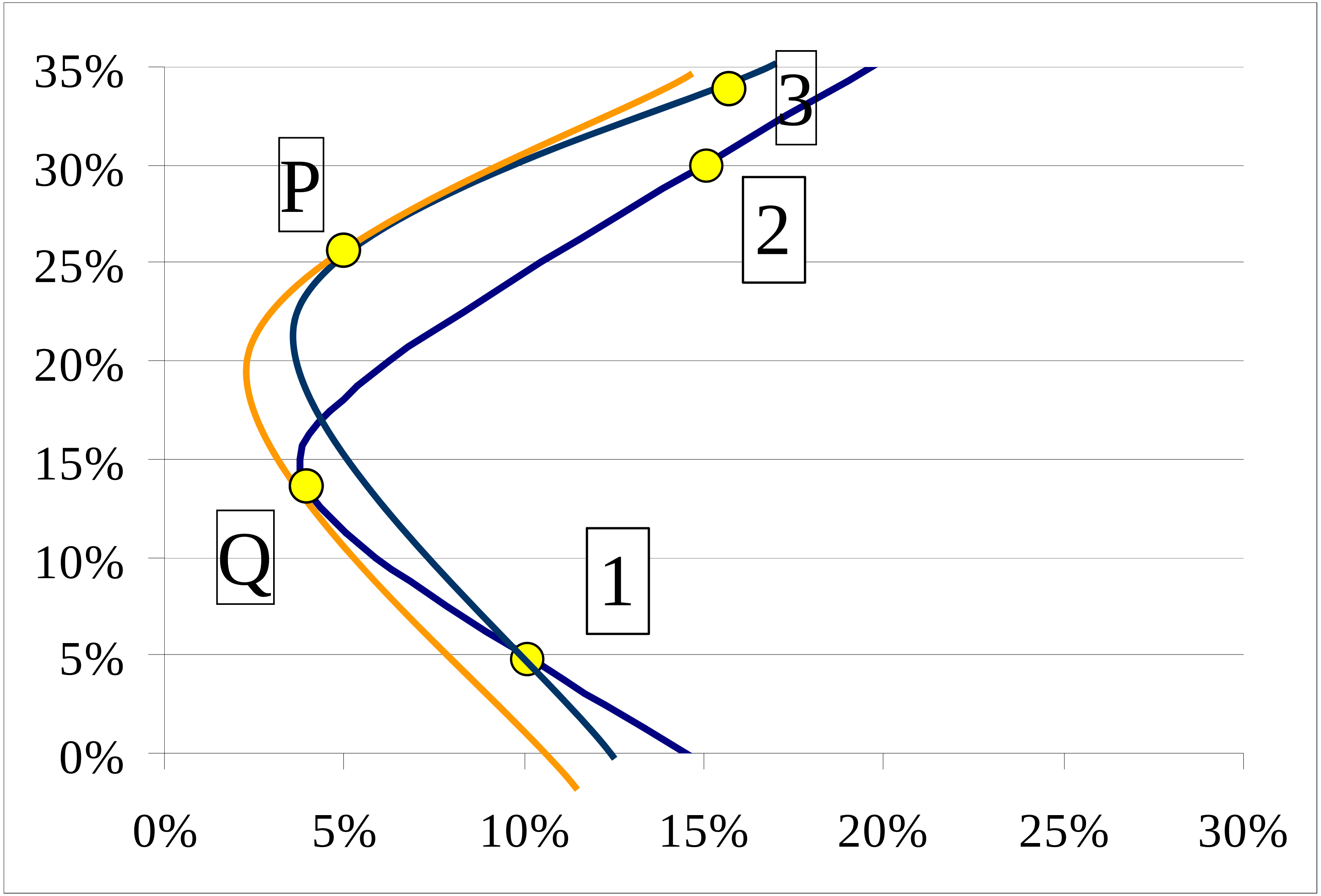
Portfolio of two risky assets (Imperfect Correlation)



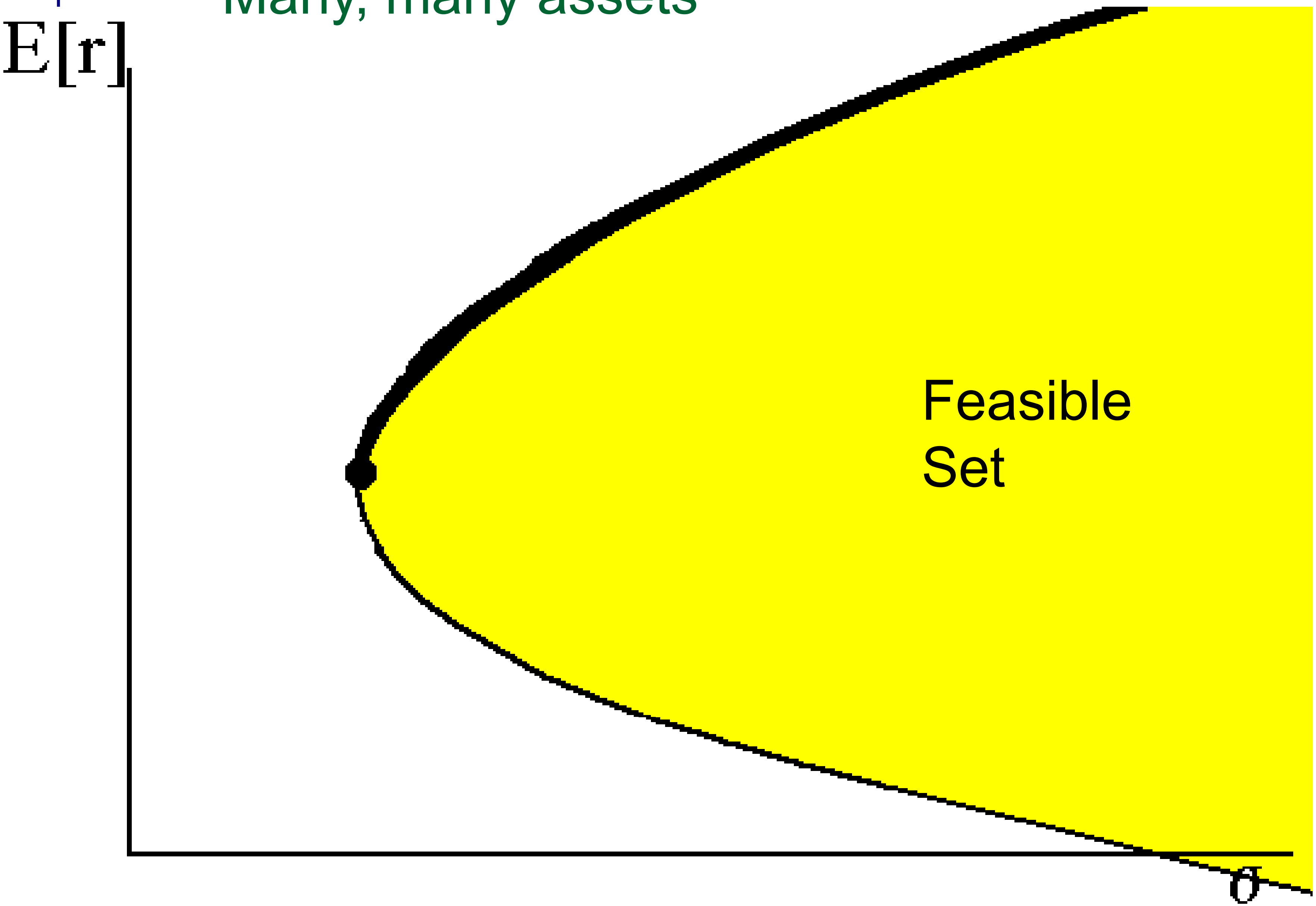
New Asset



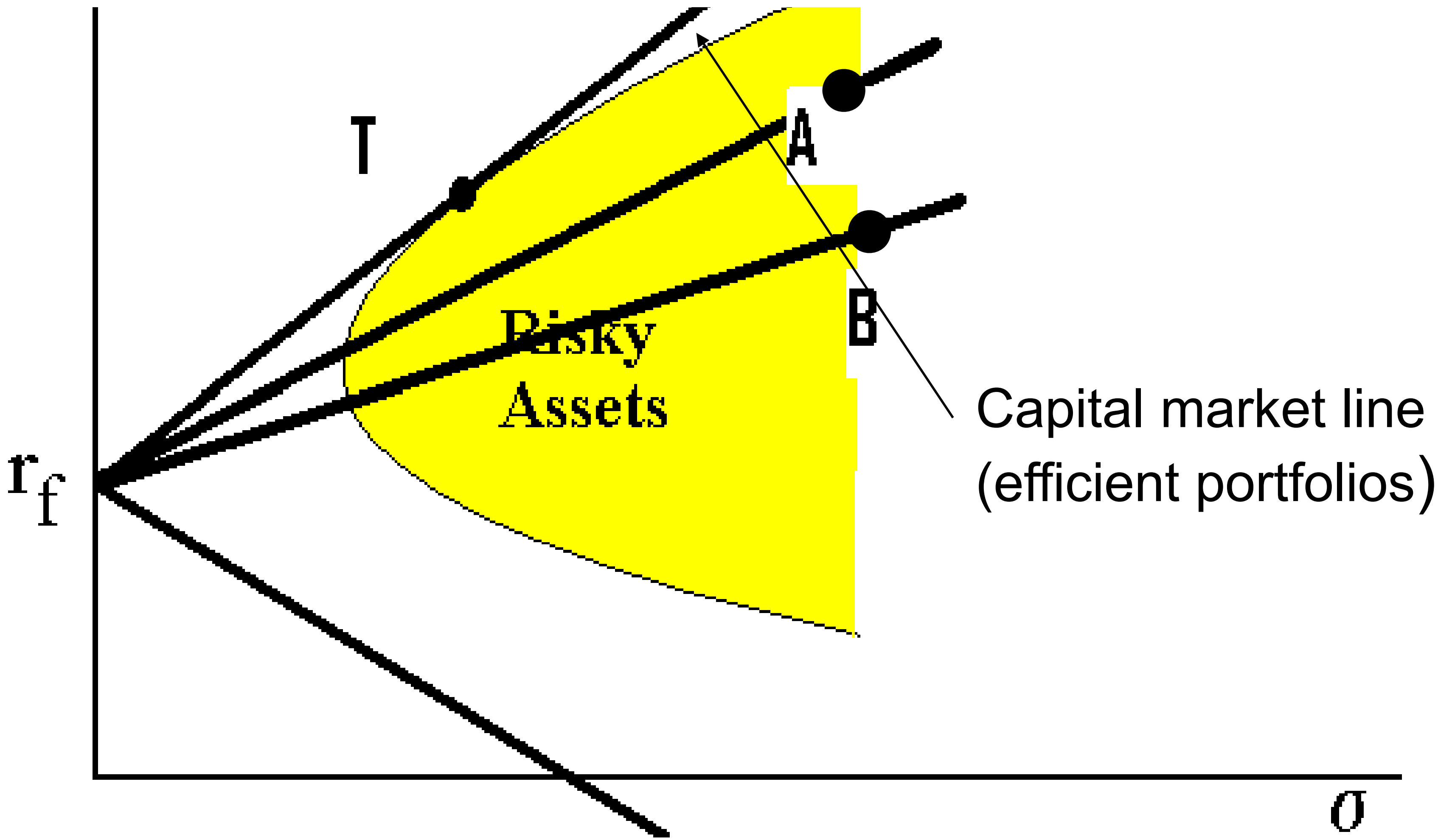
Portfolio of Portfolios is another Portfolio!

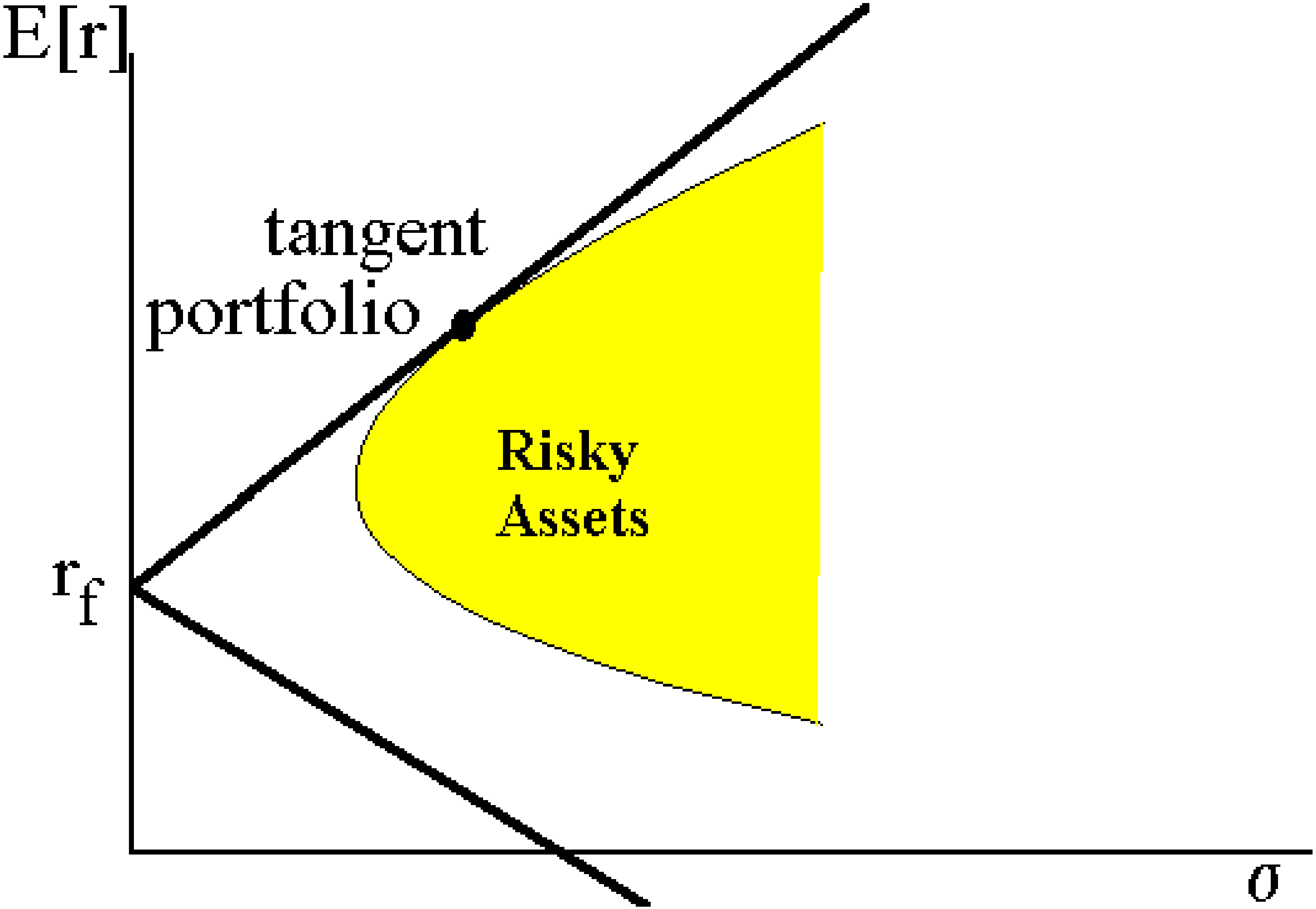


Many, many assets



Adding a Risk-free Asset




E[r]
tangent
portfolio
Risky
Assets
r_f
σ

Nice Mathematical Result: Risk & Return

- For any investment *i*, it *can be shown* that

$$E[r_i] - r_f = \beta_{iT} (E[r_T] - r_f) \text{ where } \beta_{iT} = \frac{\sigma_{i,T}}{\sigma_T^2}$$

$$E[r_i] = r_f + \beta_{iT} (E[r_T] - r_f) \text{ where } \beta_{iT} = \frac{\sigma_{i,T}}{\sigma_T^2}$$

- Tangency portfolio key to relate expected return on any investment with a measure of its risk: the covariance.

- Allows us to estimate risk premium of any asset!

Finding the Tangency Portfolio

- Derive solving complex matrix algebra:
  - Relatively easy for investment across countries or asset types
  - Low number of investments and good parameter estimates

- But may be computationally demanding (or impossible):
  - Need to estimate all means and covariances
  - Difficult for individual investment selection: there are loads!

- CAPM tell us which should be the tangency portfolio

The CAPM

Assumptions and Conclusion

1. Markets are frictionless
2. There is a risk-free asset that returns r_f
3. All investors want to hold efficient frontier portfolios;
4. Supply equals demand in financial markets (we are in *equilibrium*)
5. Investors have homogenous beliefs about means and st deviations

- These assumptions are sufficient to apply previous results
- And, we also get: tangent porfolio is given by the market portfolio (m):
  - the portfolio of all risky assets, where the weight of each asset is the market value (market capitalisation) divided by the total market value
  - Examples of rough "approximations" of the market portfolio?

- Substituting in the previous formula

$$E[r_i] = r_f + b_i (E[r_m] - r_f) \quad \text{where } b_i = b_{mi} = \frac{\sigma_{mq}}{\sigma_m^2}$$

Computing the market portfolio

Three-stock economy: HP, IBM, CPQ and US Treasuries.

	HP	IBM	CPQ
Price per share	$33	$95	$20.25
Shares outstanding	2 bill	1.758 bill	1.7 bill

- Market portfolio: (UST, HP,IBM,CPQ)=(0, 0.25, 0.62, 0.13)
  Risk free asset: (UST, HP,IBM,CPQ)=(1, 0, 0, 0)

- All investors should hold a combination of the risk-free and the market portfolio (i.e. the same relative positions in risky assets)

Extensions: Factor Models and APT

Factor models

- The return of a risky investment is determined by:
  - Common factors (e.g. interest rates, inflation, productivity…)
  - A firm-specific component (new R&D results, fire in a plant,…)
- Return variances of large portfolios are determined by common factors, firm-specific ones can often be ignored
- Common factors do not affect all investments equally: each has its sensitivities to the factors (“factor betas”)
  - E.g stock of car company more sensitive to changes in interest rate than stock of a soft drink firm
  - Car companies are highly affected by interest rate (factor) risk
- Factor models can be used to estimate the expected rate of return of an investment, as an alternative to the CAPM:
  - Arbitrage pricing theory: relation of factor risk to expected return

A One-Factor Model: the Market Model

- Run the following regression:

$$r_{Dell} = \alpha_{Dell} + \beta_{Dell} r_{S\&P500} + \varepsilon_{Dell}$$

- If $r_{S\&P}$ and ε_{Dell} are uncorrelated:

$$\sigma^2_{Dell} = \beta^2_{Dell} \sigma^2_{S\&P500} + \sigma^2_{\varepsilon_{Dell}}$$

- Risk can be divided in two:
  - Systematic, market risk: part explained by market movements
  - Unsystematic risk: part not explained by market movements

Unsystematic and Diversifiable Risk

- Unsystematic risk may be related to other factor risks:
  - Car company highly affected by interest rate risk
  - Part of this effect shows up in the residual of previous equation
  - As a result, not all unsystematic risk is diversifiable
- However, if for all the investments i we had

$$r_i = \alpha_i + \beta_i r_{\mathrm{m}} + \varepsilon_i$$

such that all ε_i were uncorrelated then ε_i would be firm-specific and therefore the related risk would be diversifiable

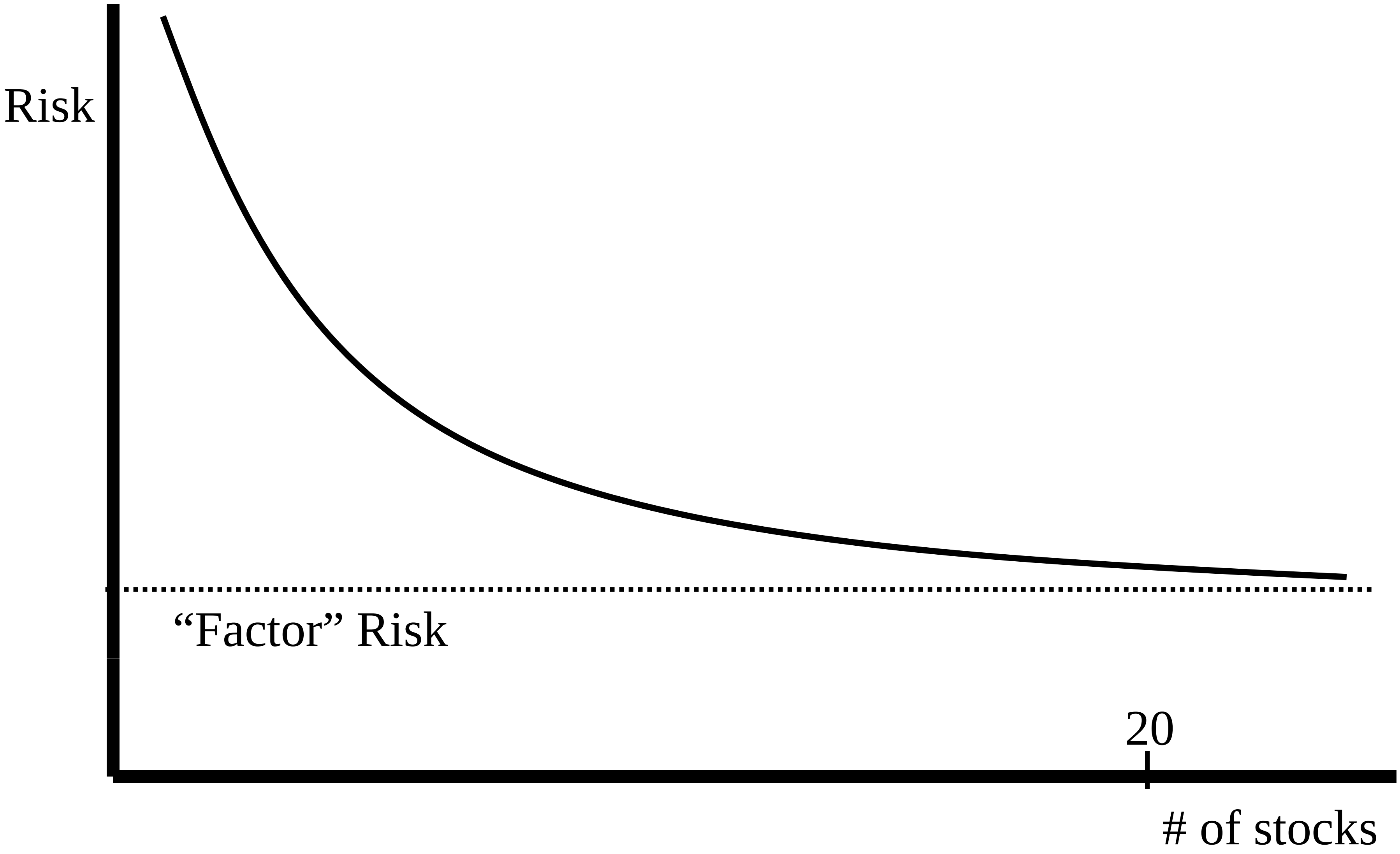
More Factors

- The last assumption however is unrealistic
- Need more factors. More generally, one could specify:

$$r_i = \alpha_i + \beta_{i,1} r_{\text{Factor 1}} + \ldots + \beta_{i,K} r_{\text{Factor K}} + \varepsilon_i$$

  - Returns are assumed to be generated by relatively small number of factors
  - Betas are the sensitivities to each factor
  - ε_i are uncorrelated firm-specific components
- Factors:
  - Other examples: industrial production, oil prices,..
  - Usually rescaled to have mean of zero
- Risk from…
  - Common factors cannot be eliminated by diversification
  - Unique factors can be eliminated and should be ignored

How Large are Diversification Benefits?



Arbitrage Pricing Theory

- Under a set of assumptions:

$$\bar{r}_i - r_f = \beta_{i,1}(\bar{r}_{\text{Factor 1}} - r_f) + \ldots + \beta_{i,K}(\bar{r}_{\text{Factor K}} - r_f)$$

- A diversified portfolio with 0 sensitivity to each macro factor…
  - Is essentially risk-free and should offer no market premium
  - If the return is higher or lower than the risk-free rate then profits can be made by arbitrage

- A diversified portfolio with sensitivity to the factors…
  - Should offer a risk premium proportional to its sensitivity to the factor
  - Otherwise, profits from arbitrage can be made!

Example: Arbitrage Pricing Theory

Estimated risk premiums for taking on risk factors (1978-1990)

Factor	Estimated Risk Premium $(r_{factor} - r_f)$
Yield spread	5.10%
Interest rate	-.61
Exchange rate	-.59
Real GNP	.49
Inflation	-.83
Market	6.36