# Corporate Finance

# Lecture 3: Mean Variance Analysis

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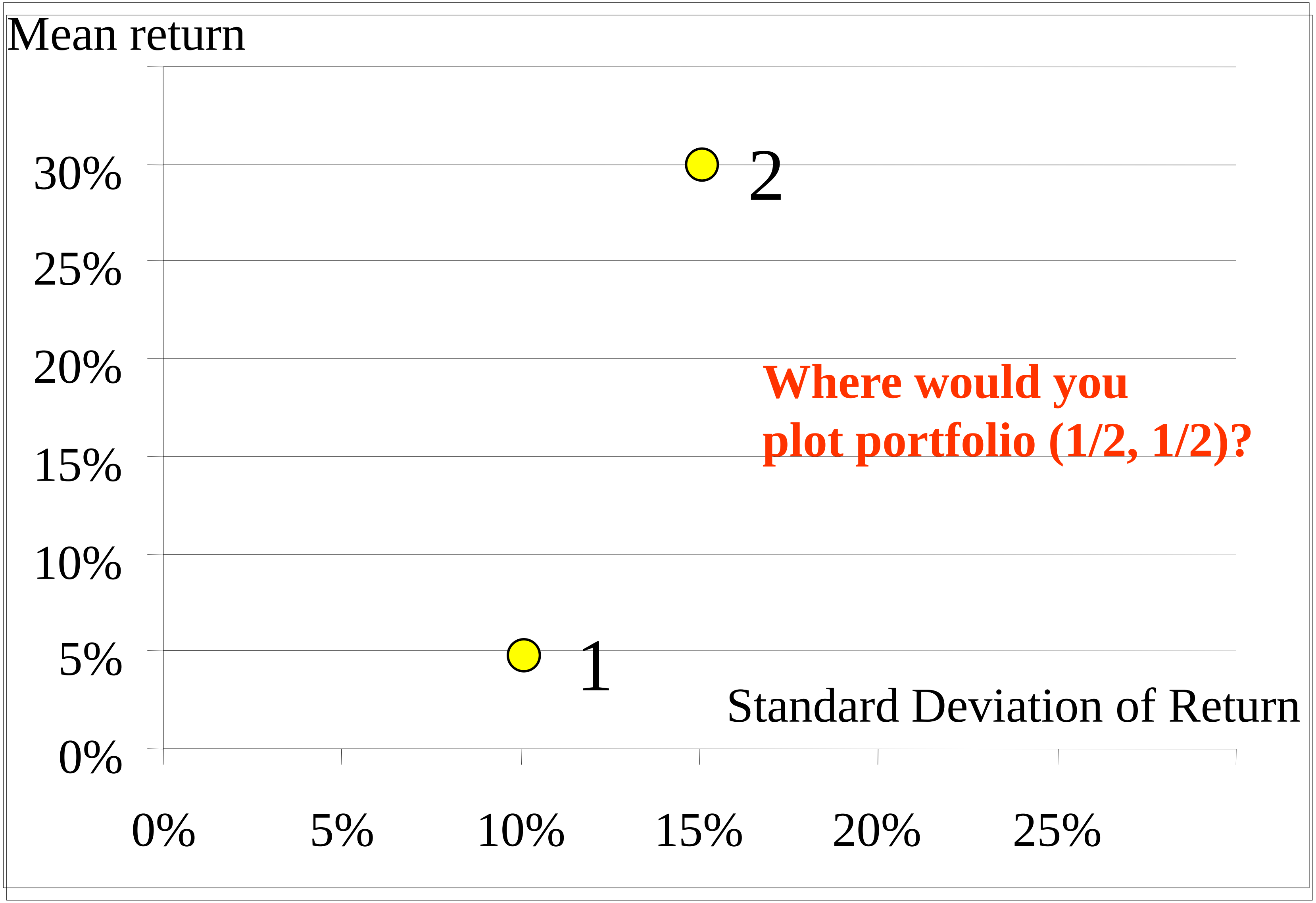
# Portfolio Problem

- How best to combine many assets in order to…
  - maximize expected return for a given variance (i.e. risk)
  - minimize variance (i.e. risk) for a given expected return.
- In other words, how to construct the set of “mean-variance efficient” portfolios?
- We assume frictionless markets:
  - all investments are tradable in any quantity (no restrictions on short-positions)
  - no transaction costs, regulations or tax consequences
- Two cases: a risk-free asset is not and is available

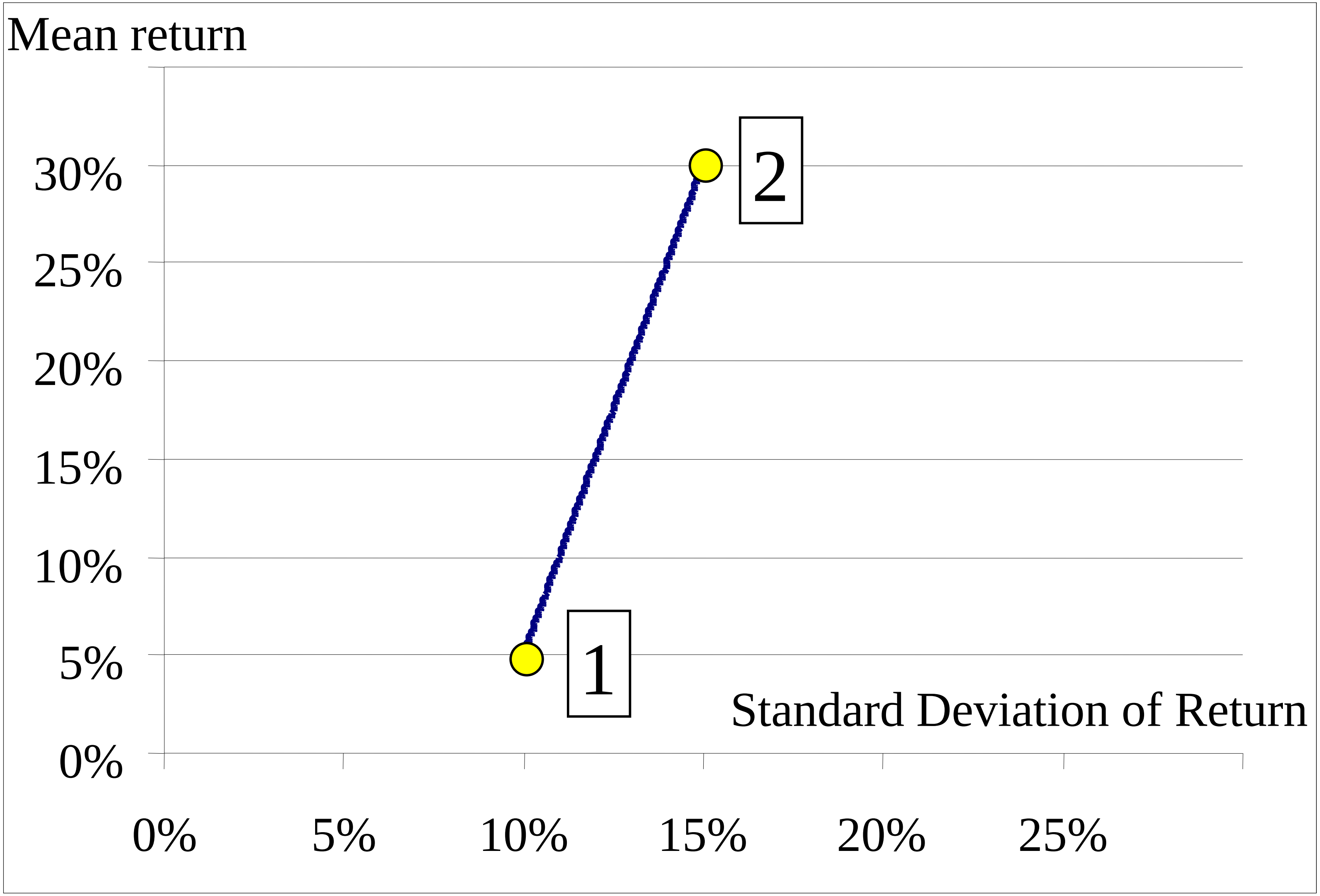
# This Chapter

- Risky environment:
  - Representing risky portfolios
  - Minimum variance portfolio and efficient frontier
  - Properties
- Introducing a risk-free asset:
  - Representing portfolios including a risk-free asset
  - New efficient frontier
  - Properties
- The risk and return equation

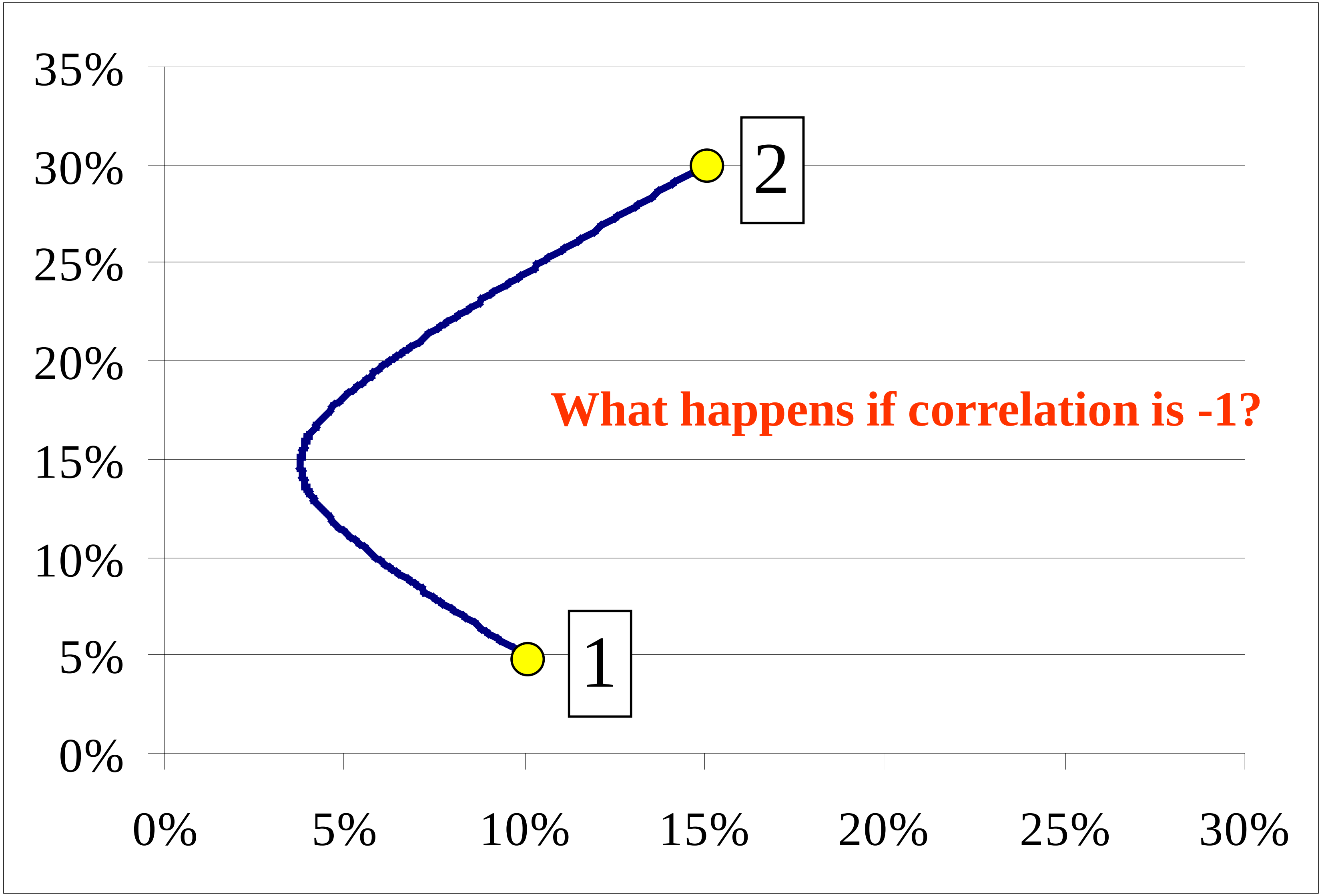
# Representation: Mean-Variance Diagrams



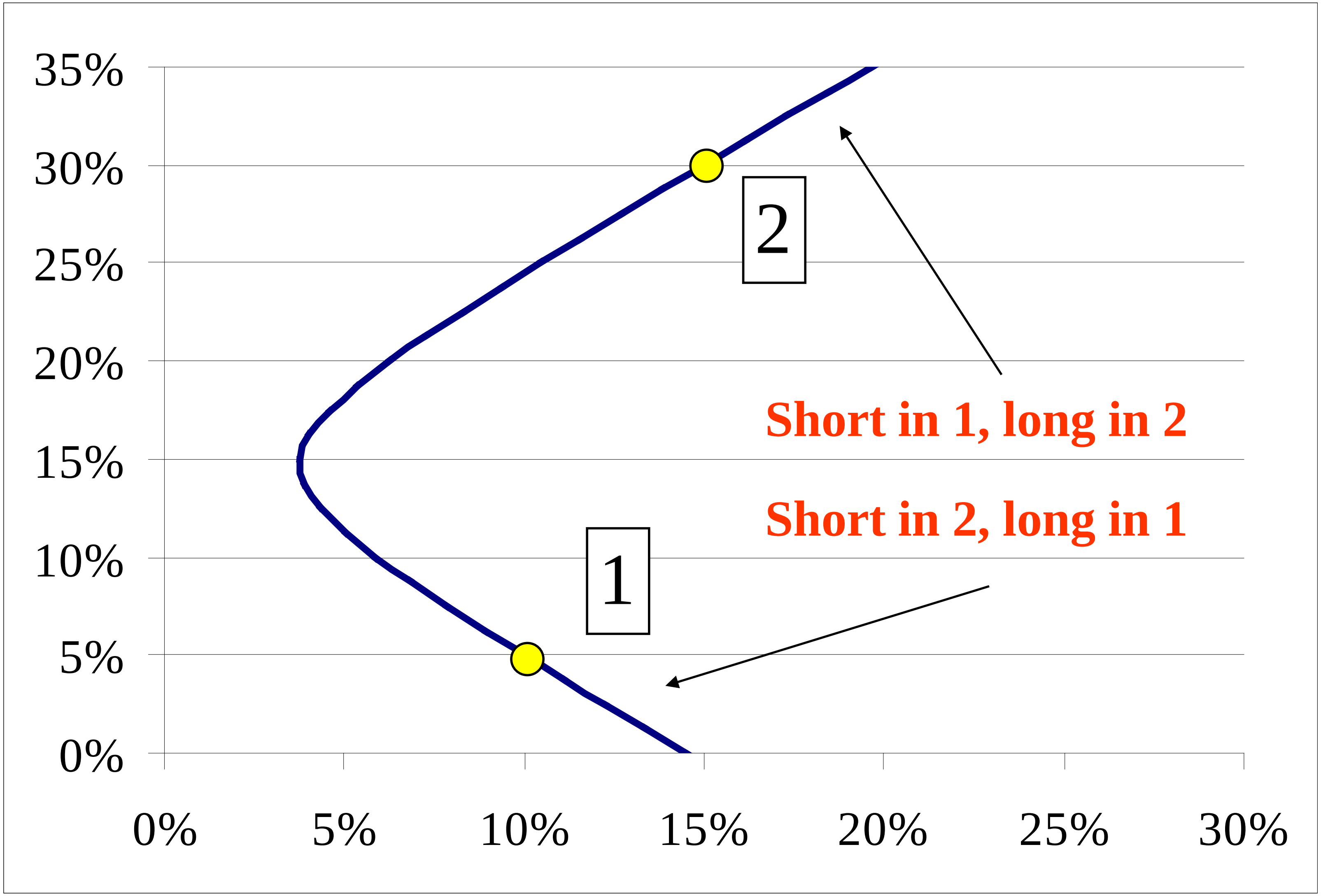
# Two-Asset Portfolio (Perfect Correlation)



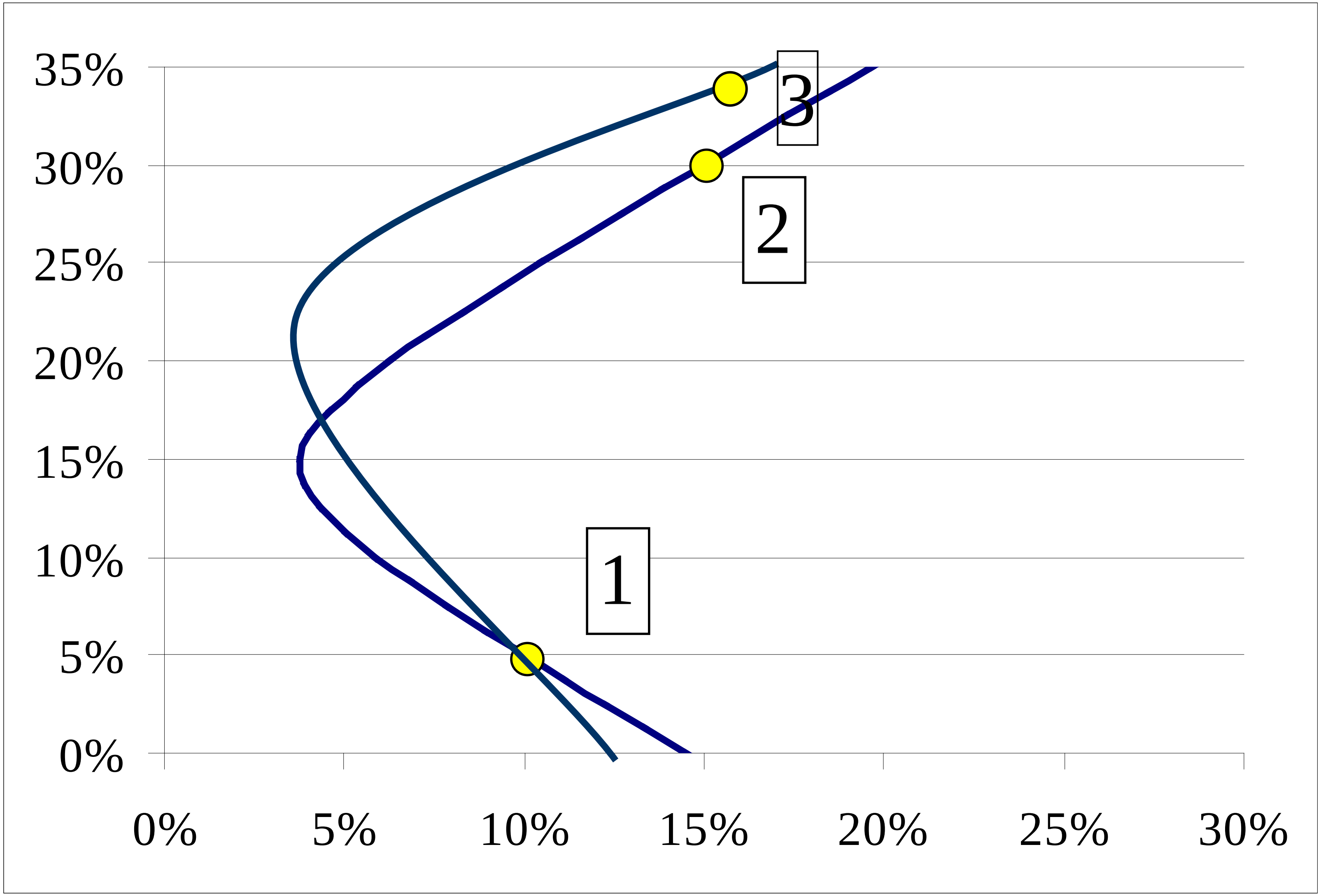
# Two-Asset Portfolio (Imperfect Correlation)



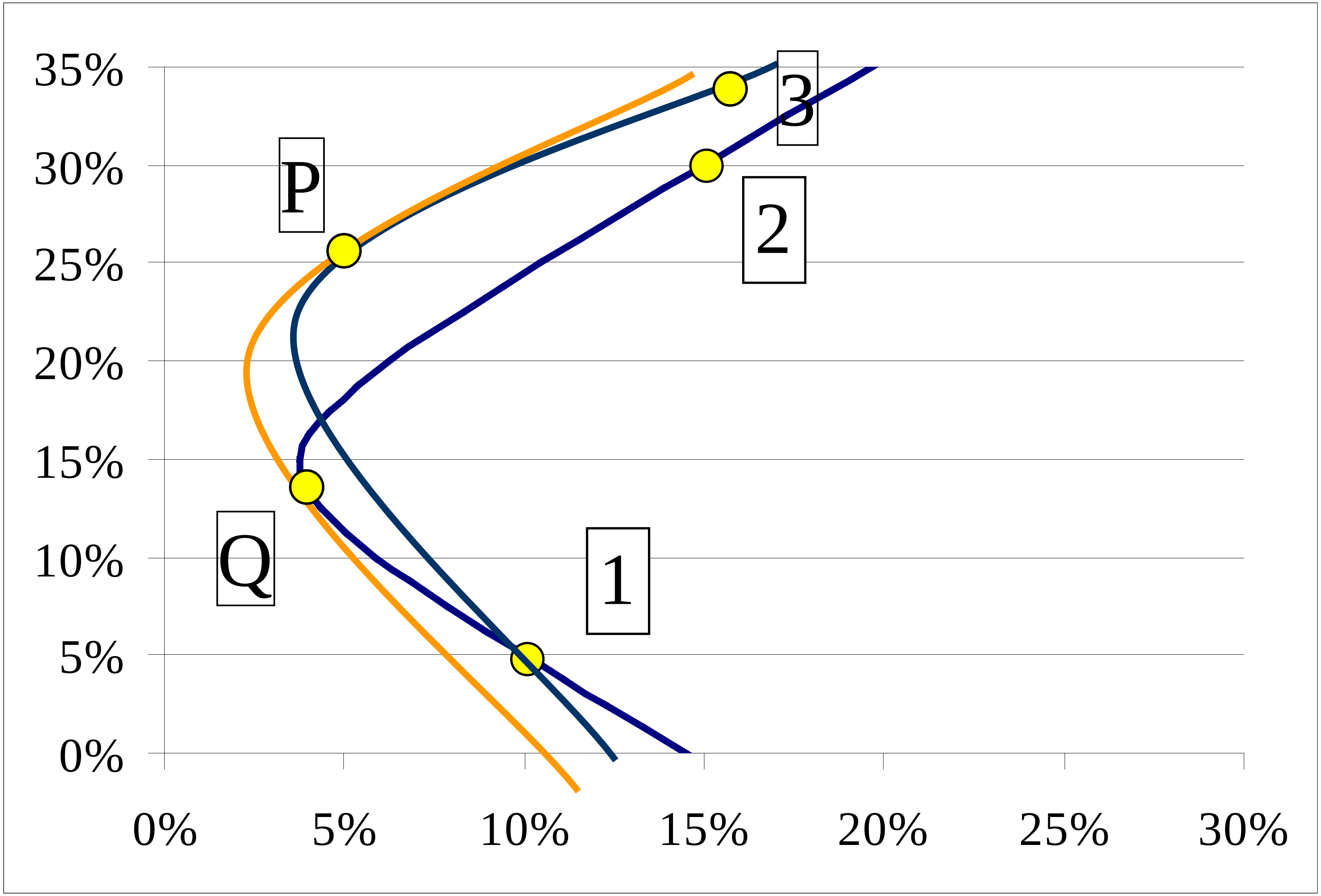
# Two-Asset Portfolio (with Short-sales)



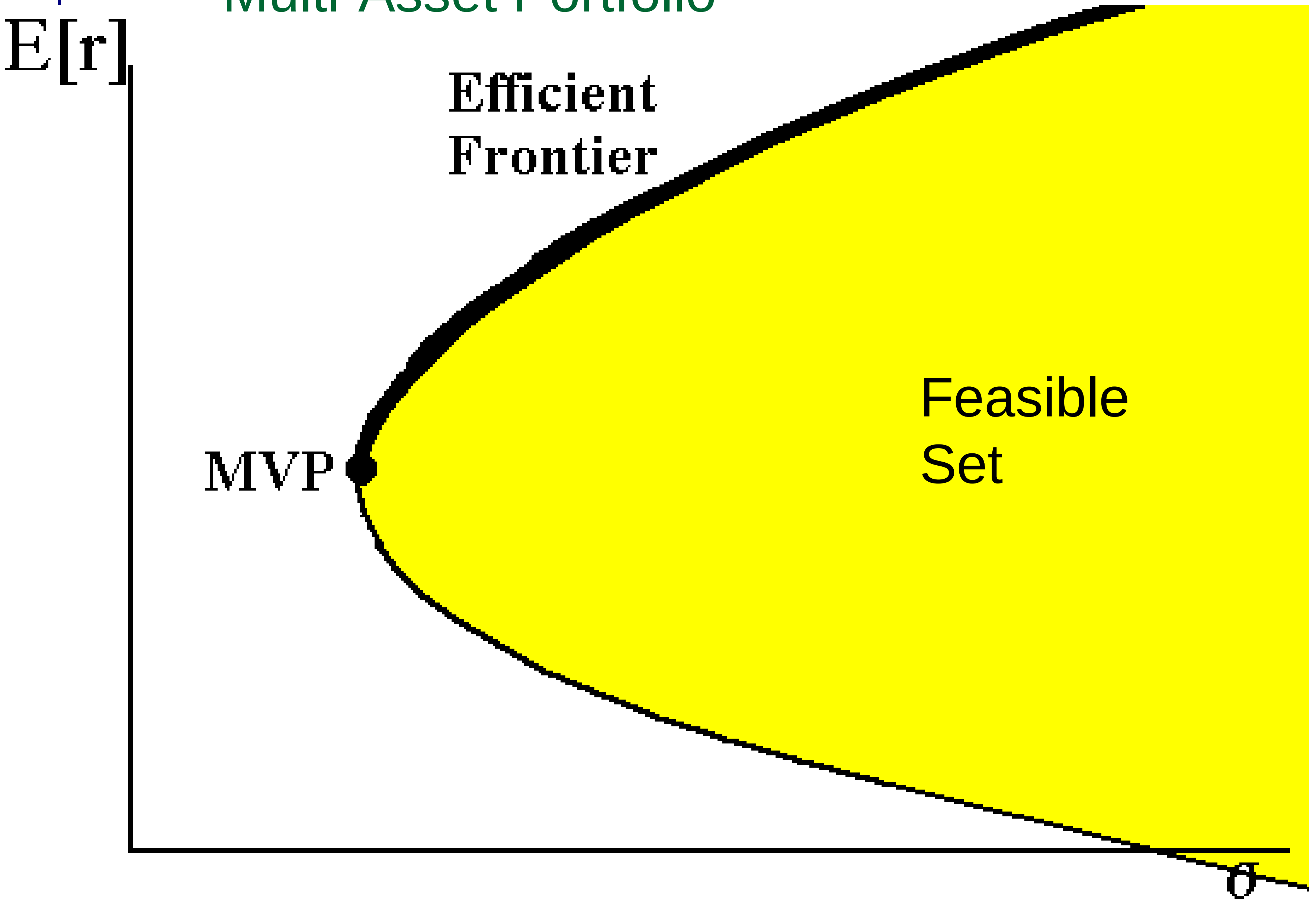
# New Asset



# Portfolio of Portfolios is another Portfolio!



# Multi-Asset Portfolio



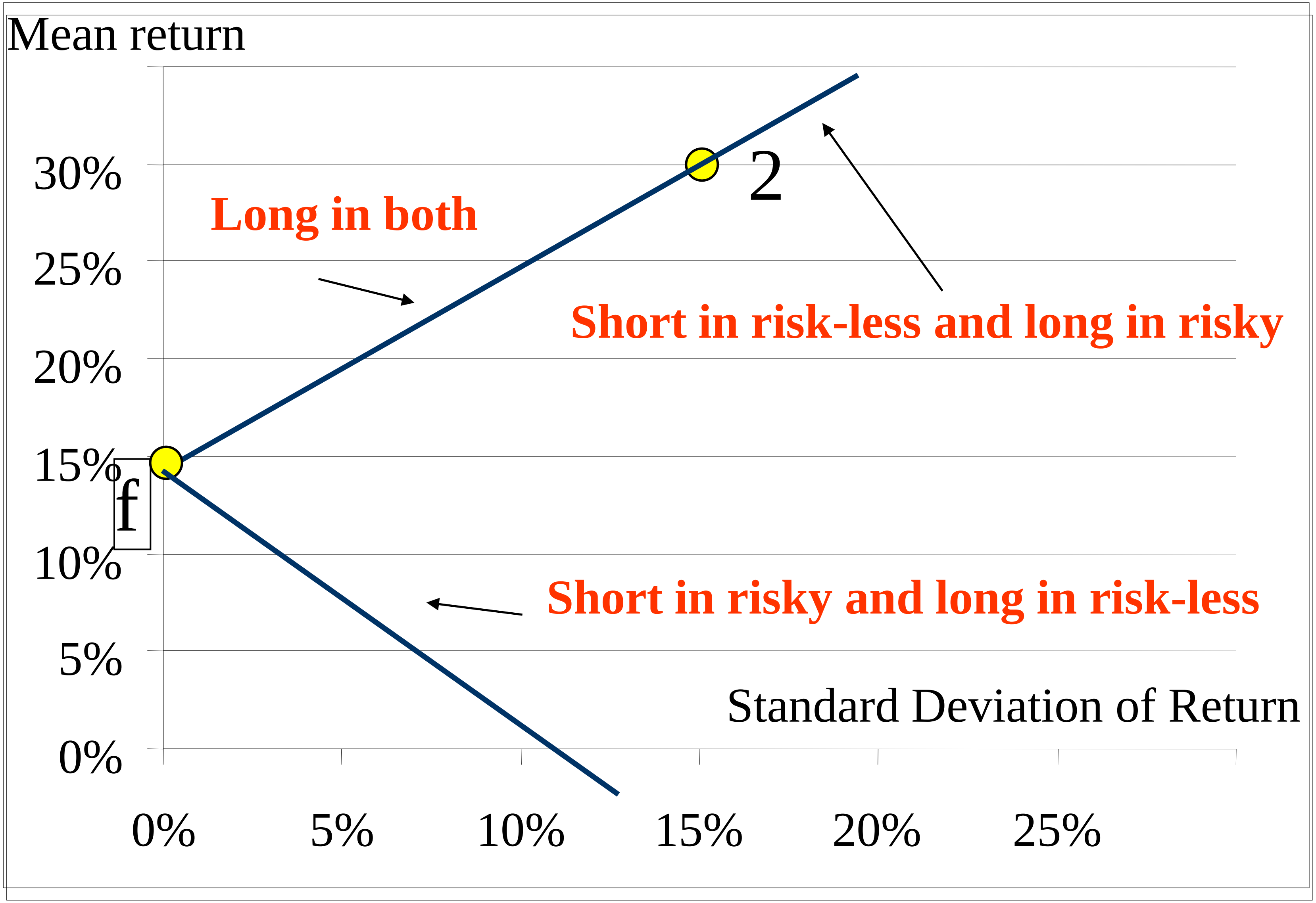
# Properties

- Portfolio frontier:
  - "The locus of points in mean-standard deviation space of all portfolios minimising standard deviation for a given expected return"
  - The portfolio frontier is a hyperbola
  - **All** risky assets and **all** portfolios lie inside the portfolio frontier ("feasible set")
  - A portfolio on the frontier is called a frontier portfolio
  - Frontier above the Minimum Variance Portfolio is called efficient

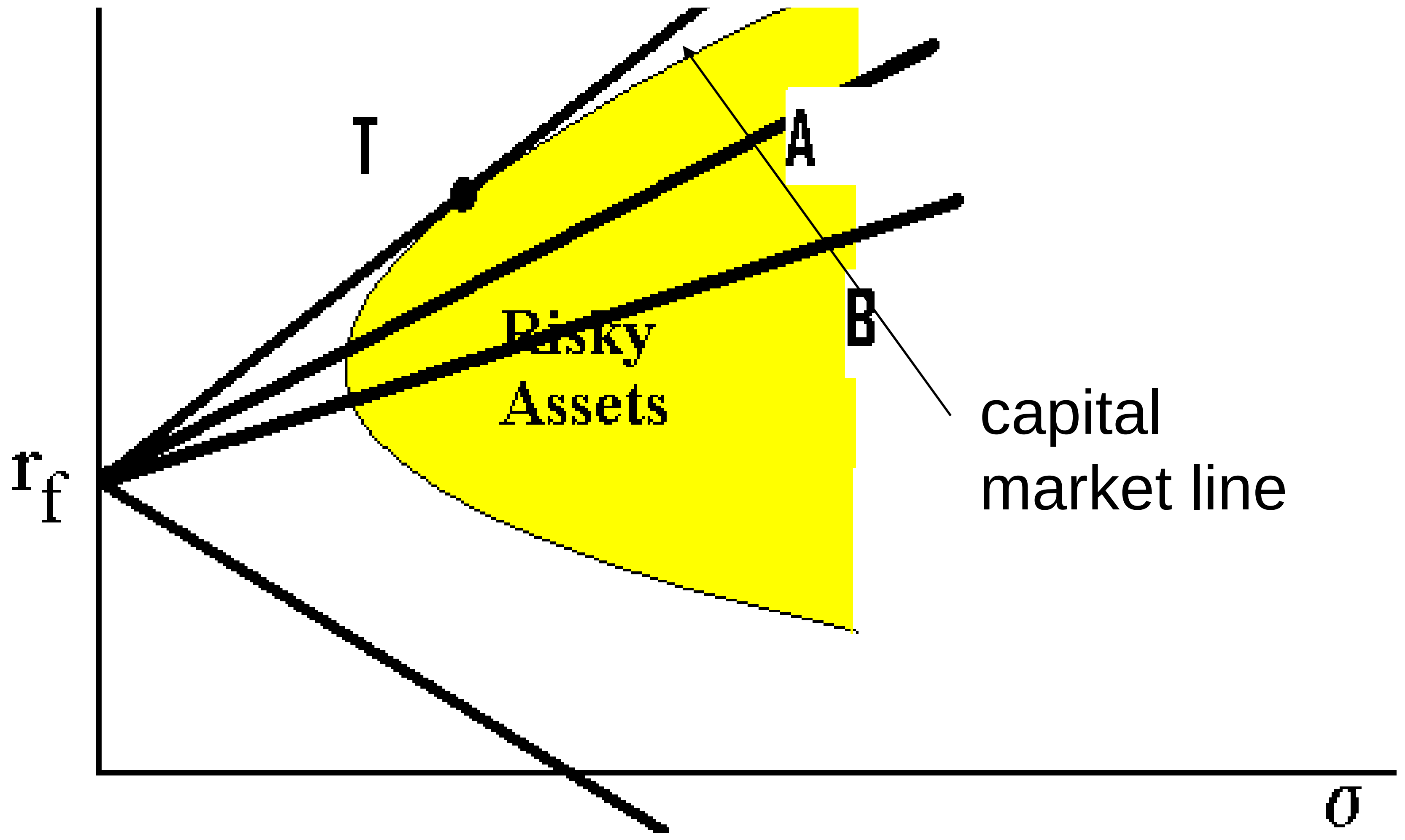
- All investors want to hold efficient portfolios

- **All** possible efficient portfolios can also be created by taking **any** 2 efficient portfolios and combining them…
- and vice versa, any combination of the two is going to be either in the boundary (***two-fund separation property)***

# Portfolios of a Risk-less Asset and a Risky Asset

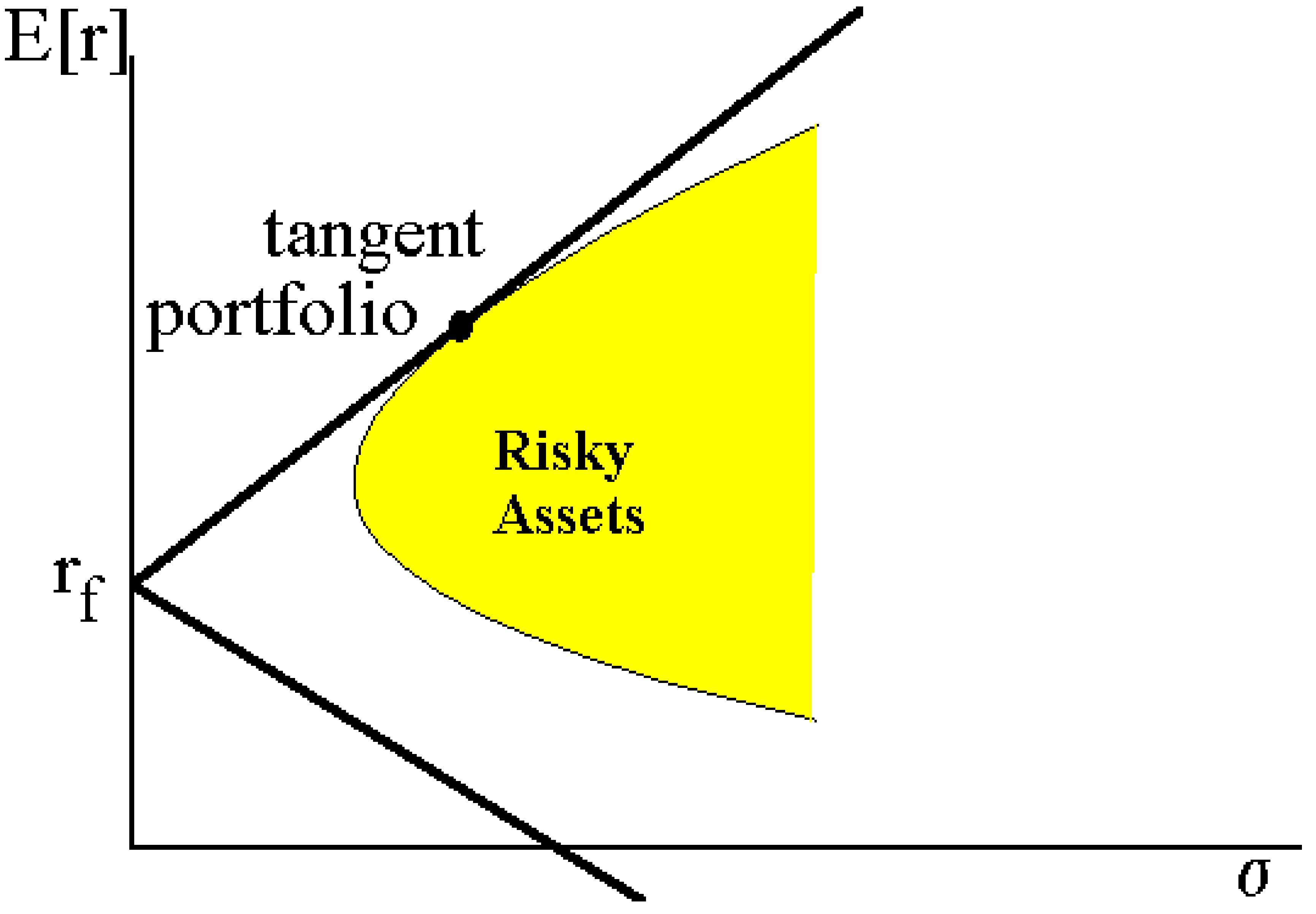


# Adding a Risk-less Asset to the Risky Set?



# Properties (with a Risk-free Asset)

- The frontier is no longer the hyperbola, but it consists of 2 half lines emanating from $r_f$, symmetric about $r_f$
- "Capital market line": Half line with positive slope tangent to hyperbola of risky assets
- Frontier portfolios above $r_f$ are efficient
- The two portfolios out of which all efficient portfolios can be created are now the risk free asset and the **tangent portfolio**
- All investors should hold the same proportion of risky assets:
  - Example: suppose that tangent is (BT,BP)=(2/3,1/3)
    What are the set of efficient portfolios?


E[r]
tangent
portfolio
Risky
Assets
$r_f$
σ

# Nice Mathematical Result: Risk & Return

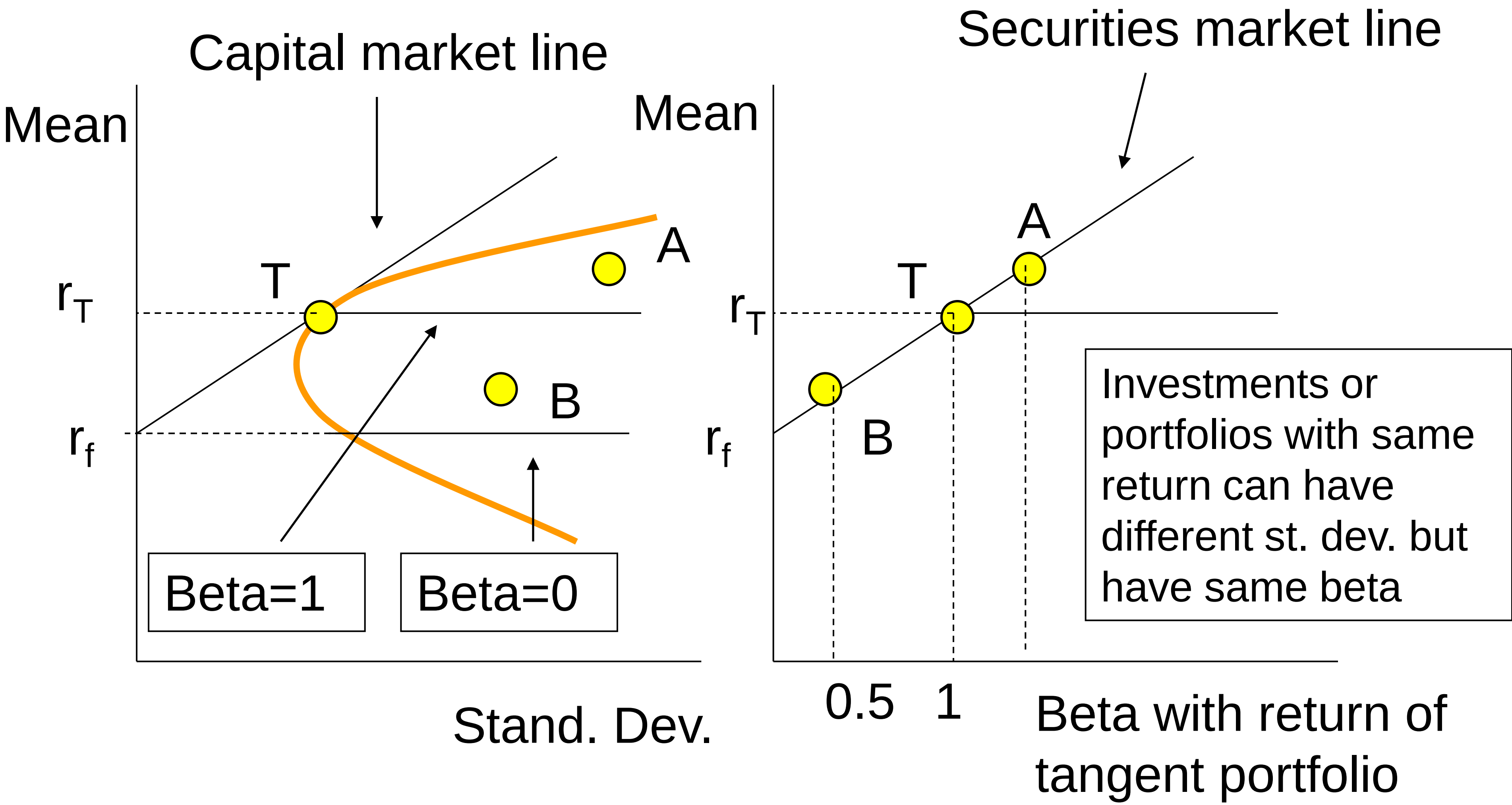
- For any investment i we have,

$$E[r_i] - r_f = \beta_{iT} (E[r_T] - r_f) \text{ where } \beta_{iT} = \frac{\sigma_{i,T}}{\sigma_T^2}$$

$$E[r_i] = r_f + \beta_{iT} (E[r_T] - r_f) \text{ where } \beta_{iT} = \frac{\sigma_{i,T}}{\sigma_T^2}$$

- *Tangency portfolio key to relate expected return on any investment with a measure of its risk: the covariance.*
- *Covariance is the relevant measure of risk*
- *Allows us to use return risk estimates to estimate return*

# Capital and Security Market Lines



# Finding the Tangency Portfolio

- Derive as shown in the appendix:
  - Relatively easy for investment across countries or asset types
  - Low number of investments and good parameter estimates

- But may be computationally demanding individual investment selection:
  - Loads of individual investments!
  - Need to estimate all means and covariances

- CAPM tell us which should be the tangency portfolio

# Appendix

# Finding the MVP of Risky Assets

- Method to obtain MVP:
  - Find "weights" ($w_1$,.., $w_N$) of a "portfolio" w such that:
    - $Cov(r_w, r_i) = 1$ for all i=1,N
    - (weights may not add up to 1)
    - N equations with N unknowns
  - Rescale them to sum 1

- Intuition:
  - We look for a portfolio whose return has an equal covariance with every individual stock return
  - If they were not, we could reduce the variance by increasing the weight on the low covariance stock and reducing that of the high covariance stock

# Example

A company wants to invest in the following countries at the minimum risk.

| | India | Russia | China |
|---|---|---|---|
| India | .002 | .001 | 0 |
| Russia | .001 | .002 | .001 |
| China | 0 | .001 | .002 |

What are the “weights” that make the covariance equal to 1 for each country?

What is the MVP?

# Finding the tangency portfolio

- Method to obtain T:
  - Find "weights" ($w_1$,.., $w_N$) of a "portfolio" w such that:
    - $Cov(r_w, r_i) = E(r_i) - r_f$ for all i=1,N
    - weights may not add up to 1
    - N equations with N unknowns
  - Rescale them to sum 1

- Intuition:
  - We look for portfolios whose covariance with the return of each stock is equal to the risk premium of the stock
  - If they were not, we could increase the mean while lowering the variance

# Example (continued)

Suppose that India, Russia and China have expected returns of 15%, 17% and 17% resp. and the risk-less asset is 6%.

- What are the “weights”?
- What is the tangent portfolio?

# Finding Efficient Frontier of Risky Assets

- We only need to find two assets in the frontier (two-fund separation)
  - Hypothesise a risk-less asset with lower return than MVP
  - Compute the hypothetical tangency portfolio using the previous return as a risk-free asset
  - Take weighted averages of the hypothetical tangency portfolio and the MVP

- What is the efficient frontier for the previous example?