Corporate Finance

Lecture 2: Basic Tools for Portfolio Analysis

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In this Chapter…

- Basic tools to deal with risk-return trade-off
- Objective: financial asset valuation

- Building “portfolios”:
  - Definition: combination of investments
- Portfolio theory:
  - Maximise return given risk
  - Suppose investors care only about risk and variance
- Diversification:
  - Holding many securities may lessen risk
- Reading: GT Chapter 4

Portfolio Weights

- Portfolio weight for stock j: $x_j = \frac{\text{Dollars held in stock j}}{\text{Dollar value of the portfolio}}$
  - Example of a portfolio: £100 in BT and £300 in BP

$$(x_{BT}, x_{BP}) = (1/4, 3/4)$$

- Properties:
  - Weights should add up to 1
  - Weights can be either positive ("long position") or negative ("short")
- Short position:
  - A sells shares that does not own to B (formally from C)
  - Position will be closed when A buys from C
  - Example of a portfolio: £400 short in BT and £800 long in BP

$$(x_{BT}, x_{BP}) = (-1, 2)$$

Remember?

- Investment return (historical return) : $r_{i,t} = \dfrac{p_t + d_t - p_{t-1}}{p_{t-1}}$

$$r_1, r_2, r_3, \ldots r_N \text{ or } r_{BT}, r_{BP}$$

- Expected return (forward looking):

$$\overline{r_1}, \ldots \overline{r_N} \text{ or } \overline{r_{BT}}, \overline{r_{BP}}$$

- Variance and standard deviation of an investment:

$$\mathrm{var}(r_i) = \sigma_i^2 = E[(r_i - \overline{r_i})^2] \qquad \sigma_i = \sqrt{\mathrm{var}(r_i)}$$

  - Standard deviation has same units as returns
- Covariance of two investments 1 and 2:

$$\mathrm{cov}(r_i, r_j) = \sigma_{i,j} = E[(r_i - \overline{r_i})(r_j - \overline{r_j})]$$

  - Interpretation: measure of relatedness. Move together?
  - Depends on units but…

Remember?

- A useful measure of the co-movement of two returns is the correlation coefficient ρ.

- $$\rho_{i,j} = \frac{\operatorname{cov}(r_i, r_j)}{\sigma_i \sigma_j} = \frac{\sigma_{i,j}}{\sigma_i \sigma_j}$$ and $\rho_{i,j}$ ε [-1,1]

  - When $\rho_{i,j} = 1$ (or -1), the assets' returns are perfectly positively (or negatively) correlated, i.e. always move together (or in opposite directions)

  - When $\rho_{i,j} = 0$, the assets' returns are uncorrelated

Obtaining the expected return, standard deviation, and covariances from historical data

Suppose that you have T observations on any two stocks, $r_{i,1} \ldots r_{i,T}$ and $r_{j,1} \ldots r_{j,T}$.

An estimate for the expected return is $$E(r_i) = \overline{r_i} = \frac{1}{T}\sum_{t=1}^{T} r_{i,t}$$

An estimate for the variance is $$\sigma_i^2 = \frac{1}{T-1}\sum_{t=1}^{T}(r_{i,t} - \overline{r_i})^2$$

The standard deviation is the square root of the variance.

An estimate for the covariance is $$\sigma_{i,j} = \frac{1}{T-1}\sum_{t=1}^{T}(r_{i,t} - \overline{r_i})(r_{j,t} - \overline{r_j})$$

Expected Return

- Portfolio return:
  - Portfolio-weighted average of returns of assets in the portfolio
  - Example: if BT's return has been 10% and BP's 5% and portfolio (0.25,0.75) then…
  - Portfolio return: 0.25*0.10+0.75*0.05=0.0625 or 6.25%

- Expected portfolio return:
  - Portfolio-weighted average of expected returns
  - If the portfolio is $P=(x_1,\ldots, x_N)$ then

$$E(r_P) = \sum_{t=1}^{N} x_i \overline{r_i}$$

- How can you maximise expected portfolio return? (e.g. in example above)

Variances and Covariances of a Portfolio

- For any two-stock portfolio…

$$\sigma_p^2 = \mathrm{var}(x_1 r_1 + x_2 r_2) = x_1^2 \sigma_1^2 + x_2^2 \sigma_2^2 + 2 x_1 x_2 \sigma_{1,2}$$

- Hence… larger covariance leads to higher portfolio variance

$$= x_1^2 \sigma_1^2 + x_2^2 \sigma_2^2 + 2 x_1 x_2 \rho \sigma_1 \sigma_2$$

$$\leq x_1^2 \sigma_1^2 + x_2^2 \sigma_2^2 + 2 x_1 x_2 \sigma_1 \sigma_2 = (x_1 \sigma_1 + x_2 \sigma_2)^2$$

- With strict inequality if ρ<1
- Thus..

$$\sigma_p \leq x_1 \sigma_1 + x_2 \sigma_2$$

How Large Diversification Benefits are?