Problem Set 2

1.- Exercise 9.C.3 in Mas Colell et al.

2. A worker is on the job market. While the potential employers do *not know* the worker's skill, the worker knows her own skill. The worker can either be "smart" ($\theta_S = 2$) or "analytical" ($\theta_A = 1$), with $\Pr(\theta = \theta_A) = q$. Taking a degree of level e requires effort cost $e/4$ to a smart worker θ_S, and effort cost e to an analytical worker θ_A. Once hired, the worker can be assigned to one of two tasks: task 1 is a "managerial" task, while task 2 is a "technical" task. When working for any employer, a worker of ability θ produces output of value $-3 + 3\theta$ in the managerial task and θ in the technical task. The worker does not care about the task to which he/she is assigned, but only about the salary net of the cost of education.

The timing is a follows. First, the worker decides the level of education e. Second, the potential employers (suppose for simplicity that there are only two of them) observe the education chosen and simultaneously make an unconditional wage offer. The employer whose offer is accepted allocates the hired worker to the task deemed most appropriate.

(a) Find the optimal allocation of the worker to the task as a function of the employer's belief μ that the worker is analytical.

(b) Characterize the least-cost separating equilibrium.

(c) In this setting the information revealed by signaling allows a better allocation of workers to task. The value of this information is defined as the increase in expected production due to the information revealed. What is the value of information revealed in a separating equilibrium? How does it compare to the resources destroyed through the signaling activity?

3.- Consider the following situation. Players 1 and 2 choose simultaneously between actions a and b but they do not the consequences. **None of them** knows whether they are playing G_1 or G_2, where

G_1	a	b		G_2	a	b
a	$3,3$	$0,5$	and	a	$3,3$	$5,0$
b	$5,0$	$1,1$		b	$0,5$	$1,1$

a) What is the (pure strategy) Nash Equilibria if they knew that they are playing G_1? And if they knew that they are playing G_2? And if they assign probabilities 0.2 to G_1 and 0.8 to G_2?

Suppose now that there is a third player, that knows which game is played (we have now a game of incomplete information). Players 1 and 2 take the same actions as before and these actions have the same payoffs for them in each of the two possible games. However, their actions have also consequences for the third player. The previous payoffs for each action and each game have been expanded to include this player's payoffs:

G_1	a	b		G_2	a	b
a	$3, 3, 3$	$0, 5, 2$	and	a	$3, 3, 3$	$5, 0, 2$
b	$5, 0, 1$	$1, 1, 5$		b	$0, 5, 1$	$1, 1, 5$

The third player sends a public message to the other players from the message set $M = \{m_1, m_2\}$ before players 1 and 2 choose their actions. The payoffs to the three players are independent of the message sent. Suppose that we transform this game of incomplete information into a Bayesian game: it is common knowledge that 0.2 is the prior probability assigned by players 1 and 2 that the payoffs are represented according to G_1 (and 0.8 according to G_2).

b) Represent in extensive form the Bayesian game.

c) Find all (pure strategy) separating weak Perfect Bayesian equilibria of the full game.

d) Find all (pure strategy) pooling weak Perfect Bayesian equilibria.

e) Does the Cho-Kreps intuitive criteria eliminate any of these equilibria? Explain briefly.

f) Now suppose that the message m_2 "costs" player I one unit of utility. That is, the payoffs given above still describe his payoffs if he sends message m_1, but his payoff if he sends message m_2 is the payoff above minus 1. Answer (b) through (f) for this new game.