

## Problem Set 1

1.- Exercise 8.F.2 in Mas Colell et al.

2.- Exercise 8.D.9 in Mas Colell et al.

3.- Consider the following game:

	$a$	$b$	$c$
$A$	5, 3	5, 5	3, 4
$B$	4, 10	9, 9	4, 11
$C$	3, 3	11, 4	5, 5

Completely characterize the set of correlated equilibria for this game. Are there any correlated equilibria that are not Nash equilibria? Explain your answer. (Hint: is the game dominance solvable?)

4.- (a) Consider the following game  $G_1$  (often called the *centipede* game). There are two players, 1 and 2. The players each start with 1 dollar in front of them. They alternate saying "stop" or "continue", starting with player 1. When a player says "continue", 1 dollar is taken by a referee from her pile and 3 dollars are put in her opponent's pile. As soon as either player says "stop", play is terminated, and each player receives the money currently in her pile. Alternatively, play stops if both piles reach 99 dollars (i.e. after 48 times saying "continue").

(a.1) Represent this game in extensive form.

(a.2) Find all the subgame perfect Nash equilibria.

(a.3) Find all the Nash equilibria of this game.

(b) Consider now a similar game  $G_2$ . Player 1 starts with 0 dollars in front of her and obtains 1 dollar each time any of the two players says "continue". Player 2 has the same payoffs as before: she starts with 1 dollar in front of her and the referee puts 3 dollars in her pile each time player 1 says "continue" and takes 1 dollar each time herself (player 2) says "continue". Again play is terminated as soon as either player says "stop" or, alternatively, if the pile of player 2 reaches 99 dollars (i.e. after 48 times saying "continue").

(b.1) Represent this game in extensive form.

(b.2) Find all the subgame perfect Nash equilibria.

(b.3) Find all the Nash equilibria of this game.

(c) Suppose now that player 2 is not informed about which game,  $G_1$  or  $G_2$ , they are playing. She gives a probability  $\lambda$  that  $G_1$  is played (and this is common knowledge).

For simplicity, we are going to modify the play and make it shorter. Independently on which game is actually played, it finishes for sure after Player 1

has decided for the second time (i.e. after 3 times saying "continue"). Player 2, therefore, will not, in any case, play twice.

**(c.1)** Represent this situation as a Bayesian game.

**(c.2)** Solve for the weak perfect Bayesian Nash equilibrium of this game.

**(d)** One might interpret the game in part (c) as follows: Player 2 is playing the centipede game against an opponent, but is not certain that her opponent understands the game. The opponent is aware of player 2's uncertainty in this regard.

**(d.1)** Does behavior unravel, as in the simple centipede game with complete information? What happens as  $\lambda$  approaches 1?

**(d.2)** What does this tell you about how people might play the centipede game when they are unsure about each others' motives?