Production intermittence in spot markets*

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June 2011

Abstract

This paper analyses the influence of production intermittence on spot markets. We use both game theory and an adaptation of the Camerer and Ho (1999) behavioral model. Controlling for costs, we find that intermittent technologies yield lower prices when incumbents have individual market power, but higher when they do not have it. This happens when firms are risk-neutral and risk-averse, and also under different intermittence and ownership configurations. Replacing high-cost assets with low-cost ones results in higher prices than letting them co-exist. The findings have implications for, among others, wholesale electricity markets, in which wind power is increasingly important.

KEYWORDS: behavioral economics, experience-weigthed attractions (EWA), intermittence, production technology, spot markets.

1 Introduction

In many markets supply is intermittent due to factors such as the weather, political instability, and infrastructure failures. For example, short-term weather variability influences the supply of agricultural futures; natural gas markets suffer sudden disruptions due to accidents and political disputes (e.g. recent cases affecting Lybian and Russian flows into Europe); financial markets experience unforeseen liquidity reduction episodes; changes in sea currents affect fish auctions; and electricity generation is becoming increasingly dependent on intermittent technologies like wind and solar power.[1] The importance of exogenous production intermittence is expected to grow in

*Earlier versions of this paper have been presented in the following conferences and universities: Universität Duisburg-Essen, ESMT-Berlin, IESE, INFORMS-2010 (Austin), Universitat Pompeu Fabra, University College London and WEHIA-2011 (Ancona).

[1] For studies of specific cases, see Changnon (1999 - the effect of El Niño on various commodities), Baba and Packer (2009 - the Lehman Brothers collapse and FX markets), Wilson (1980 - New England fish markets) and Albadi and El-Saadany (2010 - impact of wind power intermittence on power markets).

the future, as economic liberalisation, global competition and environmental concerns induce firms to search for greener, lower cost production methods, which are often more unreliable.

Typically, markets with significant supply intermittence also include more expensive back-up production plants. For example thermal units complement wind power in most electricity markets. Yet, such reliable producers face substantial risk at the time of price formation, as their sales depend heavily on how much of the intermittent production is realized. They may sell large quantities at a high price if the realization is low, but will be out-of-the-money if it is substantial. Still, we know little about the influence of intermittence on trading dynamics and, therefore, spot prices.

In this paper, we analyse the effects of intermittent production on prices. The introduction of low-cost, intermittent capacity may influence them in two ways. Adding low-cost capacity tends to decrease prices and, due to pivotal dynamics, may lead to structural breaks in the capacity-to-price relationship. A firm is pivotal if the quantity demanded exceeds the sum of production capacities of all other firms and, as a result, it is necessary to fulfill demand (e.g. Genc and Reynolds, 2005). The second possible effect is explicitly due to intermittence: prices may be different when adding reliable and intermittent capacities. Intermittence may blur pivotal dynamics as firms might or might not be pivotal depending on the uncertainty realisation. Similarly, new low-cost capacity may compress prices closer to marginal costs and reduce their volatility, while the intermittence feature should tend to increase it.

Our set-up is a standard multi-unit, uniform-price auction with two technology types: one is cheap and intermittent, the other is expensive and reliable. As a benchmark, we also consider a low-cost but reliable technology. Despite its simplicity, the setting features multiple non-Pareto ranked Nash equilibria, as in the "battle of the sexes" game. Yet, crucial differences across them allow us to draw predictions about the new capacity effects. The game-theoretical analysis shows that prices decrease with the capacity, both when it is intermittent and reliable. Further, the relationship between intermittent capacity and prices is quasi linear. Finally, the relationship between reliable capacity and prices features a structural break due to the presence of pivotal dynamics.

We use simulations to address the equilibrium multiplicity and study some extensions.[2] The simulations are based on the Camerer and Ho (1999)'s Experience-Weighted Attractions (EWA) model. This technique is flexible, but also transparent, simple, and easy to replicate. Moreover, it generates price time series with their own econometric features. Further, it accomodates well-known evolutionary game theory paradigms that do not require strong behavioural assumptions. Finally, its outcomes match standard game-theory predictions in several market contexts. As a consequence, it is increasingly used to analyse policy-relevant questions.

The game-theoretical and simulation approaches are consistent. The introduction of intermittent capacity reduces simulated prices moderately and in a quasi-linear fashion. When the new

[2] As such, this paper is part of a steadily growing market simulations literature (Noe et al., 2003, 2006, and Pouget, 2007). Examples of more losely related work include papers by Gode and Sunder (1993) on double auctions, and Weisbuch et al. (2000) on fish markets.

capacity is reliable, simulated prices drop substantially as firms cease to be pivotal and this leads to structural breaks. Intermittent technologies yield lower prices when incumbents have individual market power, but higher when they do not have it. We attribute the effects to the coordination difficulties resulting from the intermittence characteristic. Replacing high-cost assets with low-cost ones results in higher prices than letting them co-exist. The results are similar both when the new assets are owned by new entrants and the incumbent firms, and also under risk aversion. Finally, they appear under both uniform and skewed intermittence distributions as well as for different algorithm parametrisations.

As the introduction of wind power is one of the main applications, this paper is related to numerical work by Baldick (2011), Botterud et al. (2011), Sioshansi (2011) and Somani and Zhao (2011). It is also in the spirit of electricity papers such as those by Anderson and Cau (2009), Garcia et al. (2007) and Garcia et al. (2009). Yet, we have chosen not to include congestion, long-term contracts, feed-in tariffs, or ancillary and capacity payments. We believe that such electricity-specific elements would increase the model's complexity, without altering the main insights, and would limit its applicability to other contexts.

In section 2, we outline the setting. Section 3 discusses the theoretical results. Section 4 introduces the simulation procedure, and Section 5 the results. Section 6 contains a short discussion of how the findings relate to different wholesale markets, as well as some concluding remarks.

2 The basic setting

Consider a market for a homogeneous good with inelastic, certain demand, consisting of a vertical line at a level Q. Production comes from low- and high-cost plants. Two symmetric, risk-neutral firms share equally the high-cost capacity K, i.e. each owns $K/2$ units. High-cost plants can reliably produce any level of output up to capacity at an exogeneous, constant, marginal cost $c > 0$.

In the main framework, a competitive fringe owns some low (marginal) cost capacity.[3] We disentangle the intermittence effect with two cases. In one, the low-cost technology is "intermittent": its actual production level, w, is variable, uncertain and uniformly distributed between zero and its installed capacity, $2\overline{W}$, i.e. $w \sim U(0, 2\overline{W})$, $E(w) = \overline{W}$ (W is a mnemonic for "wind"). In the other, the technology is reliable: its production is constant and equal to the installed capacity N (mnemonic for "nuclear"). Without loss of generality, we assume that both technologies have zero marginal costs. We also assume some system overcapacity ($K > Q$), and that neither the low-cost, nor the high-cost capacity held by an individual firm are able to cover the demand on their own ($2\overline{W} < Q$ and $K/2 < Q$).

[3] Regulators often require incumbent electricity retailers to buy wind generation at a legally determined “feed in tariff," so wind generation takes a default first place in the merit order, which is similar to being a competitive fringe. For example, the European Union has recently approved a directive prompting all member states to ensure that generators "will be able to sell and transmit the electricity from renewable energy sources [...], whenever the source becomes available" (European Commission, 2009).

For the comparison, we set $N = W$. Under intermittence, the market capacity is $K + 2\overline{W}$ and its expected value is $K + \overline{W}$. When the new capacity is reliable, both are $K + N$.

Producers sell in a multi-unit, compulsory, uniform-price auction. Low-cost plants have dispatch priority and enter the market supply function at a price below c. The two high-cost firms compete for the residual demand, choosing a bid between c and a price cap Ψ, at which they are willing to sell their capacity, as in von der Fehr and Harbord (1993). An independent auctioneer determines the uniform market price p, as well as individual high-cost production levels q_i, $i = 1, 2$, by intersecting the supply and demand functions. The auctioneer assigns full sales to the bids below p; the remaining volume is distributed equally among the plants bidding exactly p; and any bids above p receive nothing. Profits for each high-cost producer are

$$\pi_i = q_i \cdot (p - c). \tag{1}$$

3 Game-theoretical analysis

In our set-up, the equilibrium price predictions depend on individual capacities and their influence on pivotal dynamics (see also Banal-Estanol and Ruperez Micola, 2009, 2010). First, we formally define "pivotality". Then, we pinpoint the pure-strategy (NE) and mixed-strategy Nash equilibria (MSE) and derive predictions on the capacity introduction effects. We consider both the reliable and intermittent cases and focus on their relationship with prices. Proofs are in the Appendix.

3.1 Pivotal dynamics

A firm is pivotal if it is necessary to satisfy the quantity demanded:

Definition 1 *A firm is* ***pivotal*** *if the level of demand exceeds the sum of production capacities of the remaining sellers in the market.*

According to the definition, the high-cost firms are pivotal if and only if $N < Q - K/2$ when the low-cost technology is reliable. For example, if $Q = 200$ and $K = 350$, firms are pivotal if and only if $N < 25$. When the technology is intermittent, pivotality may depend on the realisation, w. Firms are always pivotal if $\overline{W} < (Q - K/2)/2$. But if $\overline{W} > (Q - K/2)/2$, firms are pivotal if and only if $w < Q - K/2$. For instance, if $Q = 200$ and $K = 350$, they are pivotal if $\overline{W} < 12.5$, or if $\overline{W} > 12.5$ and $w < 25$. Firms are pivotal for an expected level of low-cost production if $\overline{W} < Q - K/2$ or, in the example, if $\overline{W} < 25$.[4]

3.2 Adding reliable capacity

Now, we examine the market equilibria and how they are influenced by the addition of reliable capacity. For tractability, we assume that the price-setting firms can only bid b_1 and b_2, where

[4] Firms would not be pivotal for any intermittence realization if $Q < K/2$, which we have ruled out by assumption.

$b_1 > b_2$. We also assume that b_1 and b_2 are close enough for profitable undercutting, $(b_1 - c)/2 < (b_2 - c)$.[5]

Proposition 2 *Suppose that firms can bid b_1 or b_2, where $2(b_2 - c) > b_1 - c > b_2 - c \geq 0$. Then there is a unique N^* $(< Q - K/2)$, such that the NE are (b_2, b_1) and (b_1, b_2) if $N < N^*$, and (b_2, b_2) if $N > N^*$. If $N < N^*$ there is also a symmetric MSE in which firms select b_1 with probability q^* and b_2 with probability $1 - q^*$, where:*

$$q^* \equiv \frac{(b_1 - b_2)(Q - N) - (b_1 - c)(K - Q + N)}{(b_1 - b_2)(Q - N)}. \tag{2}$$

The price is low ($p = b_2$) when hight-cost producers are not pivotal or low-cost capacity is large ($N > N^*$). In contrast, prices in the NE are high ($p = b_1$) when firms are pivotal and low-cost capacity is small ($N < N^* < Q - K/2$). Prices might still be lower due to both NE coordination difficulties and the presence of MSE.

Closer inspection allows us to derive predictions on the capacity introduction effects. In the NEs, one firm keeps prices high while the other submits b_2 and sells its full capacity. As a result, one firm obtains a relatively low profit and the other gets the maximum. This is similar to the "battle of the sexes", a standard coordination game with two NE and asymmetric payoffs. Experimental evidence shows that coordination, even in simple cases, can be lower than 50% (Cooper et al., 1990). Coordination is especially difficult when there are equilibrium payoff asymmetries because the focal point is not clear (Crawford et al., 2008). In our setting, the difficulties grow in the amount of low-cost capacity. Thus, we conjecture that prices decrease in the amount of low-cost capacity. This effect is reinforced by the decreasing prices that result from the MSEs:

Corollary 3 *The probability of playing high in the MSE, q^*, is lower as the market includes more low-cost capacity.*

`<<F1: Reliable capacity additions and predicted prices>>`

Overall, the game-theoretical analysis suggests that prices will be highest when $N = 0$, decreasing in N while $N < N^*$ and equalto b_2 beyond that point. Figure 1 summarises the price predictions of the NE and the MSE as a function of N.

[5] If this equality does not hold then, when a rival chooses b_1, a firm would always prefer to also choose b_1 and split the total quantity, as profits would be greater sharing the market at a high price b_1 rather than selling the full capacity at a price b_2.

3.3 Adding intermittent capacity

Figure 2 provides a summary of the intermittence price predictions as a function of W. We start with the NE when the low-cost capacity is intermittent.

Proposition 4 *Suppose that firms can only bid b_1 or b_2, where $2(b_2 - c) > b_1 - c > b_2 - c \geq 0$. Then there exist a unique $\overline{W}^*$ ($< Q - K/2$), such that the NE are (b_2, b_1) and (b_1, b_2) if $\overline{W} < \overline{W}^*$, and (b_2, b_2) if $\overline{W} > \overline{W}^*$. If $\overline{W} < \min\{\overline{W}^*, (Q - K/2)/2\}$ there is also a MSE, which consists in selecting b_1 with probability r^* and b_2 with probability $1 - r^*$ where:*

$$r^* \equiv \frac{(b_1 - b_2)(Q - \overline{W}) - (b_1 - c)(K - Q + \overline{W})}{(b_1 - b_2)(Q - \overline{W})}. \tag{3}$$

If $(Q - K/2)/2 < \overline{W} < \overline{W}^*$, there is a MSE resulting from r^{**} where:

$$r^{**} = \frac{(b_1 - b_2)2\overline{W}(Q - \overline{W}) + (b_1 - c)\left[(Q - K/2)^2 - 2\overline{W}(Q - \overline{W})\right]}{(b_1 - b_2)\left[(Q - K/2)(Q + K/2) - 2\overline{W}(Q - \overline{W})\right]}. \tag{4}$$

<<F2: Intermittent capacity additions and predicted prices>>

For $W < \overline{W}^*$ there are two NE whose asymmetry increases in W. When firms are pivotal for all realizations ($\overline{W} < (Q - K/2)/2$), the MSE is the same as in the reliable case, replacing N for $\overline{W}$. If firms are not always pivotal ($\overline{W} > (Q - K/2)/2$), the MSE expected prices are still decreasing in $\overline{W}$:

Corollary 5 *The probabilities of playing high r^* and r^{**} decrease as the market includes more intermittent capacity.*

3.4 Intermittent vs. reliable capacity

One of our main objectives is to isolate the effects one can attribute to the intermittence feature. To that purpose, the following result compares the reliable and intermittent NE and MSE.

Corollary 6 *If $\overline{W}^* < (Q - K/2)/2$, the prices in the NE and the expected prices in the MSE of the intermittent case for any $\overline{W}$ are the same as those of the reliable case (and in particular $\overline{W}^* = N^*$). In contrast, if $\overline{W}^* > (Q - K/2)/2$, the prices in the NE and the expected prices in the MSE of the intermittent case are strictly higher if $(Q - K/2)/2 < \overline{W} < \overline{W}^*$ (and in particular $\overline{W}^* > N^*$). In terms of the exogenous parameters, $\overline{W}^* > (Q - K/2)/2$ if and only if*

$$\frac{2(Q - K/2)}{Q + K/2} > \frac{b_2 - c}{b_1 - c}.$$

The theory suggests differences in the price impact of reliable and intermittent capacity introductions. The intermittent technology yields lower prices when incumbents have individual market power, but higher when they do not have it. The effect is stronger when the low bid b_2 is relatively smaller than b_1 and if Q is large relative to K (i.e. there is tight excess capacity).

Summarising, the game-theoretical analysis suggests the following stylised facts. First, the introduction of low-cost capacity, both intermittent or reliable, decreases prices. Second, they decrease more slowly under intermittence. Third, prices become competitive when there is sufficient reliable capacity but the intermittence feature hinders equilibrium coordination on the competitive market outcomes.

4 Simulation procedure

The preceeding analysis has two important limitations. First, the model includes several simplifying assumptions that improve its tractability, but also limit its applicability to existing spot markets. For example, there are only two possible bids, rather than a quasi-continuum, as in most real situations (e.g. Biais et al., 2011). Further, new plants may not be added to the current production mix, as we have assumed, but replace outdated, perhaps more polluting, capacity. Moreover, many high-cost firms are not specialists but hold multi-technology portfolios that include low-cost technologies. Finally, intermittent production realisations might follow asymmetric probability distributions, and firms often are risk-averse rather than risk-neutral.

The second limitation is that multiple equilibria may coexist when firms are pivotal, raising the issue of how to select the relevant prediction. There are two broad types of equilibrium selection methods (Haruvy and Stahl, 2004). On the one hand, there is deductive selection, based on reasoning and coordination. Until recently, deductive principles have dominated the equilibrium selection literature. However, our setting is particularly unsuited to them, as the equilibria lack clear focal points. Moreover, deductive principles have been shown to do poorly in experiments (see e.g. van Huyck et al., 1990). On the other hand, there is inductive selection based on simple learning dynamics. Inductive selection methods have the advantage of not requiring coordination points. In addition, their predictive performance is often better than the deduction selection principles (e.g. Camerer and Ho, 1999; Erev and Roth, 1998; Haruvy and Stahl, 2004; Roth and Erev, 1995). Finally, one can use simulations to consider policy-relevant settings outside the stylised framework. For these reasons, we complement the game-theoretical analysis with simple learning dynamics' simulations.

4.1 Behavioral algorithm

We use an adaptation of the Experience-Weighted Attraction (EWA) mechanism put forward by Camerer and Ho (1999). EWA is a general behavioral model that nests standard paradigms, e.g. reinforcement learning and best-response, and that has already been used to model financial

(Pouget, 2007) and energy markets (Banal-Estanol and Rupérez Micola, 2009). Other applications include supply chain (Kalkanci et al. 2011), social network (Chang, 2011) and marketing issues (Ho et al., 2006).

EWA has been shown to offer successful predictions vis-a-vis experimental results (Camerer and Ho, 1999; Kalkanci et al. 2011). In addition, it is based on prominent evolutionary game theory paradigms that do not require strong behavioural assumptions. It can also be implemented to closely match existing market features, it is flexible but transparent, simple and easy to replicate. Moreover, it is theoretically sound, matches game theoretical results in simple settings (e.g. Banal-Estanol and Rupérez Micola, 2011; Pouget, 2007), endogenously produces price time series and is flexible enough to accomodate risk aversion, industry ownership configurations, and accurate probability distributions for the uncertain variables.

In our implementation, bids have attractions that determine their selection probability. In each round, producers submit bids according to firm-specific distributions. Once the auctioneer determines the price and individual quantities, attractions are adjusted with the behavioural rule and mapped into probabilities. This process is repeated until the simulation converges. In the following, we describe how firms use experience to update the attractions, and how these lead to choices. Then, we specify the initial and convergence specifications.

4.2 Attractions and probabilities

The feasible bid domain is approximated by a discrete grid. High-cost producers choose among S possible prices, equally spaced between marginal costs c and the price cap Ψ, at which they are willing to sell,

$$S_i = \{c + s\,(\Psi - c)\,/S,\ s = 1, ..., S\}\,.$$

Each "action s" corresponds to a price bid. Lower actions are closer to marginal costs, i.e. more competitive.

Each bidding "action" s of each high-cost producer i has an "attraction" $A_{i,s}(t) > 0$ *after* period t (≥ 1). Attractions are updated with

$$A_{i,s}(t) = \frac{\phi N(t-1)A_{i,s}(t-1) + [\delta + (1-\delta)I(s, r_i)]\,\pi_i(s, r_{-i})}{N(t)}, \tag{5}$$

where r_i and r_{-i} denote the bid chosen in period t by firm i and the others, $I(x, y)$ is an indicator function with value 1 if $x = y$ and 0 if $x \neq y$ and $N(t) = \rho N(t-1) + 1$. $N(0) = 0$, represents the number of "observation-equivalents" of past experience. The EWA parameters δ, ϕ, and ρ denote the weight placed on foregone payoffs, a parameter to depreciate previous attractions, and one that weights the impact of previous against future experience.

The next step is to map $A_{i,s}(t)$ into bid choice probabilities. The probability of choosing a specific bid in $t+1$ is its attraction divided by the sum of all attractions,

$$P_{i,s}(t+1) = \frac{A_{i,s}(t)}{\sum\limits_{k=1}^{S}(A_{i,s}(t))}. \tag{6}$$

When $\delta = 0$, the model behaves like in a widely used class of reinforcement learning (RL) models (e.g. Roth and Erev, 1995; Erev and Haruvy, 2001; Sandholm, 2002, and Hopkins, 2008). When $\delta = 1$, it corresponds to standard best-response dynamics. The standard literature suggests that RL is a priori suitable in unstable environments, but recent research suggests that it might be too simplistic to fully capture the strategic opportunities available to humans (Erev et al., 2007; Ert and Erev, 2007). In BR, actions are determined by the best response to what opponents did in the immediately preceding period. It is more suitable when agents know about their opponents behavior, and infer how it influences their own profits, but might be less effective than RL in unstable markets. Banal-Estanol and Rupérez Micola (2011) show that both RL and BR are widely consistent with pivotal dynamics in a stylised market. For expositional ease, we use RL in the main body of the results and devote the last section of the paper to show that the results also appear under BR.[6]

4.3 Initial conditions and convergence

The first period probabilities, $P_{i,s}(1)$, should be understood to reflect pre-game attractions, $A_{i,s}(0)$. We define $A_{i,s}(0)$ as the profit that each bid s renders when each firm i believes that the others will bid competitively. Banal-Estanol and Rupérez Micola (2011) show that market simulations under this prior converge to the game-theoretical predictions.

The simulation starts at $t = 1$ and continues until agents reach a stationary state, when their strategy profile variations are sufficiently small. For a given τ (small), a simulation run has converged to a mixed strategy profile z in period t if for any potential action profile a in time $t+1$, the probability distribution adjustments are such that

$$|P_{i,s}(t+1) - P_{i,s}(t)| < \tau, \ \forall i, s \tag{7}$$

In practice, we select the lowest probability bid in period t. Then, we compute the hypothetical probability that would result from assigning maximum profits to it, and minimum profits to all other bids. The run has not converged as long as the difference between present and hypothetical probabilities is higher than τ. It has converged when it is lower. The smaller τ, the more stringent the threshold and the higher the necessary t.

The end price is computed from the agents' mixed strategy profile z. Convergence is compatible with the survival of several feasible trading actions, as in mixed strategies. Price volatility may not

[6] EWA yields other paradigms, such as fictitious play, hybrids of the three, and there are other algorithms that are also popular in the literature. Results under several of these parametrisations are available upon request.

be zero even if there is a steady state. Moreover, EWA bidding depends on a stochastic process and, as a result, simulation runs for the same parameters might lead to different prices, i.e. the end price standard deviation across simulations may be different from zero.

4.4 Parameters

The simulations approximate existing markets by allowing high-cost firms to choose from an equally spaced grid of $S = 100$ prices between marginal costs, $c = 2$, and the price cap, $\Psi = 200$ (i.e. $2, 4, ..., 200$). Recall that in the theoretical part we had to assume that traders could only bid b_1 and b_2, where $b_1 > b_2$, but that most markets display quasi-continuous price grids with small monetary increments (e.g. Biais et al., 2011).

Low-cost producers are modelled as a passive competitive fringe with zero marginal cost. Their realised production is sold in full. The high- and low-costs are very close to ensure that strong technological discrepancies do not drive the price effects. Demand is set at $Q = 200$ and the high-cost capacity is $K = 350$, so that each of the two producers holds $K/2 = 175$ units. We perform simulations for fifteen low-cost capacity cases, $N = \overline{W} = \{0, 5, 10, .., 70\}$, corresponding to demand coverage from 0% to 71.1%. The case in which $N = \overline{W} = 0$ is a no low-cost capacity benchmark.

We perform one-hundred simulation runs for each N and $\overline{W}$ levels of "reliable" and "intermittent" technologies. Once a run converges, we calculate the expected price and volatility resulting from the agents' choice distributions. Hence, we have $2 \cdot 15 \cdot 100 = 3,000$ observations for each specification.

5 Simulation results

5.1 Intermittence and prices

The red lines in Figure 3 report average prices and volatilities for different intermittent capacity values. High-cost producers are always pivotal when $\overline{W} < 12.5$, and they are pivotal for the expected intermittent production if $\overline{W} < 25$. For comparison, the blue lines provide the averages for the equivalent amounts of low-cost reliable technology, where high-cost firms are pivotal as long as $N < 25$.

When $N = \overline{W} = 0$, prices are around 150. This value corresponds to about 75% of the maximum price the two producers could achieve. The remaining 50 monetary units can be interpreted to measure their lack of coordination. The addition of reliable assets reduces them to about 120 when $N = 20$. Then. there is a noticeable structural break at $N = 25$ coinciding with the pivotal regime change. At that point, prices decrease drastically to competitive levels, $c = 2$.

Intermittence prices are similar for $\overline{W} < 15$. Thereafter, their descent continues without a structural break. This leads to comparatively higher price values when $\overline{W} \geq 25$. Intermittent prices remain above competitive levels even for large amounts of new capacity.

<<F3: Average price and volatility levels - intermittent and reliable capacity additions>>

The coordination difficulties grow in the amount of low-cost capacity. Before the structural break, the residual demand tightens and the producers face an increasing pay-off asymmetry . This leads to coordination difficulties, which intermittence exacerbates. The post-break reliable case does not feature coordination difficulties. In contrast, some difficulties remain in the intermittent case because firms are pivotal for some (sufficiently low) realised production values. As the coordination focal point is the competitive outcome, this translates into higher intermittence prices. Overall, the simulations are consistent with the theoretical analysis in that intermittent technologies yield lower prices when incumbents have individual market power, but higher when they do not have it.

In the following subsections, we consider alternative parameters to provide a richer picture of the relationship between intermittence and prices.

5.2 Capacity replacement

So far, we have assumed that the new assets are added on top of the existing capacity. However, new technologies often have advantages (e.g. in terms of cost, or the environment) that lead to the medium-term obsolescence of the conventional plants. Now, we consider this replacement situation.

For ease of comparison, we assume a constant expected market production. Each low-cost unit replaces one high-cost unit. In the reliable case, each producer holds $K/2 - N/2$ high-cost units and both the market capacity and expected production are K. When $Q = 200$ and $K = 350$, firms are pivotal for $N < 2(Q - K/2) = 50$. The amount of new capacity necessary to induce competition is larger than under the addition baseline. When intermittent capacity replaces high-cost assets, producers hold $K/2 - \overline{W}/2$ units. The expected market production is K, but total capacity is $K + \overline{W}$.[7] Producers are pivotal for all intermittent realisations as long as $\overline{W} < 2(Q - K/2)/3 = 16.667$.

<<F4: Average price levels and volatilities under capacity replacement - intermittent and reliable technologies>>

[7]Here, one also needs to assume that demand can be covered with the expensive technology ($Q < K - \overline{W}$) to avoid complications due to lack of supply. Our simulations paramters are chosen to satisfy this constraint as $\overline{W} < K - Q = 150$.

Figure 4 reports average price and volatility levels under capacity replacement. Reliability prices decrease slightly between $N = 0$ and $N = 45$. At that point, there is an abrupt reduction to c for $N = 50$. In contrast, the intermittent price reduction is almost linear, leading to lower pre-break prices, $p_W < p_N$. Post-break, $p_W > c$. Intermittence also increases the price time-series volatility. For $N \leq 15$, $\hat{\sigma}_W = \hat{\sigma}_N$, beyond that point, $\hat{\sigma}_W > \hat{\sigma}_N$.

This fits nicely both with the previous simulations and the game-theoretical results. Thus, we conclude that replacing high-cost assets with low-cost ones results in higher prices than letting them co-exist.

5.3 Joint high- and low-cost asset ownership

So far we have worked under the assumption that a third party holds the low-cost assets. This has the advantage of isolating the intermittence effects. Hence, in this section we analyse whether the game-theoretical insights persist under a joint-ownership assumption. To simplify, we focus on the case in which low-cost capacity is evenly split between the two high-cost producers, so that profits are

$$\pi_i = q_i(p - c) + wp/2. \tag{8}$$

Figure 5 presents the results. The upper panels report capacity addition and the lower panels focus on replacement. The relationships are very similar to those under third party ownership (differences statistically not significant, $p > .10$). The fact that the are similar both when the new assets are owned by new entrants and the incumbent firms indicates that who holds the assets does not have too much bearing on the price results.

`<<F5: Average price levels under joint low- and high-cost ownership: Capacity addition & replacement - intermittent & reliable technologies>>`

5.4 Risk aversion

So far we have assumed risk neutrality, but it is possible that firms are risk-averse. Hence, in this section, we represent traders with a mean-variance preference specification. Their utility is

$$U_i = E(\pi_i) - RV(\pi_i), \tag{9}$$

where E and V are the expectation and variance functions, R is the risk aversion coefficient and π_i is defined in (1).

`<<F6: Average price levels under risk aversion: Capacity addition & replacement - intermittent & reliable technologies>>`

As one of several configurations that we have tried, Figure 6 reports prices under $R = 1$. As in the benchmark, we assume a low-cost competitive fringe. Risk-aversion penalizes higher prices because they are associated to higher volatilities. As in Asplund (2002), risk-averse firms give relatively greater weight to realisations with low profits. Hence, simulation outcomes are more competitive than under $R = 0$ (prices around 90 for $\bar{W} = 0$). Pre-break intermittence and reliability prices are similar, but they differ post-break. The structural breaks are clearly noticeable when capacity is reliable but not under intermittence. This suggests that risk attitude assumptions do not vary the qualitative nature of the results.

5.5 Skewed distributions

In the theoretical model, intermittent realisations are drawn from a uniform distribution. This is helpful to simplify the analysis but does not apply to many markets. For example, wind power is negatively skewed. To address this issue, we propose an alternative formulation in which realised production is drawn from a triangular distribution function. We define the density function as[8]

$$f(x) = \begin{cases} \frac{2\left(3\overline{W}-x\right)}{\left(3\overline{W}\right)^2}, & \text{if } 0 \leq x \leq 3\overline{W} \\ 0 & \text{otherwise,} \end{cases},$$

whereby the lower lower limit coincides with the mode (i.e. the "peak") and the upper limit is adjusted to generate the same mean as in the baseline.

<<F7: Average price levels with left-skewed intermittent production: Capacity addition & replacement - intermittent & reliable technologies>>

One can then consider both right- and left-skewness, as well as symmetrical distributions. As an example, the panels in Figure 7 report average prices under separate ownership and risk neutrality. The introduction of a skewed distribution does not change much the results. $\bar{W} = 0$ prices are around 150. Intermittent prices decrease faster than the reliable ones, up to the structural break. At that point, reliable prices drop to the competitive level while the intermittent ones conti continue a progressive descent. Therefore, we conclude that the results are not solely for the uniform distribution, and that the method can be readily used to characterise other types of uncertainty distributions

[8] The specific case of wind power is far more complex. Wind intensity (in m/s) is typically modelled as governed by a Weibull distribution, V, with a non-linear relationship to power generation: $w = \frac{\rho A V^3}{2}$, where A and ρ are constants representing land area size and air density (kg/m3). The distribution is defined as $V(x; \lambda, k) = \frac{k}{\lambda}\left(\frac{x_t}{\lambda}\right)^{k-1} e^{-(x/\lambda)^k}$, where $k > 0$ is the shape parameter and $\lambda > 0$ is the (mean) scale parameter. The higher k the more consistent the wind speed, i.e. $k < 2$ represents locations with very strong peaks and those with fairly consistent wind speeds around the median have $k \approx 3$. In most locations around the world $k \approx 2$.

5.6 Best-response dynamics

Until now, the results are based on the naïve reinforcement learning (RL) parametrisation of the behavioral algorithm. In this section, we relax it by taking a more sophisticated Cournot best-response (BR) choice rule from EWA, when $\delta = 1$, and $\rho = \phi = 0$. In BR dynamics, bids are determined by the best response to what opponents did in the immediately preceding period. Thus, BR assumes that players have access to their opponents' previous bids and sufficient reasoning power to compute best responses.

`<<F8: Average price levels under best-response dynamics: Capacity addition & replacement - intermittent & reliable technologies>>`

We report the BR results in Figure 8. Prices are only lower slightly (140 vs. 150 monetary units when $N = \overline{W} = 0$). This is fully consistent with the analysis by Banal-Estanol and Rupérez Micola (2011), who show that the main difference between RL and BR is that the latter yields more competitive outcomes when pivotal dynamics lead to equilibrium multiplicity. Moreover, the shape of the relationship with low-cost capacity is virtually the same as in the preceeding sections.

We interpret these results as suggesting that the reliability and intermittence findings do not depend on the use of a reinforcement learning model, but are robust to alternative best-response behavioral assumptions.

6 Discussion and concluding remarks

Intermittence is a feature of many markets, including those for financial assets, agricultural commodities, natural gas, fish and electricity, and bound to become more salient as firms search for greener, lower cost production methods, which are often more unreliable.

We use analytical and simulation methods to study the impact of new low-cost capacity on the market's pivotal dynamics, and how these, in turn, influence prices. Under low production conditions, firms may be able to set high prices and still sell their goods, but not if production is high. Thus, variability has at least two effects on prices. For low values, it has a small pro-competitive effect. For high values, uncertainty results in less competition and higher prices. The combination of these effects blurs the presence of different pivotal dynamics regimes. This occurs when one considers game-theoretical solutions, as well as reinforcement learning and best-response algorithms.

We believe that the introduction of wind power is an important setting, and one that is especially pertinent to apply our findings (see, e.g. Gross, 2006). Wind power is plentiful, widely distributed, has very low variable costs and leads to virtually no emissions. Its economic advantages have boosted its adoption to an annual growth rate of 30 percent. In 2008, the world's installed capacity was 121.2 GW (28 GW in the US). In Europe, it amounts to 19% of the capacity in Denmark, 11% in Spain and Portugal, and 7% in Germany and Ireland (REN21, 2009). In several countries, it routinely covers more than half of the the electricity demand.

Yet, wind power is highly intermittent. Its intensity depends on gusts, daily cycles, and annual seasonality and, as a result, turbines seldom operate at a constant load (Söder, 2002), with half of the energy arriving in about 15% of the operating time. Moreover, it is difficult for market participants to predict wind speed and direction at the precise generation locations. This has a large impact, as a 10% deviation of the wind speed corresponds to a 30% deviation in electricity generation (Ackermann and Söder, 2002). Feed-in tariff obligations in many countries further magnify the price variability (Butler and Neuhoff, 2008; Twomey and Neuhoff, 2010).

The analysis suggests that the introduction of small amounts of wind power may reduce prices as firms may find it more difficult to coordinate in high prices. However, it also indicates that large scale wind power investments can have a detrimental effect on consumers' welfare because firms may fail to compete as fiercely as one would expect in that case. This may happen irrespectively of whether the wind capacity is held by specialist firms or the incumbents.

However, as much as wind power is an important application, the findings are intended to be also applicable to other settings. Major industries are highly climate sensitive, including agriculture, transportation, construction and retailing (e.g. Changnon et al., 2007). A recent practitioners' report (Barclays, 2010) discusses the various factors that in the near future may bring "intermittent, but recurrent, supply shortages" in commodity markets such as those for coal, cocoa, copper, lead, tin and zinc. Several events have affected gas markets in recent times.[9] Similarly, Wilson (1980) discusses how crucial pieces of information in New England fish markets are only imprecisely known by the parties at the time of the transaction. These include current prices, and the fish quality, as this is only offloaded and inspected after the sale. Consequently, "at the time the parties agree to conduct the transaction there are no unambiguous, market-generated measures of value available to assess accurately the current value of the fisherman's load, nor is there a way to determine accurately the precise characteristics of the load."

Further, lack of liquidity in have recently started to also raise concerns about intermittence in financial markets and its negative effect on trading immediacy. Baba and Packer (2009) argue that the dollar liquidity problems had a contagion effect on the foreign exchange (FX) swap markets during the September 2008 Lehman Brothers collapse. At times, regulators restrict short-selling (e.g. Bris, 2008). Further, algorithmic trading may result in sudden drops in trading volume and prices. An example of this is the May 6th, 2010 "flash crash" in which stock markets suddenly plummeted, volume hit its high for the year, and volatilities skyrocketed. The medium term mutual fund outflows due these events were estimated in the $50-60 billion region (Traders Magazine,

[9] Lybian gas deliveries into Europe ceased completely due to internal violence (see e.g. New York Times, 2011). For comparison, one week of Lybian deliveries corresponds to over 15 billion cubic feet, or slightly less than the annual consumption in Belgium. Moreover, there was also a flow interruption occurred when Ukraine refused to transit Russian gas to Europe through its export pipelines in 2009 (e.g. New York Times, 2009). Both events seriously affected the European natural gas market (for more information, see e.g. Stern, 2006).

2010).

In all these cases, we interpret the results as suggesting that a small amount of intermittence is good for consumers, but that too much of it might harm them. The questions of whether there are viable alternatives and whether the respective thresholds have been reached should take into account its specific technological, economic and institutional features.

More generally, future models may incorporate a more careful characterization of the markets in which intermittence is important. For example, constant marginal costs is a reasonable assumption in the electricity industry, but it may be unrealistic in most other contexts. Moreover, one could model the different ways to reduce the impact of supply variability in specific industries. In the wind power case, important mitigation measures include the use of capacity reserves, storage, grid interconnection, distributed generation and smart grids. Other industries have their specific features.

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Appendix: proofs

6.1 Proof of Proposition 2

Part (i): Suppose first that $N < Q - K/2$ and therefore both firms are pivotal. Take one of the firms. Suppose that the other firm bids b_1 with probability q and b_2 with probability $(1-q)$. The expected profits from bidding b_1 are

$$q\frac{Q-N}{2}(b_1-c)+(1-q)\left(Q-N-K/2\right)(b_1-c)$$

whereas the payoff from playing b_2 is

$$qk(b_1-c)+(1-q)\frac{Q-N}{2}(b_2-c).$$

Simplifying playing b_1 is better as long as

$$q\left(Q-N\right)(b_1-b_2)\leq 2(Q-N-K/2)(b_1-c)-\left(Q-N\right)(b_2-c) \tag{10}$$

On the one hand, if the other plays b_2 for sure ($q=0$), then it is better to play b_1 as long as the righ-hand side is positive, or if

$$\frac{2(Q-N-K/2)}{Q-N}\geq\frac{b_2-c}{b_1-c} \tag{11}$$

The left hand side of condition (11) is decreasing in N, as the derivative is equal to $-K/(Q-N)^2$. Evaluated at $N=0$ the conditions reduces to

$$\frac{2(Q-K/2)}{Q}\geq\frac{b_2-c}{b_1-c} \tag{12}$$

which can or cannot be satisfied. Indeed, the right-hand side is between $1/2$ and 1. The left-hand side ranges from 0 (if $Q=K/2$) to 1 (if $Q=K$). Evaluated at $N=Q-K/2$, we have that the condition is never satisfied as the left-hand side is equal to 0.

On the other hand, if the other firm plays b_1 for sure ($q = 1$), then it is better to play b_2 given that the condition (10) reduces to $0 \leq Q - N - K$, which is never satisfied.

As a result,we have that if condition (11) is satisfied, we have multiplicity of equilibrium: (b_2, b_1) and (b_1, b_2) are pure-strategy Nash equilibria. Additionally, there is a mixed strategy equilibrium where each player plays b_1 with probability q^*, where

$$q^* \equiv \frac{(Q-N)(b_1-b_2)-(K-Q+N)(b_1-c)}{(Q-N)(b_1-b_2)}$$

If condition (11) is not satisfied then (b_2, b_2) is the unique equilibrium.

Each case arise under the following situations. If Condition (12) is not satisfied then condition (11) is never satisfied and we always have the unique equilibrium. Instead, if condition (12) is satisfied, then there exists N^*, $0 < N^* < Q - K/2$ such that we have the multiple equilibria if $N < N^*$ and the unique equilibrium if $N > N^*$. Both statements can be generalised saying that there exists N^*, $0 \leq N^* < Q - K/2$ such that we have the multiple equilibria if $N < N^*$ and the unique equilibrium if $N > N^*$.

Part (ii): Suppose second that $N > Q - K/2$ and therefore no firm is pivotal. Suppose again that other bids b_1 with probability q and b_2 with probability $(1-q)$. The expected profits from bidding b_1 are

$$q\frac{Q-N}{2}(b_1-c)$$

whereas those from playing b_2 are

$$q(Q-N)(b_2-c)+(1-q)\frac{Q-N}{2}(b_2-c).$$

Playing b_1 would be better if

$$\frac{q}{1+q} > \frac{b_2-c}{b_1-c},$$

but this condition is never satisfied, as the left hand side is lower than $1/2$ and the right hand side is between $1/2$ and 1, strictly. Playing b_2 is a dominant strategy.

Proof of Corollary 3

Notice that q^* can be rewritten as

$$q^* \equiv 1-\frac{(K-Q+N)(b_1-c)}{(Q-N)(b_1-b_2)}.$$

Clearly this function is decreasing as the derivative with respect to N is $-K(b_1-c)/\left(Q-N\right)^2(b_1-b_2)$. Clearly q^* is lower as K is larger.

Proof of Proposition 4

Part (i): Suppose first that $\overline{W} < (Q-K)/2$. In this case $w < 2\overline{W} < Q - K/2$ and the firms are always pivotal. Take one of the firms. Suppose that the other firm bids b_1 with probability q and b_2 with probability $(1-q)$. The expected profits from bidding b_1 are

$$\int_0^{2\overline{W}} \frac{1}{2\overline{W}} \left[r\frac{Q-w}{2}(b_1-c) + (1-r)\left(Q-w-K/2\right)(b_1-c) \right] dw$$

whereas those from playing b_2 are

$$\int_0^{2\overline{W}} \frac{1}{2\overline{W}} \left[rk(b_1-c) + (1-r)\frac{Q-w}{2}(b_2-c) \right] dw.$$

Given that $\int_0^s (Q-w)\,dw = s(Q-s/2)$, playing b_1 is better as long as

$$r(Q-\overline{W})(b_1-b_2) \leq 2(Q-\overline{W}-K/2)(b_1-c) - (Q-\overline{W})(b_2-c), \tag{13}$$

as in the previous proposition.

On the one hand, if the other plays b_2 for sure ($r=0$), then it is better to play b_1 as long as

$$\frac{2(Q-\overline{W}-K/2)}{Q-\overline{W}} \geq \frac{b_2-c}{b_1-c} \tag{14}$$

Again, the left hand side of condition (14) is decreasing in $\overline{W}$ (see proof of Proposition 2) Evavaluated at $\overline{W}=0$, it reduces to

$$\frac{2(Q-K/2)}{Q} \geq \frac{b_2-c}{b_1-c} \tag{15}$$

which can or cannot be satisfied, as the left-hand side ranges from 0 to 1 the right-hand side from 0 until 1/2. Evaluated at $\overline{W} = (Q-K/2)/2$, we have that the condition is equal to

$$\frac{2(Q-K/2)}{Q+K/2} \geq \frac{b_2-c}{b_1-c} \tag{16}$$

which can or cannot be satisfied. The left-hand side has positive derivative and it can range from 0 (if $Q = K/2$) to 2/3(if $Q = K$) whereas the right-hand side can range from 1/2 to 1. Notice that if (16) is satisfied then Condition (15) is also satisfied, as the left-hand side in the former is smaller than in the latter.

On the other hand, if the other firm plays b_1 for sure ($r=1$), then it is better to play b_2 given that the condition (13) reduces to $0 \leq Q-\overline{W}-K$, which is never satisfied.

As a result, we have that if condition (14) is satisfied, we have multiplicity of equilibrium: (b_2, b_1) and (b_1, b_2) are pure-strategy Nash equilibria. Additionally, there is a mixed strategy equilibrium where each player plays b_1 with probability r^*, where.

$$r^* \equiv \frac{\left(Q-\overline{W}\right)(b_1-b_2)-(K-Q+\overline{W})(b_1-c)}{\left(Q-\overline{W}\right)(b_1-b_2)}$$

If condition (14) is not satisfied then (b_2, b_2) is the unique equilibrium.

Each case arise under the following situations. If Condition (15) is not satisfied then condition (14) is never satisfied and we always have the unique equilibrium. At the other end, if Condition (16) is satisfied then condition (14) is always satisfied and we always have the multiple equilibria. Finally, if condition (15) is satisfied but (16) is not then there exists $\overline{W}^*$, $0 < \overline{W}^* < (Q-K/2)/2$ such that we have the multiple equilibria if $\overline{W} < \overline{W}^*$ and the unique equilibrium if $\overline{W} > \overline{W}^*$. Both statements can be generalised by saying that there exists $\overline{W}^*$, $0 \leq \overline{W}^* \leq (Q-K/2)/2$ such that we have the multiple equilibria if $\overline{W} < \overline{W}^*$ and the unique equilibrium if $\overline{W} > \overline{W}^*$.

Part (ii): Suppose now that $(Q-K/2)/2 < \overline{W} < Q/2$. If the other firm bids b_1 with probability r and b_2 with probability $(1-r)$. The profits from bidding b_1 are

$$\begin{aligned}&\textstyle\int_0^{Q-K/2} \frac{1}{2\overline{W}}\left[r\frac{Q-w}{2}(b_1-c)+(1-r)\left(Q-w-K/2\right)(b_1-c)\right]dw+\\&+\textstyle\int_{Q-K/2}^{2\overline{W}} \frac{1}{2\overline{W}}r\frac{Q-w}{2}(b_1-c)dw\end{aligned}$$

whereas those from playing b_2 are

$$\begin{aligned}&\textstyle\int_0^{Q-K/2} \frac{1}{2\overline{W}}\left[rk(b_1-c)+(1-r)\frac{Q-w}{2}(b_2-c)\right]dw+\\&+\textstyle\int_{Q-K/2}^{2\overline{W}} \frac{1}{2\overline{W}}\left[r\left(Q-w\right)(b_2-c)+(1-r)\frac{Q-w}{2}(b_2-c)\right]dw.\end{aligned}$$

In each of the two equations, the first integral computes the expected payoff in the cases in which both firms are pivotal, i.e. $0 < w < Q-K/2$, whereas the second term computes the expected payoff for the case in which none of them is, i.e. $Q-K/2 < w < 2\overline{W}$.

Given that $\int_0^s (Q-w)\,dw = s(Q-s/2)$ and $\int_s^{2\overline{W}} (Q-w)\,dw = 2\overline{W}\left(Q-\overline{W}\right) - s\left(Q-s/2\right)$, playing b_1 is better as long as

$$r(b_1-b_2)\left[(Q-K/2)(Q+K/2)-2\overline{W}(Q-\overline{W})\right] \leq (b_1-c)(Q-K/2)^2-(b_2-c)2\overline{W}(Q-\overline{W})$$

If the other plays b_2 for sure ($r=0$), then it is better to play b_1 if

$$\frac{(Q-K/2)^2}{2\overline{W}(Q-\overline{W})} \geq \frac{b_2-c}{b_1-c} \tag{17}$$

Notice that all terms on the left-hand side are positive as $(Q > Q/2 > \overline{W}$ and $Q > K/2)$. The derivative with respect to $\overline{W}$ is negative as the denominator is increasing in $\overline{W}$ (derivative $2(Q-2\overline{W}) > 0$).

Evaluated at $\overline{W} = (Q - K/2)/2$ we have that the condition is satisfied as long as

$$\frac{2\,(Q - K/2)}{Q + K/2} \geq \frac{b_2 - c}{b_1 - c} \tag{18}$$

which, is exactly the same condition as (16). Evaluated at $\overline{W} = Q - K/2$, the condition becomes

$$\frac{(Q - K/2)}{K} \geq \frac{b_2 - c}{b_1 - c} \tag{19}$$

which is never satisfied, as the left-hand side can range from 0 (if $Q = K/2$) until $1/2$ (if $Q = K$).

If the other plays b_1 ($r = 1$), then it is better to play b_1 if

$$(b_1 - c)\left[2\overline{W}(Q - \overline{W}) - (Q - K/2)\,K\right] > (b_2 - c)\left[4\overline{W}(Q - \overline{W}) - (Q - K/2)\,(Q + K/2)\right]$$

Notice that the right hand side is positive if $(Q - K/2)/2 < \overline{W}$ (indeed, it is increasing in $\overline{W}$ and equal to 0 at $Q - K/2$). Therefore we can write it as

$$\frac{2\overline{W}(Q - \overline{W}) - (Q - K/2)\,K}{4\overline{W}(Q - \overline{W}) - (Q - K/2)\,(Q + K/2)} > \frac{b_2 - c}{b_1 - c}.$$

Given that the derivative is positive and, in the limit (for $\overline{W}$ arbitrarily large) the left-hand side would at most be equal to $1/2$, we have that this condition is never satisfied.

As a result, we have that if condition (17) is satisfied, we have multiplicity of equilibrium: (b_2, b_1) and (b_1, b_2) are pure-strategy Nash equilibria. Additionally, there is a mixed strategy equilibrium where each player plays b_1 with probability r^{**}, where

$$r^{**} = \frac{(b_1 - c)\,(Q - K/2)^2 - (b_2 - c)2\overline{W}(Q - \overline{W})}{(b_1 - c)\big[(Q - K/2)\,(Q + K/2) - 2\overline{W}(Q - \overline{W})\big] - (b_2 - c)\big[(Q - K/2)\,(Q + K/2) - 2\overline{W}(Q - \overline{W}}$$

If condition (17) is not satisfied then (b_2, b_2) is the unique equilibrium.

Each case arise under the following situations. If Condition (18) is not satisfied then condition (17) is never satisfied and we always have the unique equilibrium. Instead, if condition (18) is satisfied, then there exists $\overline{W}^*$, $(Q - K/2)/2 < \overline{W}^* < Q - K/2$ such that we have the multiple equilibria if $\overline{W} < \overline{W}^*$ and the unique equilibrium if $\overline{W} > \overline{W}^*$. Both statements can be generalised by saying that there exists $\overline{W}^*$, $(Q - K/2)/2 \leq \overline{W}^* < Q - K/2$ such that we have the multiple equilibria if $\overline{W} < \overline{W}^*$ and the unique equilibrium if $\overline{W} > \overline{W}^*$.

Proof of Corollary 5

First, notice that r^* can be rewritten as

$$r^* = 1 - \frac{(K - Q + \overline{W})(b_1 - c)}{(Q - \overline{W})(b_1 - b_2)}$$

Clearly this is decrasing in $\overline{W}^*$ as the numerator of the second term is increasing and the denominator decreasing.

Second, notice that r^{**} can be rewritten as

$$r^{**} = \frac{K_1 - (b_2 - c)f(\overline{W})}{K_2 - (b_1 - b_2)f(\overline{W})}$$

where $K_1 = (b_1 - c)\,(Q - K/2)^2$ and $K_2 = (b_1 - b_2)\,(Q - K/2)\,(Q + K/2)$ are independent of $\overline{W}$ and $f(\overline{W}) = 2\overline{W}(Q - \overline{W})$. The derivative with respect to $\overline{W}$ can then be written as

$$\frac{-(b_2 - c)f'(\overline{W})K_2 + (b_1 - b_2)f'(\overline{W})K_1}{\left[K_2 - (b_1 - b_2)f(\overline{W})\right]^2}$$

Given that $f'(\overline{W}) = 2(Q - 2\overline{W}) > 0$, the derivative is positive if and only if $-(b_2 - c)K_2 + (b_1 - b_2)K_1 > 0$ which taking common factor $(b_1 - b_2)\,(Q - K/2)$ is equal to

$$-(b_2 - c)(Q + K/2) + (b_1 - c)\,(Q - K/2)\,.$$

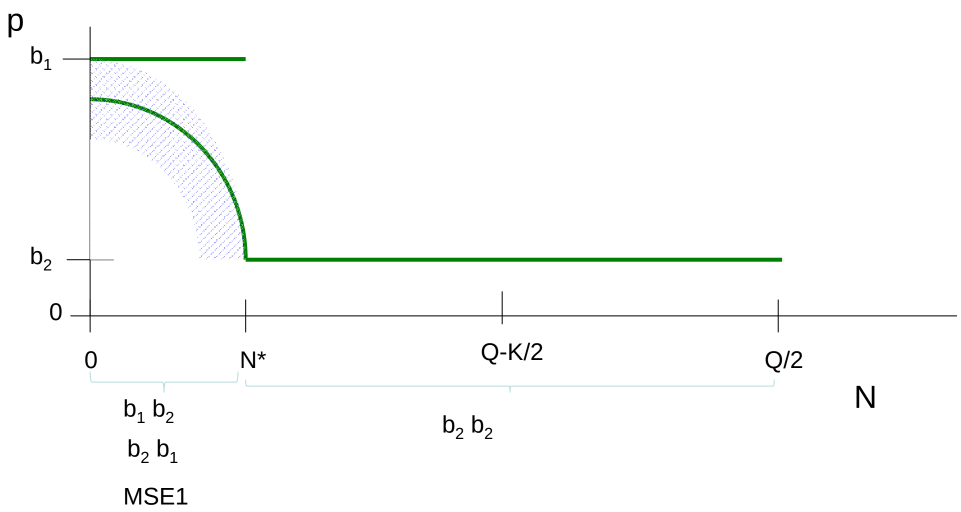
Given that $b_1 - c < 2(b_2 - c)$ then this term is lower than $-(b_2 - c)(Q + K/2) + 2(b_2 - c)\,(Q - K/2) = (b_2 - c)(-K/2 + Q - K) < 0$.

Proof of Corollary 6

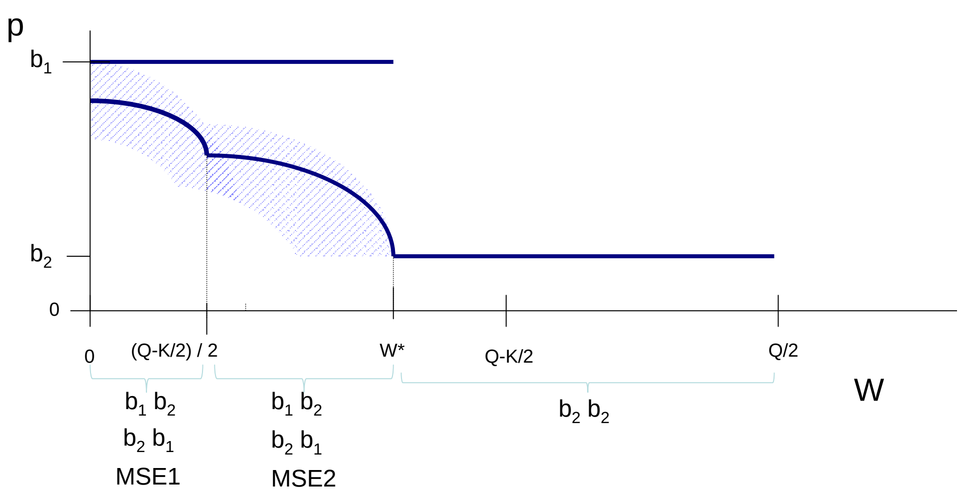
We have that the left hand side in condition (14) is smaller than the left hand side of condition (17) as

$$\frac{2(Q - \overline{W} - K/2)}{Q - \overline{W}} \leq \frac{(Q - K/2)^2}{2\overline{W}(Q - \overline{W})}$$

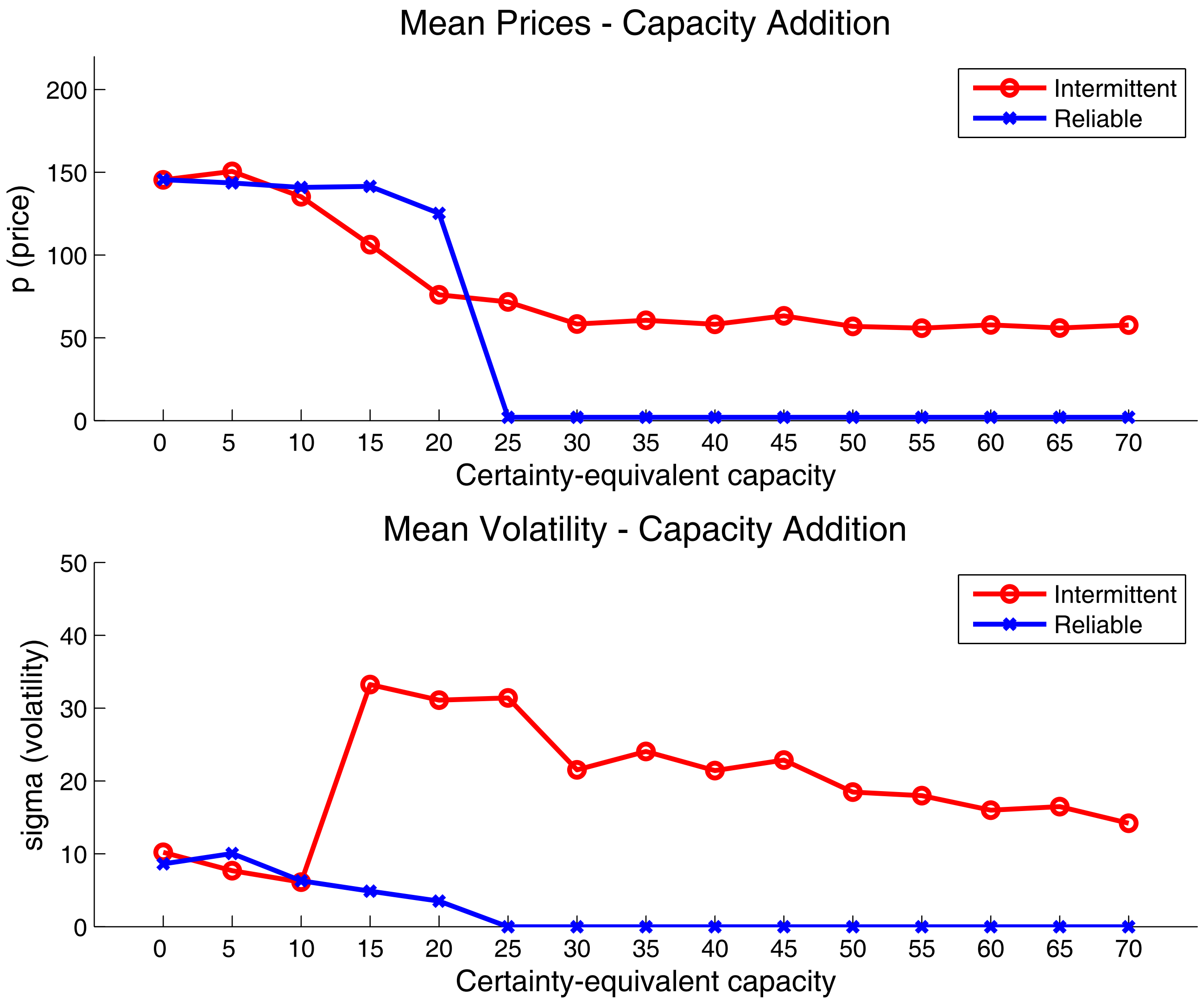
is equivalent to $\left(Q - K/2 - 2\overline{W}\right)^2 \geq 0$.



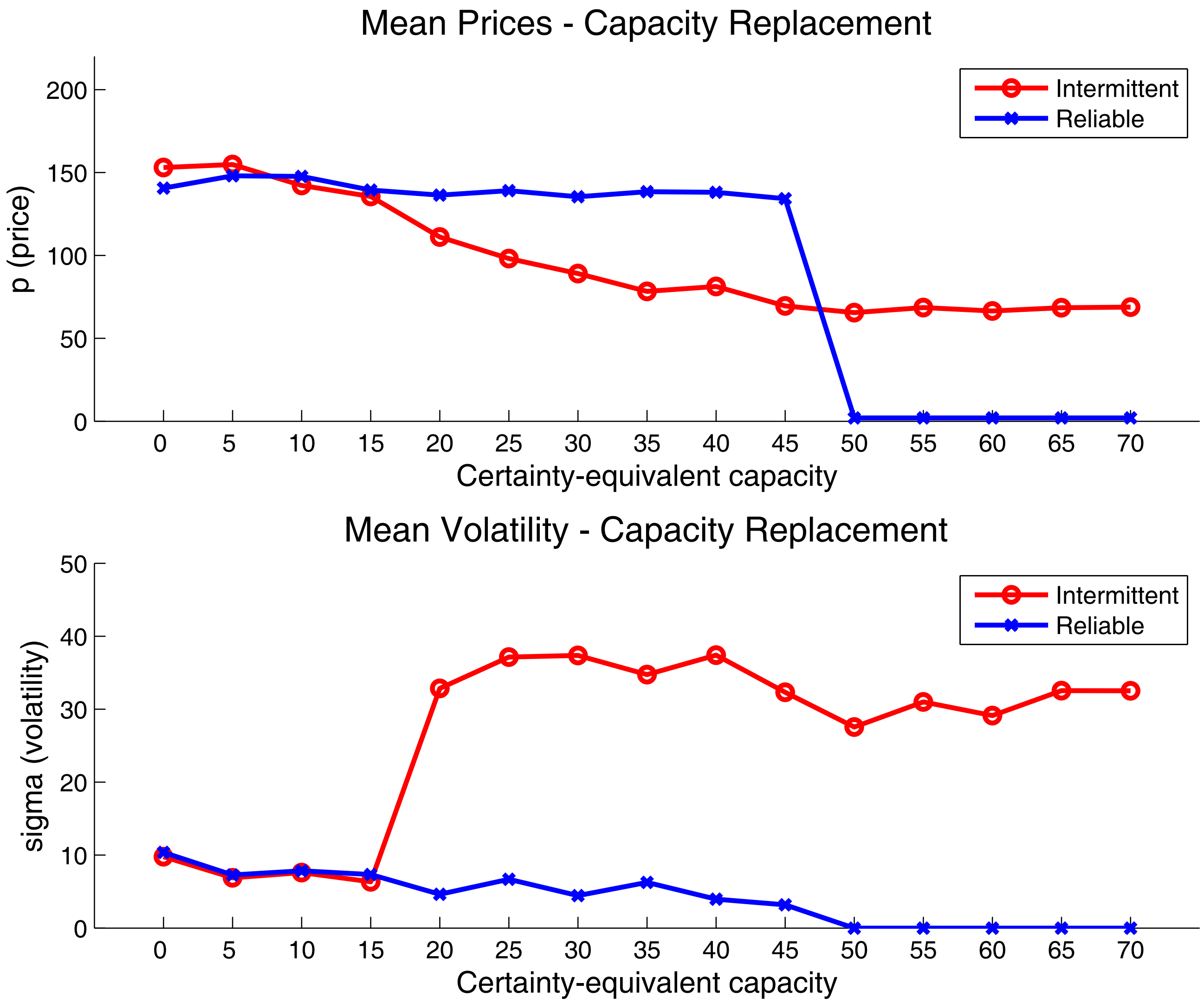
F1: Reliable capacity addition to predicted prices relationship



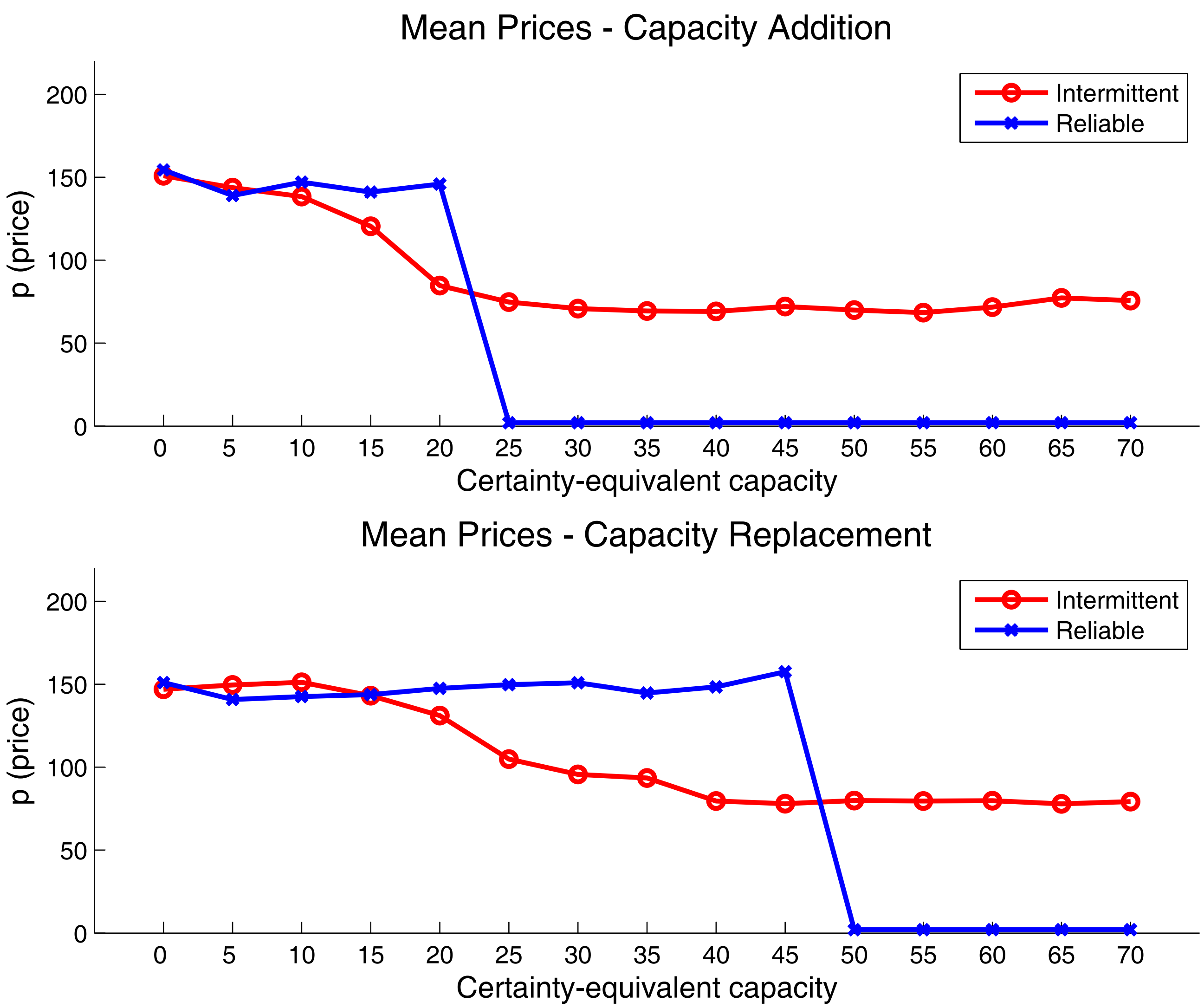
F2: Intermittent capacity addition to predicted prices relationship



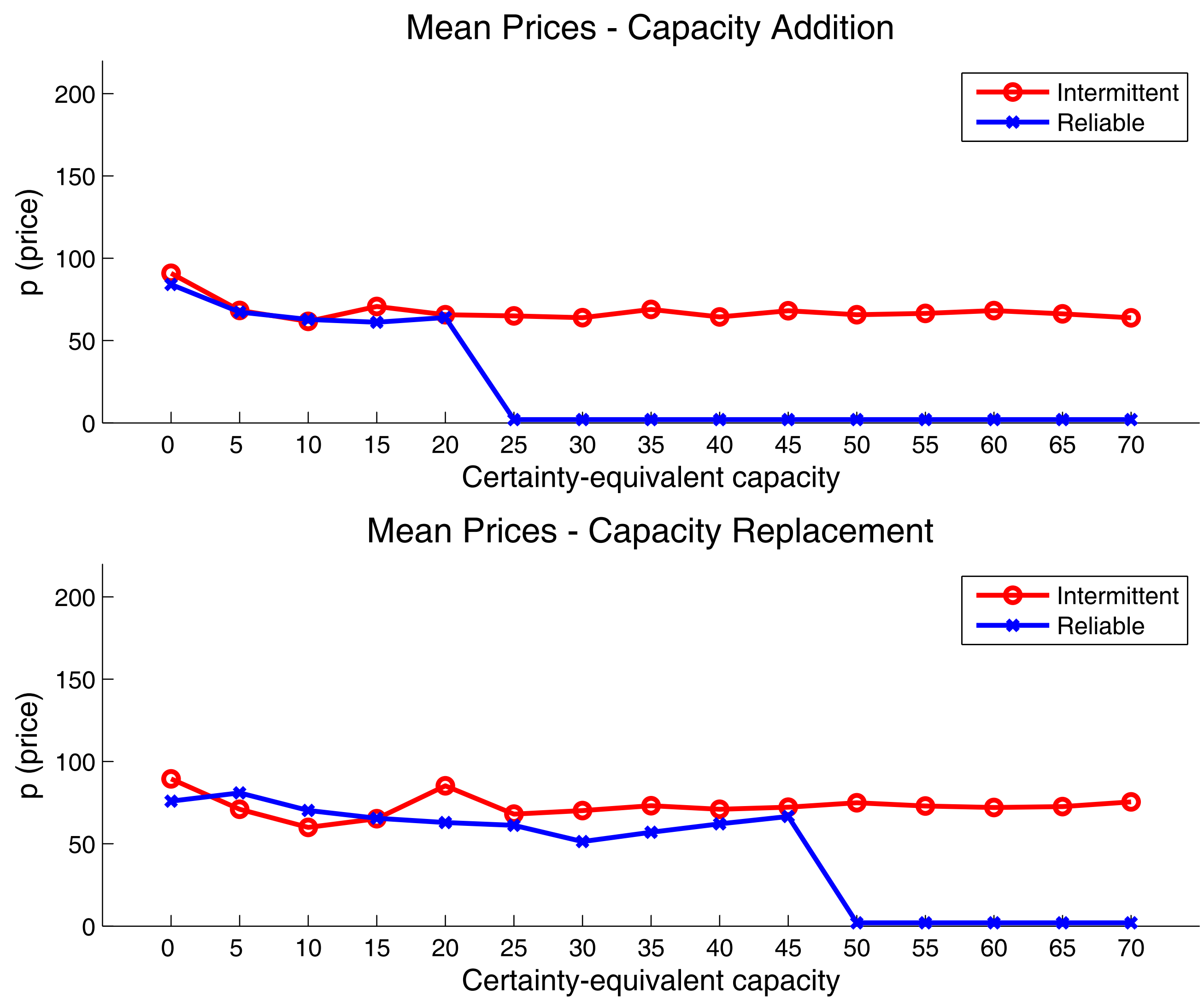
F3: Average price and volatility levels under capacity addition- intermittent and reliable technologies



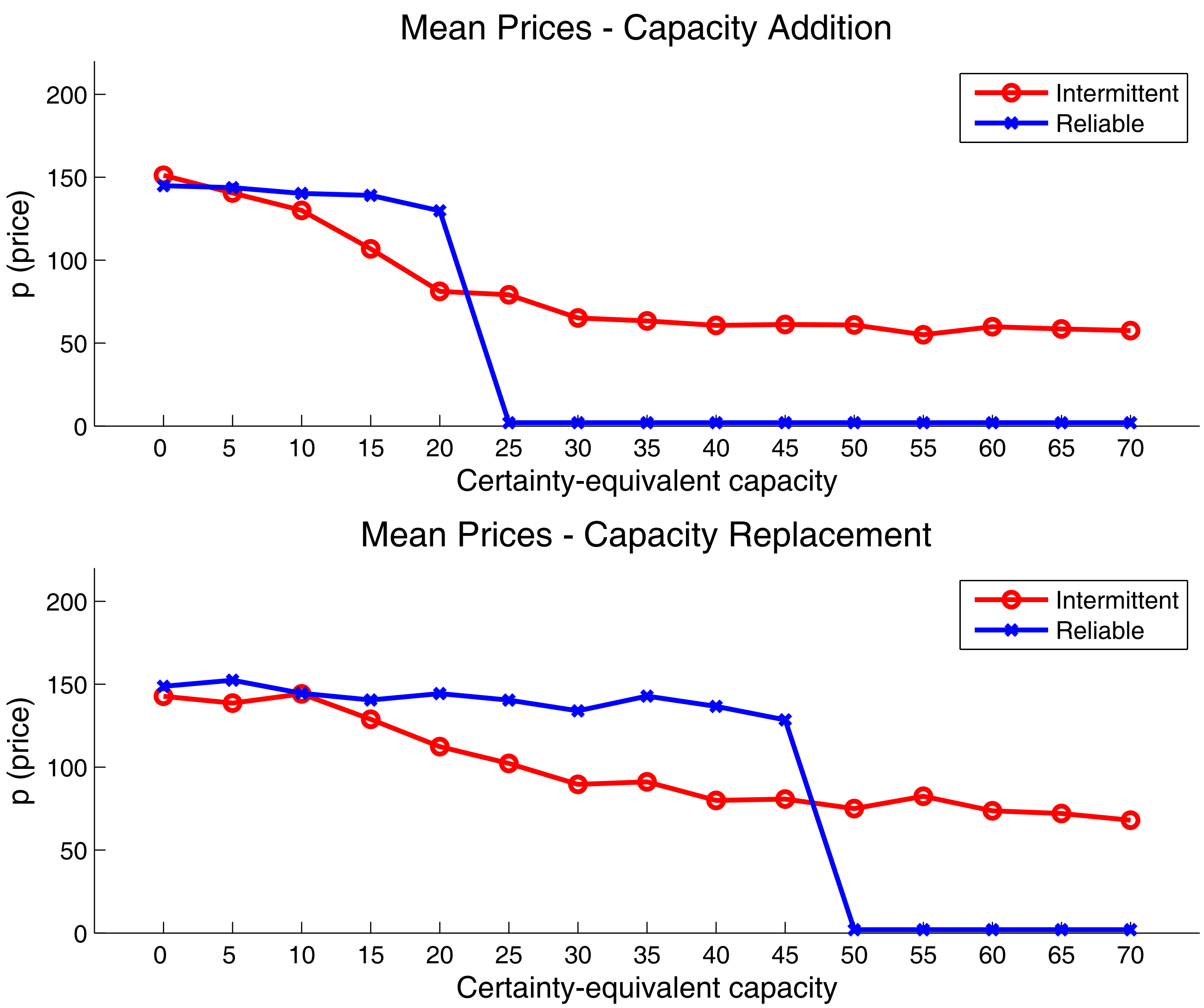
F4: Average price and volatility levels under capacity replacement-intermittent and reliable technologies



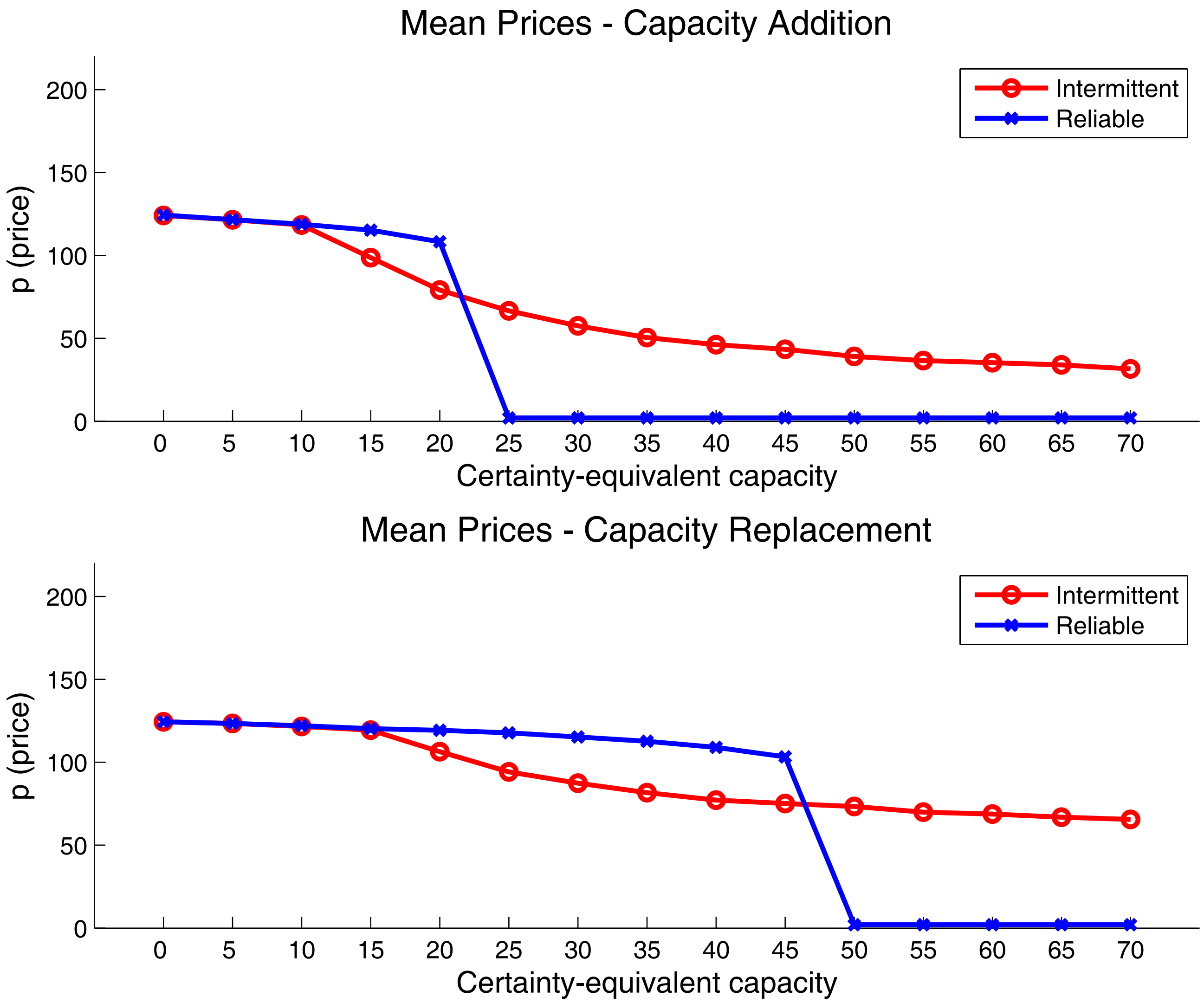
F5: Average price levels under joint low- and high-cost ownership: Capacity addition & replacement - intermittent & reliable technologies



F6: Average price levels under risk aversion:
Capacity addition & replacement - intermittent & reliable technologies



F7: Average price levels with left-skewed intermittent production:
Capacity addition & replacement - intermittent & reliable technologies



F8: Average price levels under best-response dynamics:
Capacity addition & replacement - intermittent & reliable technologies